Ankara University
Department of Geological Engineering

GEO222 STATICS and STRENGTH of MATERIALS

Lecture Notes

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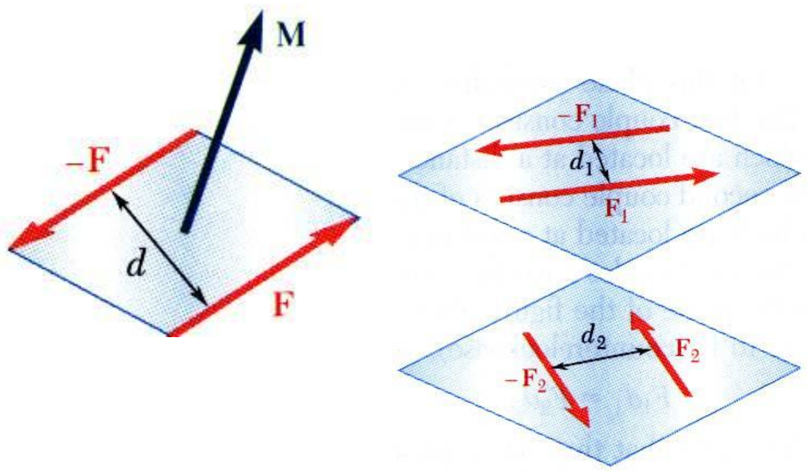
CHAPTER 4. MOMENT OF A COUPLE

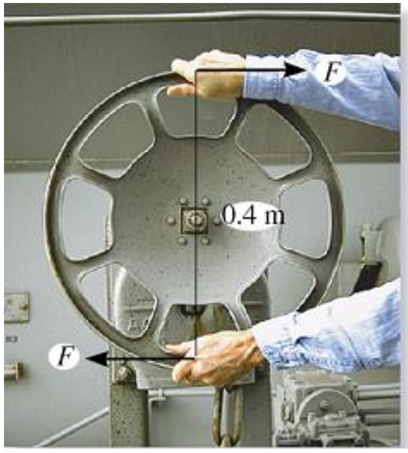
Two couples will have equal moments if

- the two couples lie in parallel planes, and
- the two couples have the same sense or the tendency to cause rotation in the same direction.



The reason we use Force Couples to analyze Moments is that; Location of the axis the Moment is calculated about does not matter. The Moment of a Couple is constant over the entire body it is acting on Equivalent couples; two couples that produce the same magnitude and direction.

- ❑ The point of action of a Couple does not matter
- ❑ The plane that the Couple is acting in does not matter
- ❑ All that matters is the orientation of the plane the Couple is acting in
- ❑ **Therefore, a Force Couple is said to be a Free Vector and can be applied at any point on the body it is acting**



Moment of couples (Hibbeler, 2010)

Example 11. Determine the resultant couple moment of three couples acting on the plate.

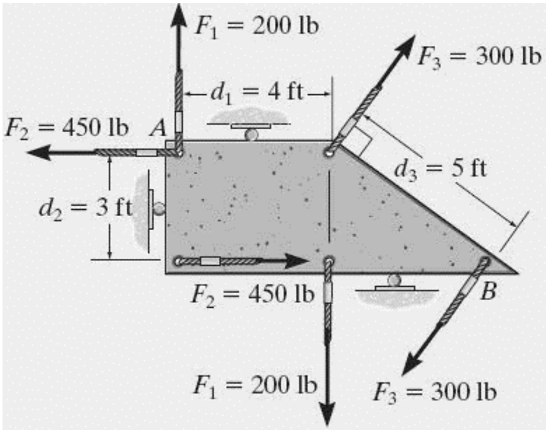


Fig. 4–30

SOLUTION

As shown the perpendicular distances between each pair of couple forces are $d_1 = 4$ ft, $d_2 = 3$ ft, and $d_3 = 5$ ft. Considering counterclockwise couple moments as positive, we have

