Ankara University
Department of Geological Engineering

GEO222 STATICS and STRENGTH of MATERIALS

Lecture Notes

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CHAPTER 5. SHEAR FORCE and BENDING MOMENT in BEAMS

When a beam is loaded by forces and couples, internal stresses arise as normal and shear. In order to determine the magnitude of stresses at any section of the beam, it is necessary to know the resultant force and moment acting at that section by applying the equations of static equilibrium.

Resisting Moment

The magnitude of "M" which states that the sum of moments of all forces about an axis and perpendicular to the plane of the page is zero. The resisting moment M is due to stresses that are distributed over the vertical section.

Resisting Shear

The vertical force is called the resisting shear. For equilibrium of forces in the vertical direction, this force is actually the resultant of shearing stresses distributed over the vertical section on a beam.

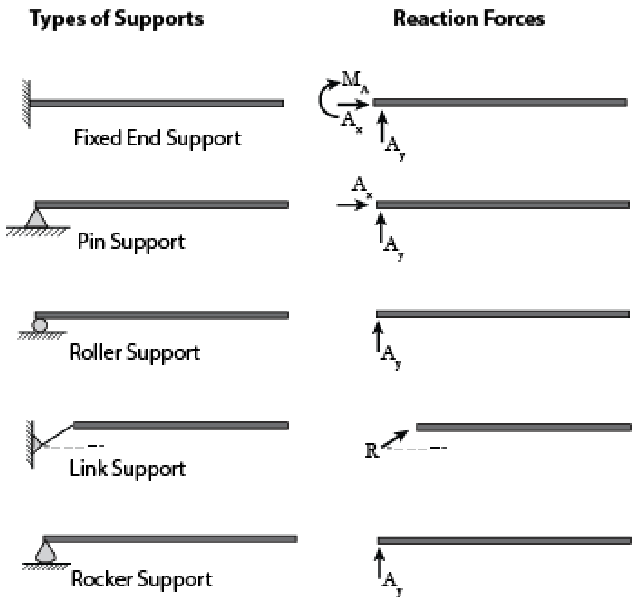
Bending Moment

The algebraic sum of the moments of external forces to one side of the vertical section of a beam about an axis. Bending moment is opposite in direction to the resisting moment with same magnitude. Bending moment rather than the resisting moment is used in calculations because it can be represented directly in terms of the external loads.

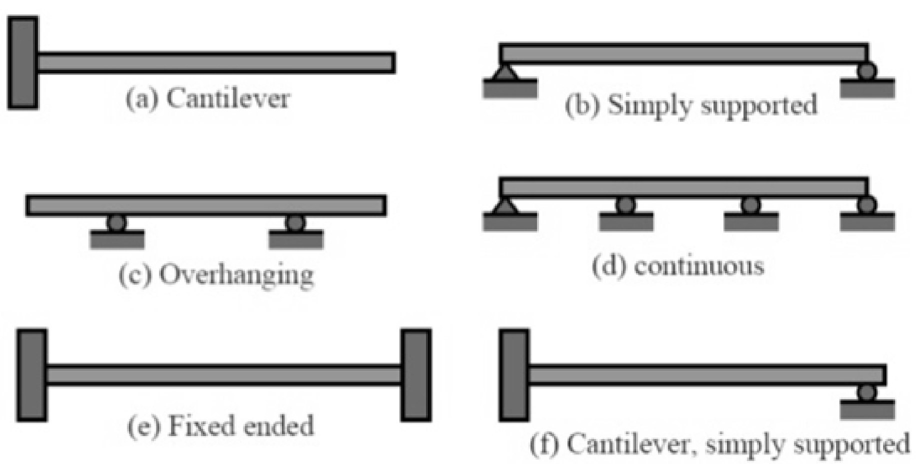
Shear Force

The algebraic sum of all the vertical forces to one side is called the shearing force at that section. Shear force is opposite in direction to the resisting shear but of the same magnitude. It is ordinarily used in calculations, rather than the resisting shear.

Support: 3D Structural members are generally justified to 2D planes to simplify the analysis. Some common support types are used for their separate resistance to forces. Types of supports are considered for use of their resisting force in any directions.



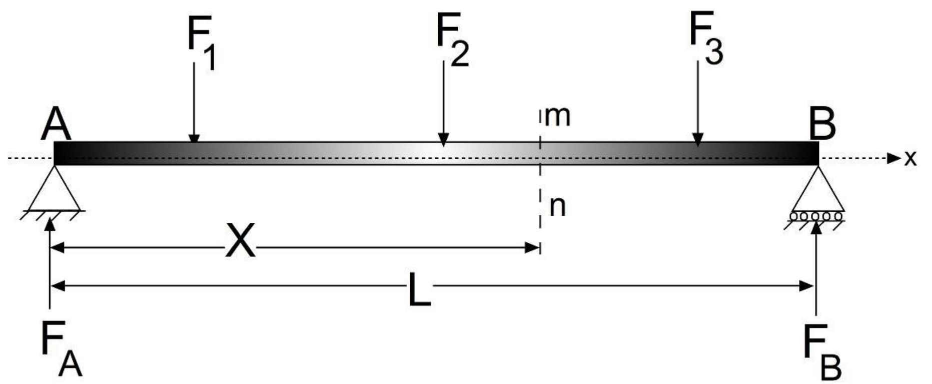
Beam: Horizontal bar which undergoes lateral load or couple which tends to bend the bar or a horizontal bar undergoes bending stress known as beam. Beams might be classified based on their use of support, cross section shape or statically determinate/indeterminate conditions. Beams are also cited as "*Slender Member*" which corresponds to structural elements whose cross sectional area is much smaller compared to their length.



Concentrated (Single, Point) Loads on Beams

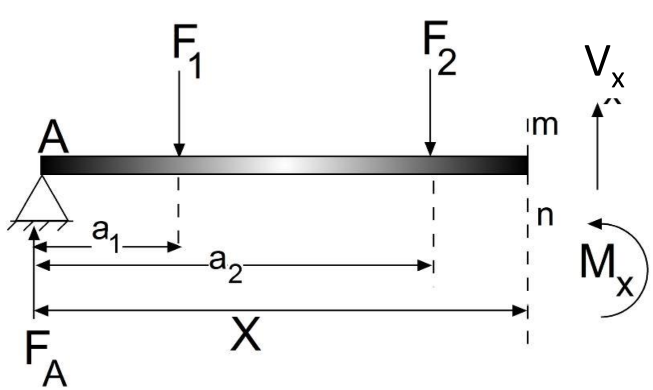
In case a single beam is loaded, "***Shear Forces***" will act on supports in opposite direction. Any shear force from "A" point in "X" distance turns out to be;

$$V_x = F_A - F_1 - F_2$$

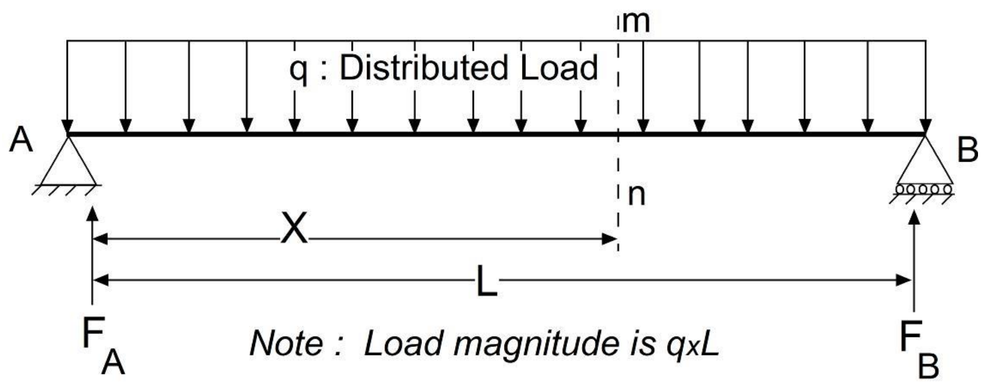


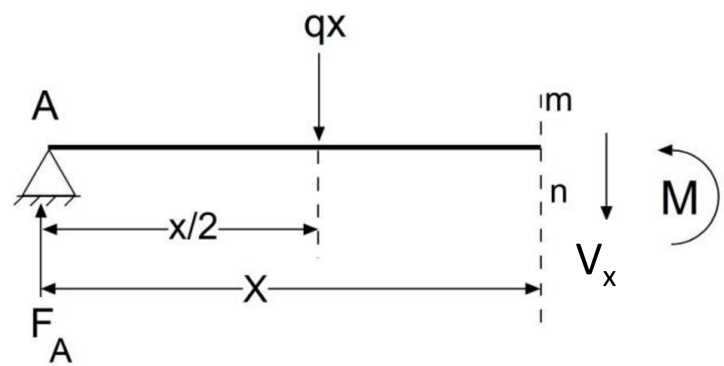
Moreover, due to the magnitude(s) of load(s), the beam might be subjected to "***Bending Moment***" on "m-n" cross section that is;

$$M_x = F_A X - F_1(X - a_1) - F_2(X - a_2)$$



DISTRIBUTED LOADS ON BEAMS





Support Reactions at “A” and “B” are equal;

$$F_B=(qL/2) = F_A$$

Shear Force on “mn” cross section is;

$$V_x=F_A-qX=(qL/2)-qX = q(1/2-X)$$

Bending Moment is;

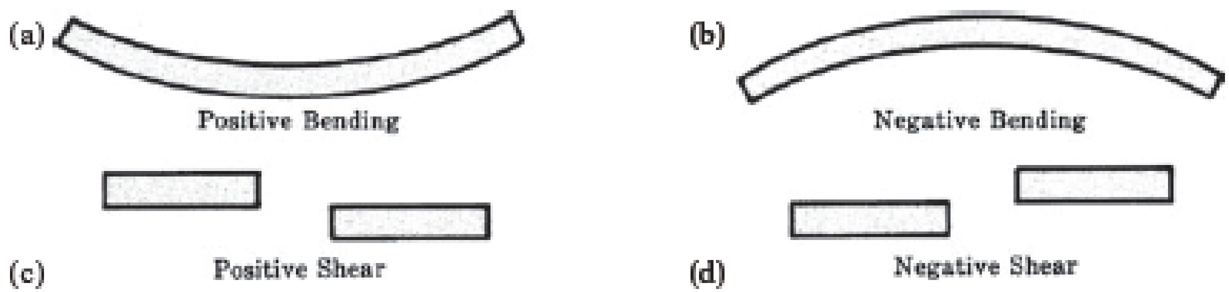
$$M_x=F_AX-(qX^2/2) = (qL/2)X-(qX^2/2) = (qX/2)(1-X)$$

SIGN CONVENTIONS

Due to the magnitude, type and location of loading, beams tend to bend in upward or downward direction.