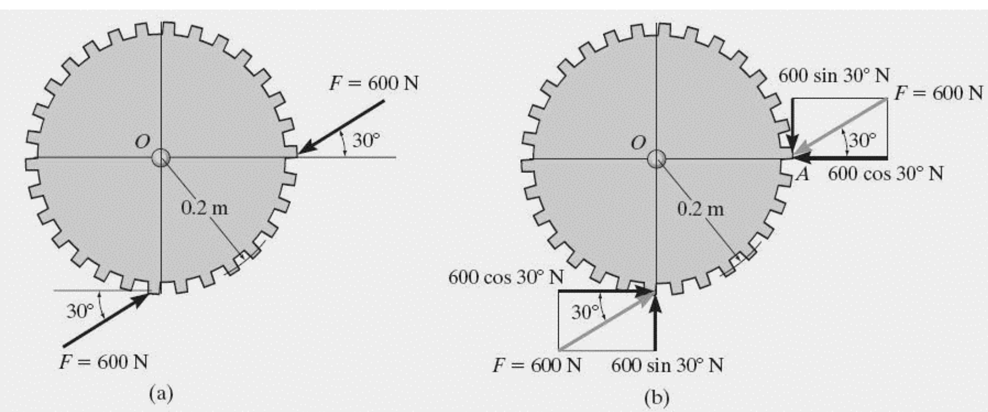
$$\circlearrowleft +M_R = \Sigma M;\; M_R = -F_1 d_1 + F_2 d_2 - F_3 d_3$$

$$= (-200\text{ lb})(4\text{ ft}) + (450\text{ lb})(3\text{ ft}) - (300\text{ lb})(5\text{ ft})$$

$$= -950\text{ lb}\cdot\text{ft} = 950\text{ lb}\cdot\text{ft} \circlearrowright \qquad \textit{Ans.}$$

The negative sign indicates that $\mathbf{M}_R$ has a clockwise rotational sense.

Example 12. Determine the magnitude and direction of the couple moment acting on the gear.



$$\circlearrowleft +M = \Sigma M_O;\; M = (600\cos 30°\text{ N})(0.2\text{ m}) - (600\sin 30°\text{ N})(0.2\text{ m})$$

$$= 43.9\text{ N}\cdot\text{m} \circlearrowleft \qquad \textit{Ans.}$$

or

$$\circlearrowleft +M = \Sigma M_A;\; M = (600\cos 30°\text{ N})(0.2\text{ m}) - (600\sin 30°\text{ N})(0.2\text{ m})$$

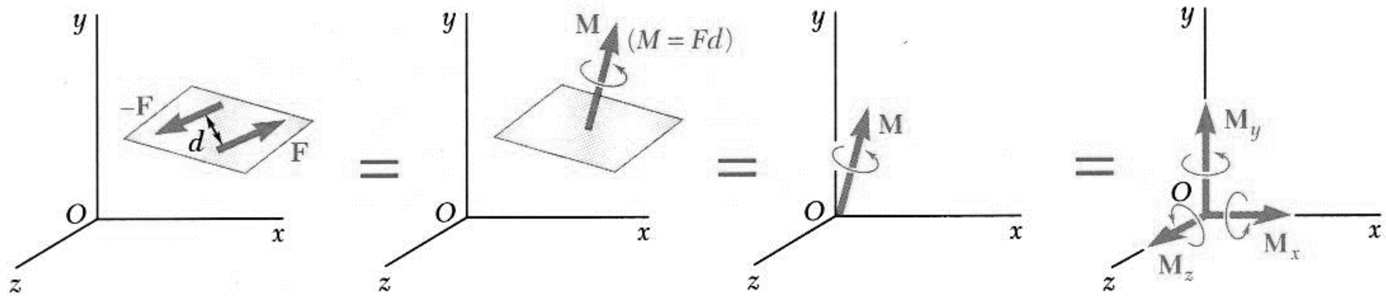
$$= 43.9\text{ N}\cdot\text{m} \circlearrowleft \qquad \textit{Ans.}$$

This positive result indicates that **M** has a counterclockwise rotational sense, so it is directed outward, perpendicular to the page.

(Hibbeler, 2010)

VARIGNON's THEOREM

Algebraic sum of several concurrent forces about any point is equal to the moments of the moments of their resultant about the point. By applying "***Varignon's Theorem***" to the Forces in the Couple, it can be proven that couples can be added and resolved as Vectors.



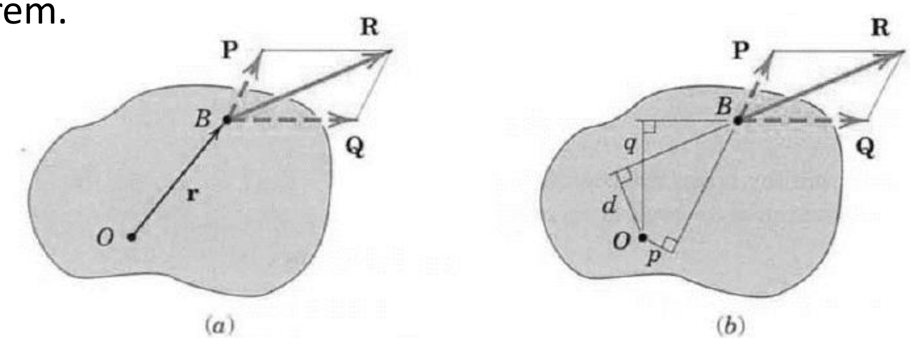
To prove Varignon's Theorem, consider the force R acting in the plane of the body as shown in the above-left side figure (a). The forces 'P' and 'Q' represent any two non-rectangular components of 'R'. The moment of 'R' about point 'O' is

$$M_o = r \times R$$

Because R = P + Q, we can write $r \times R = r \times (P + Q)$

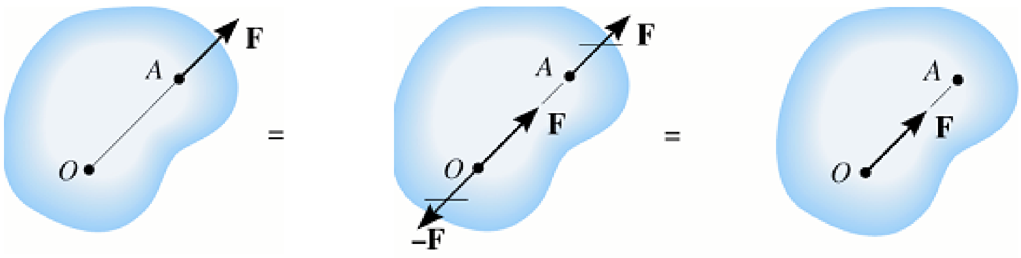
Using the distributive law for cross products, we have $M_o = r \times R = r \times P + r \times Q$

which says that the moment of 'R' about 'O' equals the sum of the moments about 'O' of its components 'P' and 'Q'. This proves the theorem.