Figure (a): Concave bending (say in direction of gravity) of the beam . Positive bending leads to produce Positive Bending.

Figure (c): Left portion of the beam is sheared upwards with respect to right portion is “Positive Shear”



BEAMS IN BENDING

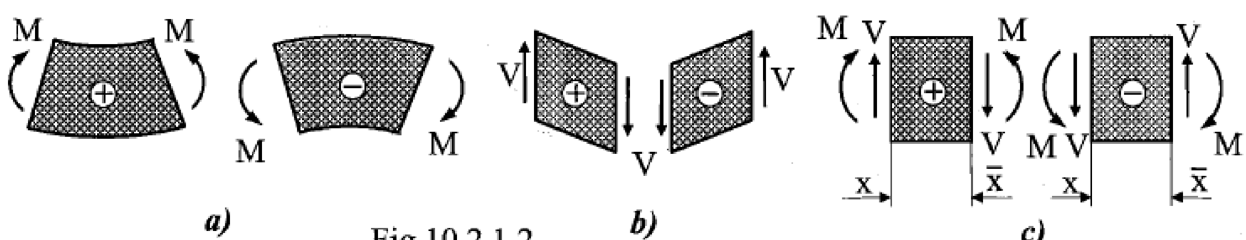
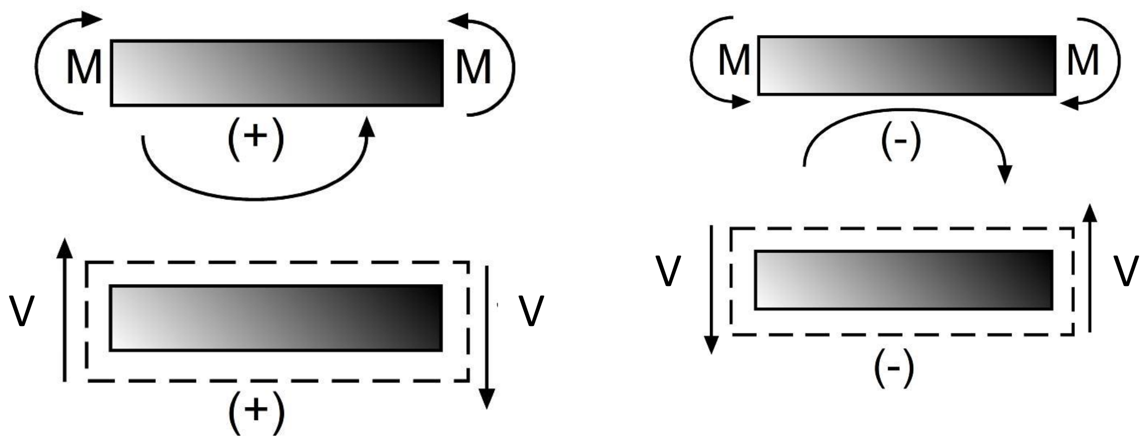


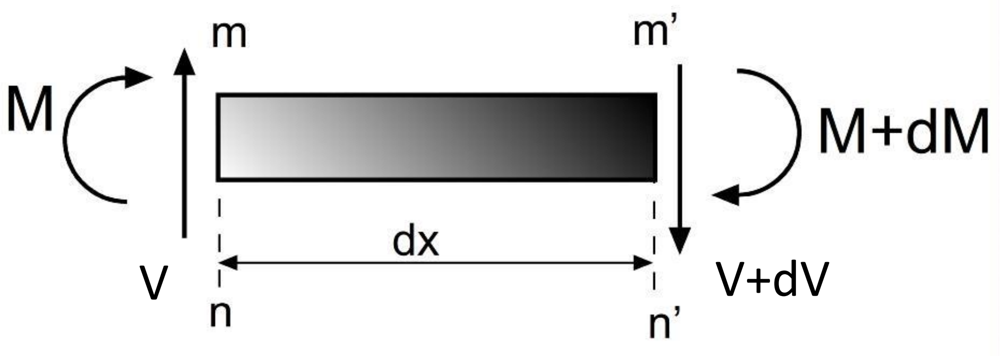
Fig.10.2.1.2



If the bending moment are positive, forces at the left of mn cross-section have CW and forces on the right of mn have CCW moment directions.

RELATION BETWEEN SHEAR FORCE and BENDING MOMENTS

SINGLE LOAD : Shear forces are equal, however moments are different **if there exist no additional forces** between mn and m’n’ sections.

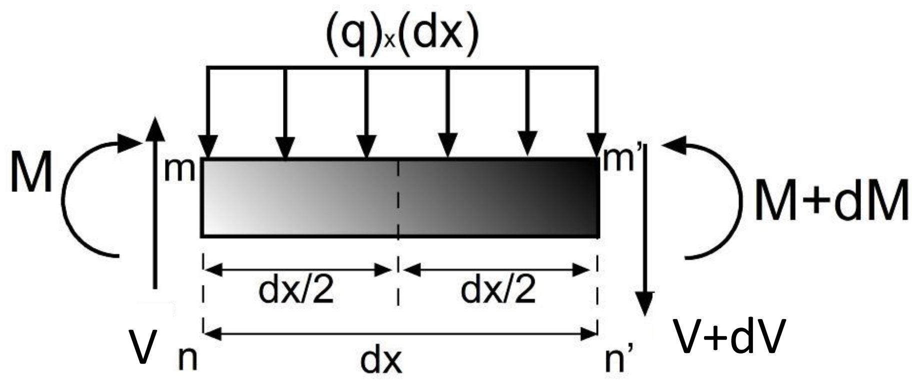


$$\Sigma M_{m'n'} = 0 \text{ ; } M + V\,dx - (M + dM) = 0 \text{ ;}$$

$$\mathbf{V = dM/dx}$$

Shear force for every single parts of beams between forces are the derivative of bending moment

DISTRIBUTED LOAD : Derivative of shear force equals to the negative magnitude of distributed load without **any additional loading** between mn and m’n’ cross sections.



Resultant of vertical loads must be zero; $\Sigma x = 0$; $T - q\,dx - (T+dT) = 0$;

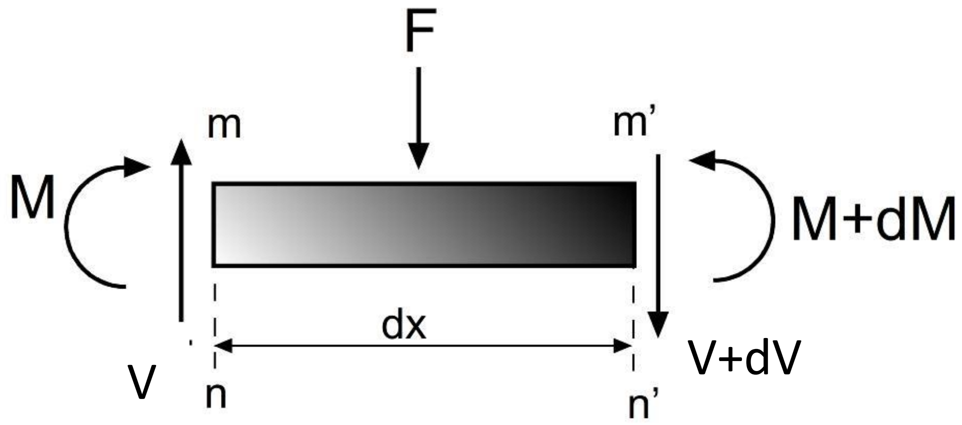
$$\mathbf{q=(-)dV/dx}$$

Moment on m’n’ section must be zero; $M-(M+dM)+V\,dx - (q\,dx^2/2) = 0$;

$$\mathbf{V=dM/dx}$$

Note: dx is negligible, $dx^2/2$ is even smaller to be accepted as zero

<u>SINGLE LOAD:</u> In case there is another "F" load between mn and m'n', the shear force will be different by the magnitude of "F" just nearby the application point of the "F" force. Due, the derivative of (*dM/dx)* will also change.



$$-q = dV/dx = d^2M/dx^2$$

$$-\int q\, dx = (dM/dx) = V$$

$$-\int\int q\, dx\, dx = \int V\, dx = M$$

ESSENTIALS of SHEAR FORCE and MOMENT DIAGRAMS

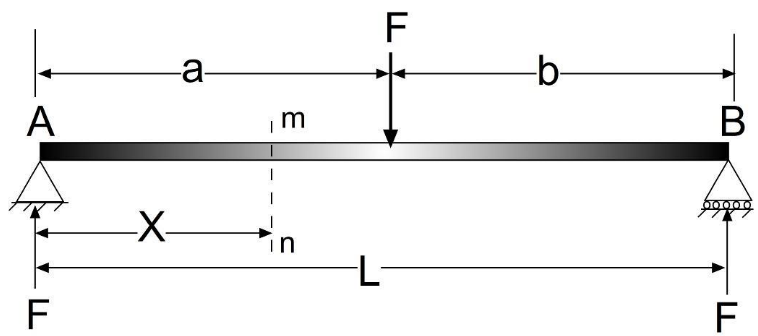
1. Bending moments are minimum or maximum where the slope of bending moments are zero. Minimum and maximum moment values indicate the zero shear forces

2. Variation of shear forces are fixed (constant in value) and variation of moments are linear where the magnitude of distributed loads are zero

3. The variation of shear forces are linear and variation of moments are parabolic for zero slope distributed loading (no angle)