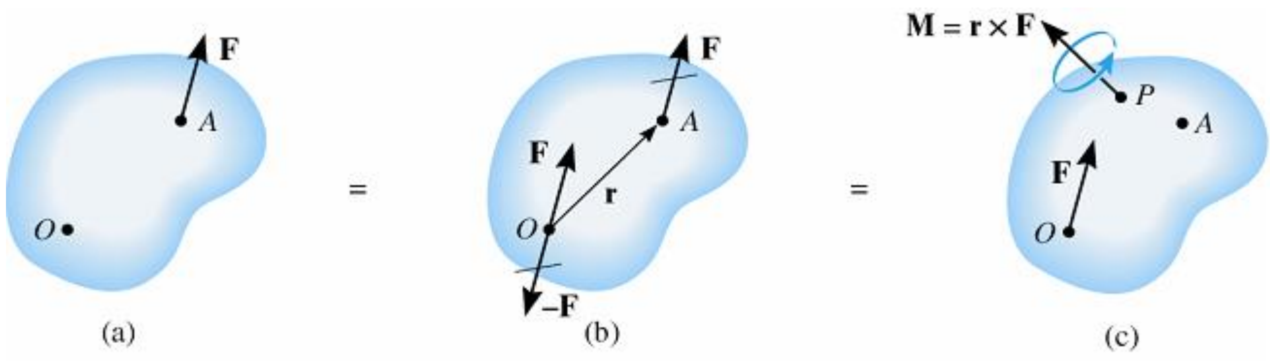


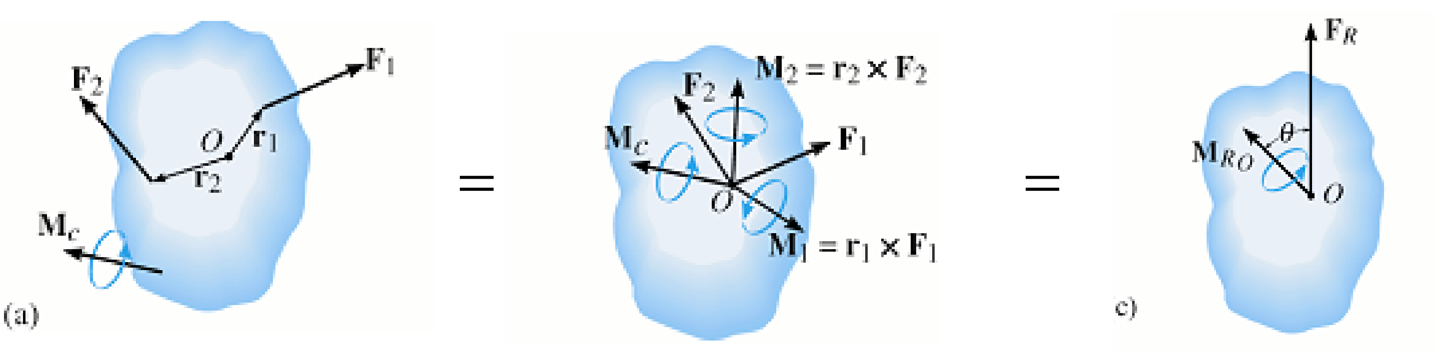
Resultant of a Force and Couple System

Extend this idea to a general 3-D case. Now, the force can be moved



Force now causes the force at any point 0 and then a couple.

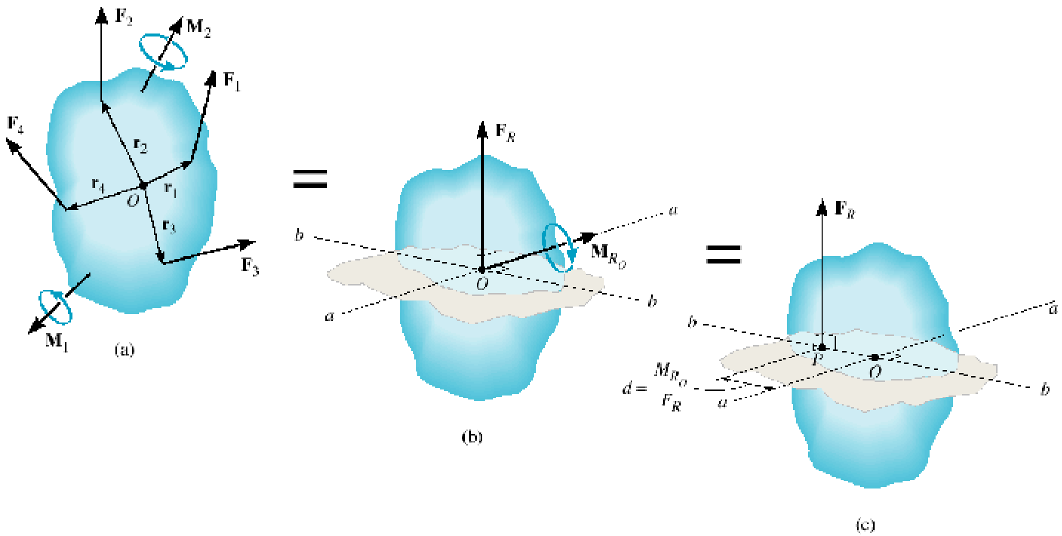




Couples resultant (Hibbeler, 2010)

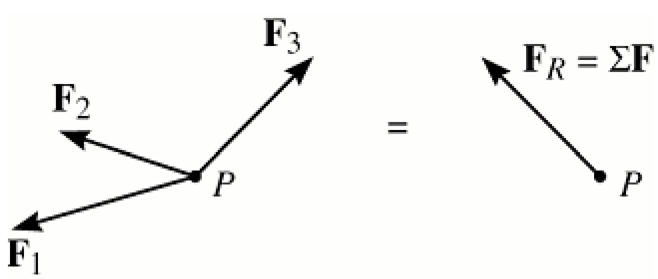
Further Reduction on Force/Couples

If resultant force $\bar{F}_R$ and moment $\bar{M}_{R_o}$ is known then it is possible to reduce them to a single force at P. d= $\dfrac{\left|\bar{M}_{R_o}\right|}{\left|\bar{F}_R\right|}$