4. In case of triangular loading; shear forces are second and moments are third degree parabolic in shape

ESSENTIALS of SHEAR FORCE and MOMENT DIAGRAMS



Total moment at "B" : $(F_A \times L)-(F \times b)=0$; $F_A=(F \times b/L)$

Total moment at "A" : $F_B \times L-F \times a=0$; $F_B=(F \times a/L)$

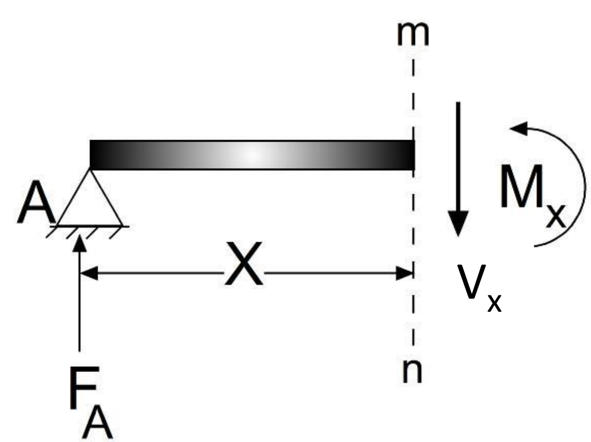
<u>Section I :</u>

Moment about mn section
$-F_A \times X = M_x$; $M_x = [(F \times b)/L]x$

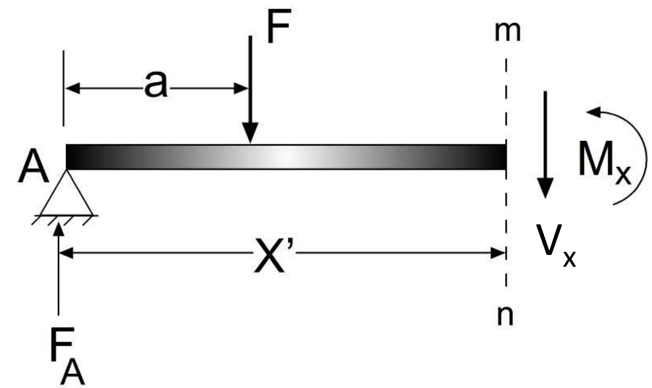
Shear Force;
$V_x = (dM_x/dx) = F_A = (F \times b)/L$

Boundary Conditions
$X = 0$; $M_x= [(F \times b)/L]x =0$ and $V_x=(F \times b)/L$

$X=a$; $M_x = [(F \times b)/L]a$ and $V_x = (F \times b)/L$



Section II :



Moment about mn section

$M'x = F_A \times X' - F\,(X'-a) = [(F \times b)/L]x' - F(X'-a)$

Boundary Conditions

$X' = a;\ M = [(F \times b)/L]a - F(a-a) = (F \times b \times a)/L$

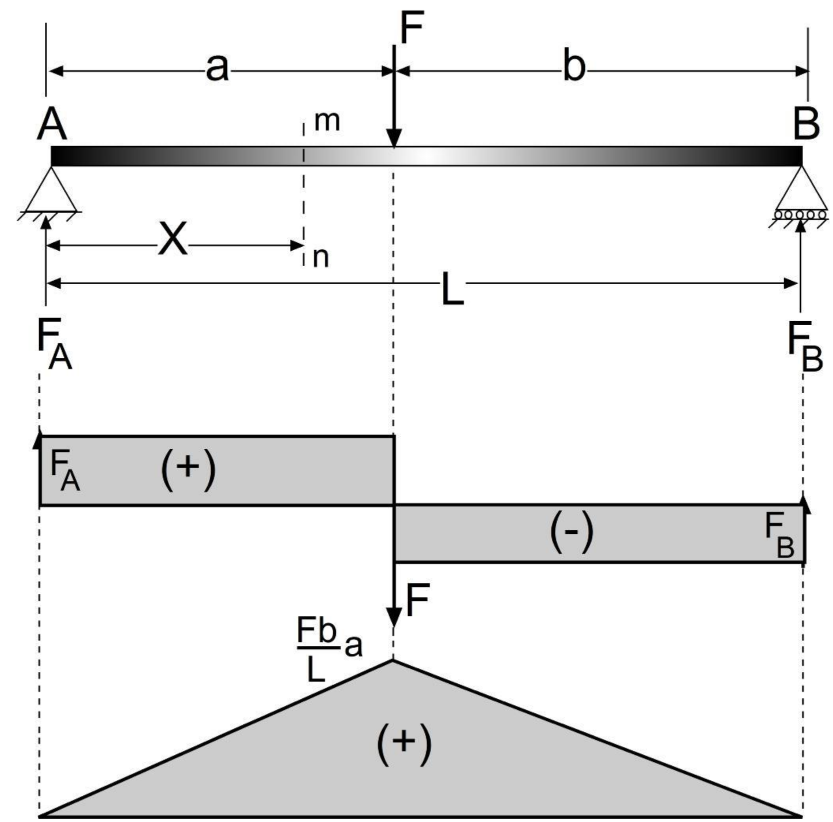
$X' = L;\ M = [(F \times b)/L]L - (F\,(L-a) = F \times b - F \times b = 0$

Shear Force;

$V_{x'} = (M_x'/dx') = [(F \times b)/L] - F = [(F \times b - F \times L)/L] = -F(a/L)$

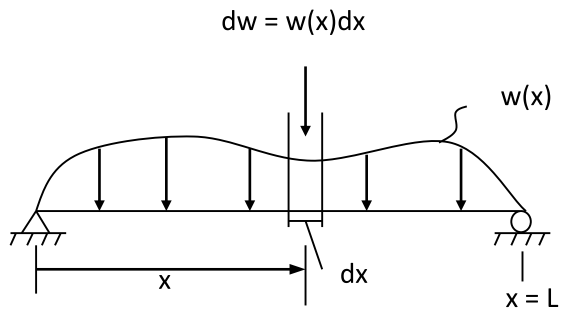
The area of shear force and moment diagram parts are equal, their total should be zero;

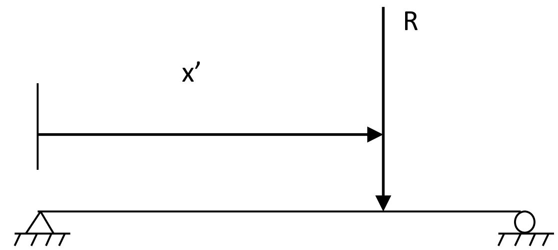
$$\int_A^B dM = \int_A^B V dx \ ;\ \int_A^B dM = M_B - M_A$$



Shear and Moment Diagram

<u>Distributed Load:</u> A simply supported beam has a distributed load applied over its entire length. The distributed load w(x) varies in intensity (height) with position x. The load intensity w(x) has units of force/length (lb/ft or kN/m).





To find R, the original load is broken into strips of width **dx** with a small force **dw = w(x)dx** centered on each strip. Equivalent force R is the sum of all small dw’s. As dx , there are more and more dw’s to add up and the sum becomes an integral. The equation to find R is then;

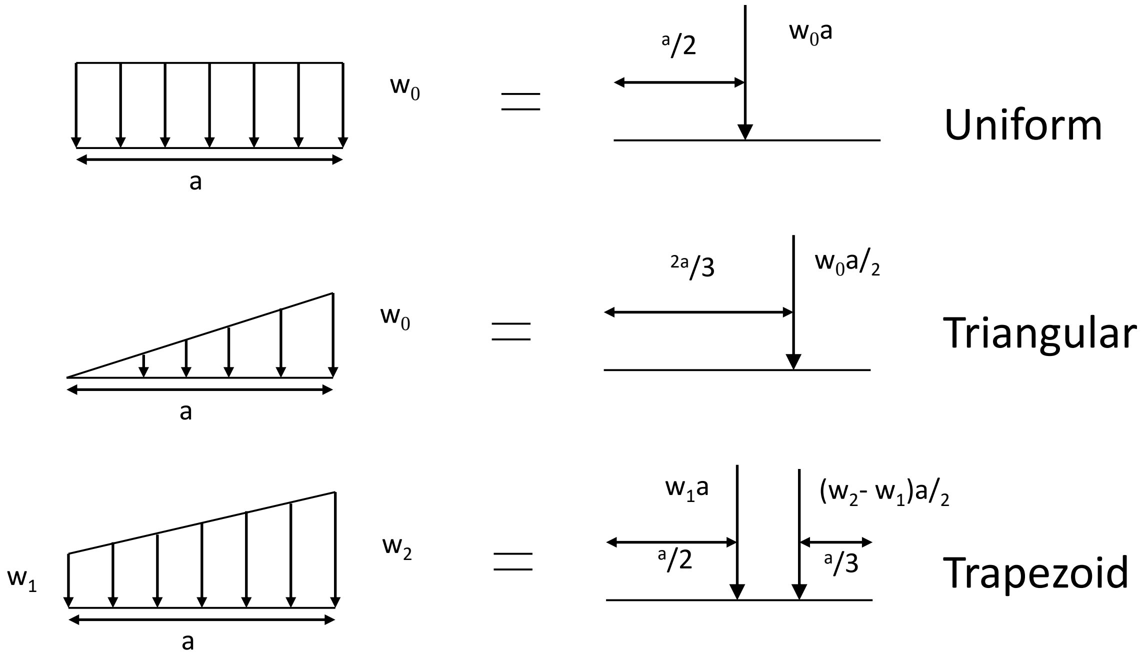
$$R = \int_0^L dw = \int_0^L w(x)dx$$

The location x’ is found based on the principle of moments. Each small dw has a moment about some point (say x = 0). The total moment of all the dw’s about this point must equal the moment of R about the same point. x’ is;