Concurrent Force Systems

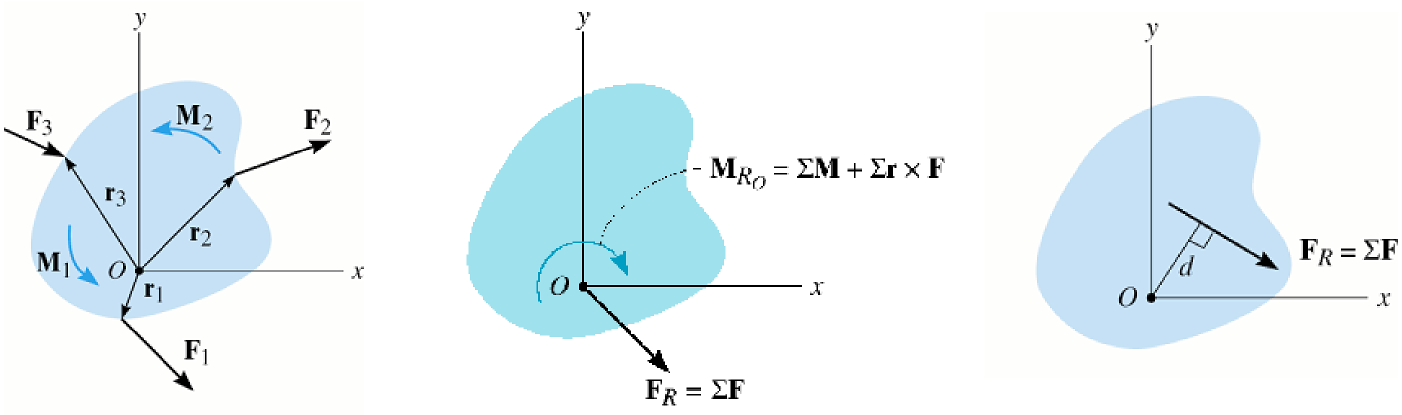
Only equivalent force



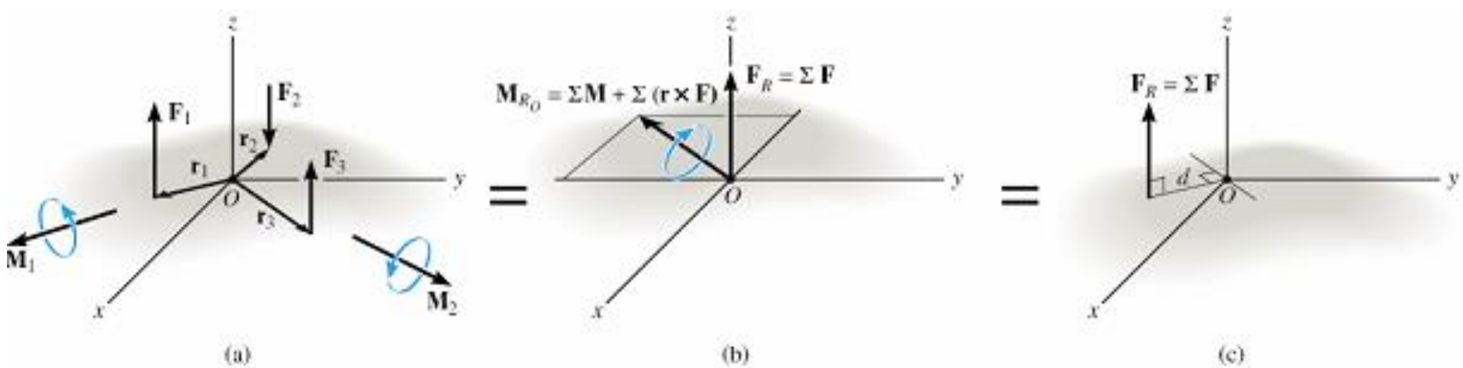
Reduction of forces (Beer, et al. 2011; Hibbeler, 2010)

Coplanar Force Systems

A single force at d from point 0



Parallel Force Systems



Here we have parallel forces and moments that are perpendicular.

Resultant moment (see b): $\sum \bar{M}_{R_o} = \sum \bar{M}_C + \sum\left(\bar{r} \times \bar{F}\right)$

A single force $\bar{F}_R = \sum \bar{F}$

Reduction of forces (Beer, et al. 2011; Hibbeler, 2010)