$$x'R = \int_0^L x\,dw = \int_0^L x\,w(x)dx$$

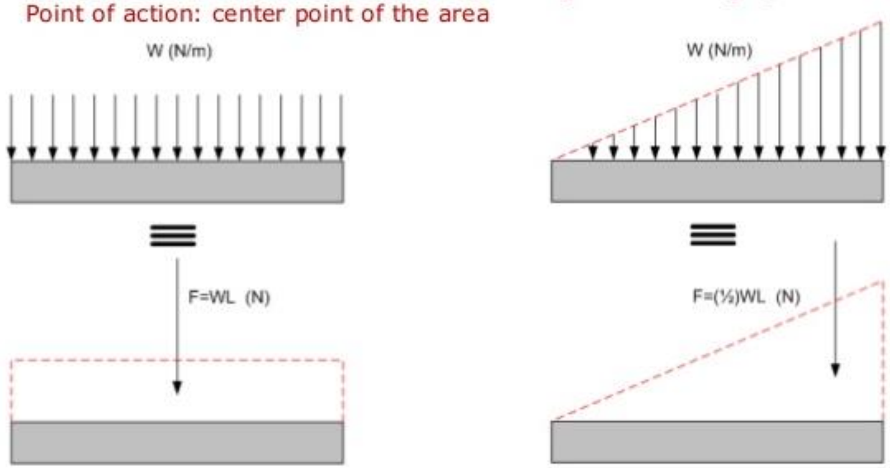
<u>The equivalent force R is equal to the area under the distributed load curve. The location of the force (given by the distance x’) is at the centroid of the distributed load area.</u>

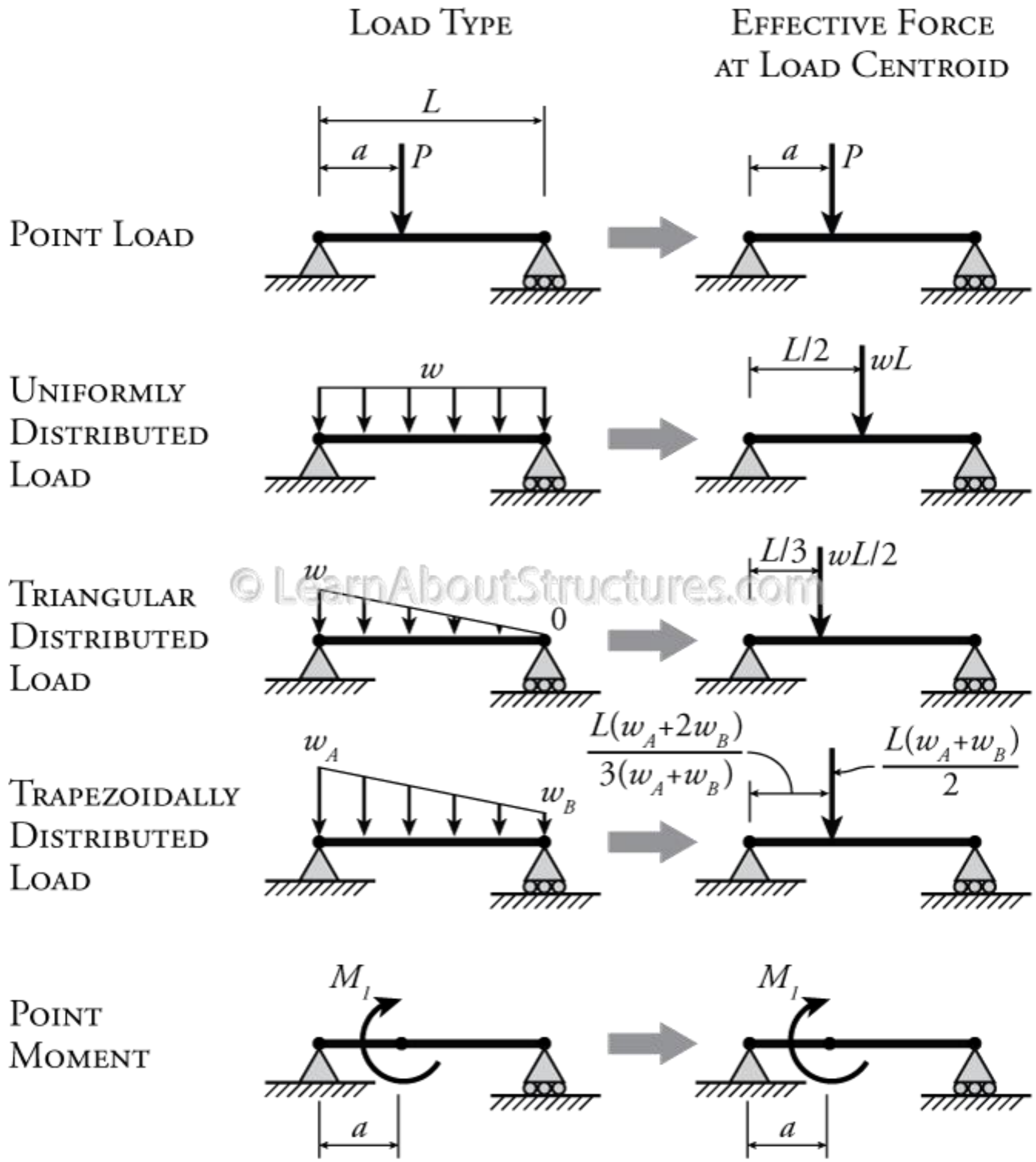
Common Types of Distributed Load



For calculation purposes, distributed load can be represented as a single load acting on the center point of the distributed area.

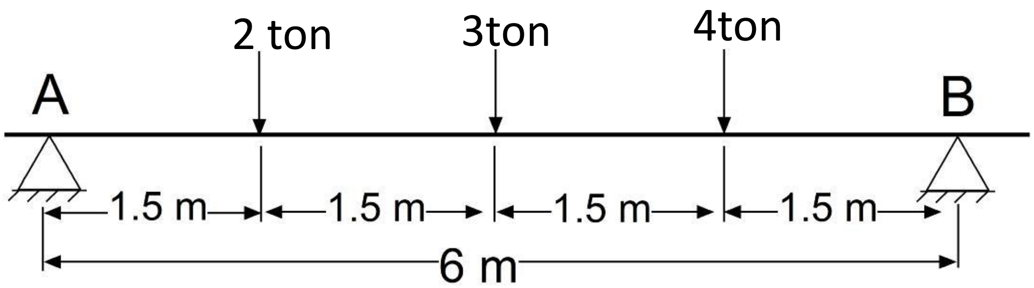
Total force = area of distributed load (W : height and L: length)
Point of action: center point of the area





(Image courtesy of ©LearnAbout Structures.com)

Example 14. Determine the support reactions of A and B pin supports

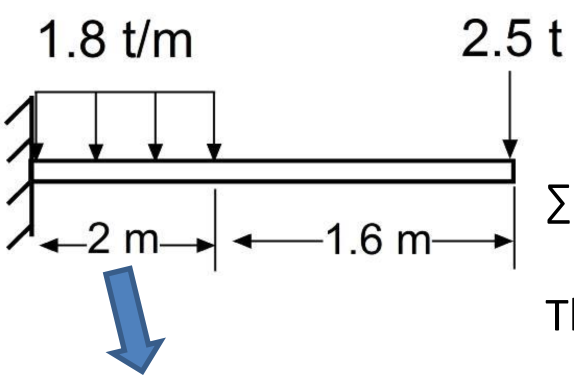


$\sum F_x=0$; Since the forces are acting along "y" axis, no reactions will develop

$\sum F_y=0$; $R_A+R_B=2+3+4 = 9$ t

$\sum M_A=0$; $(R_B x9)=(2x1.5)+(3x3)+(4x4.5)$; $\mathbf{R_B=5\ t,\ R_A= 4t}$

Example 15. Determine the support reaction on the cantilever beam



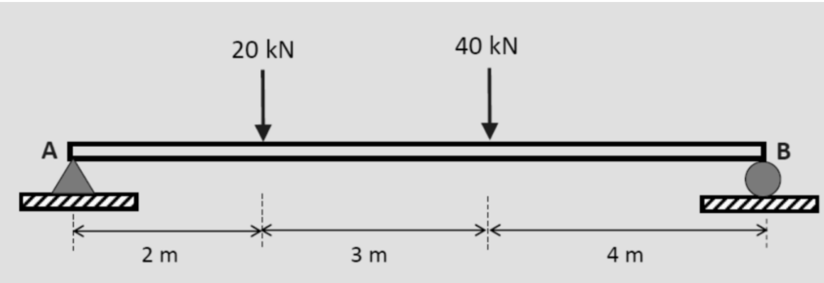
$\sum F_x=0$; Since the forces are acting along "y" axis, no reactions will develop

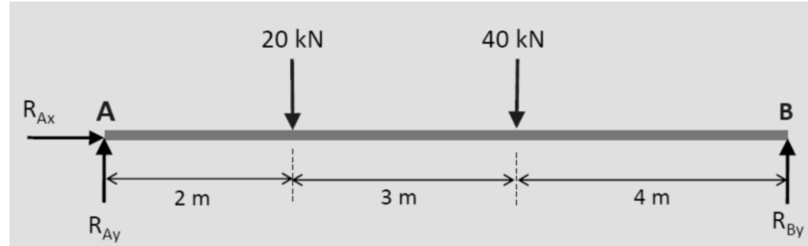
$\sum F_y=0$; $R_A=(2x1.8)+2.5$; $R_A=6.1$ t

The beam is fixed at "A" and no other support reactions will occur

Distributed load: The area of the rectangle = (1.8x2)=3.6 t

Example 16. Determine the support reactions.





Free body diagram

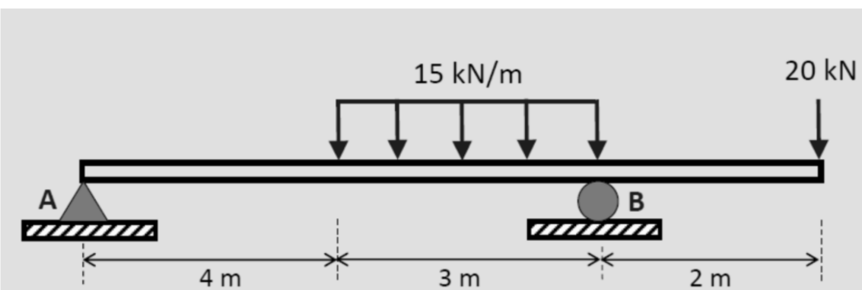
$\sum F_Y=0$; $R_A+R_B=20+40=60$ kN

Moment at "B";

$\sum M_B=0$

$(R_A x9)-(20x7)-(40x4)=0$, $R_A=33.3$ kN; $R_B=26.7$ kN

Example 17. Determine the support reactions.



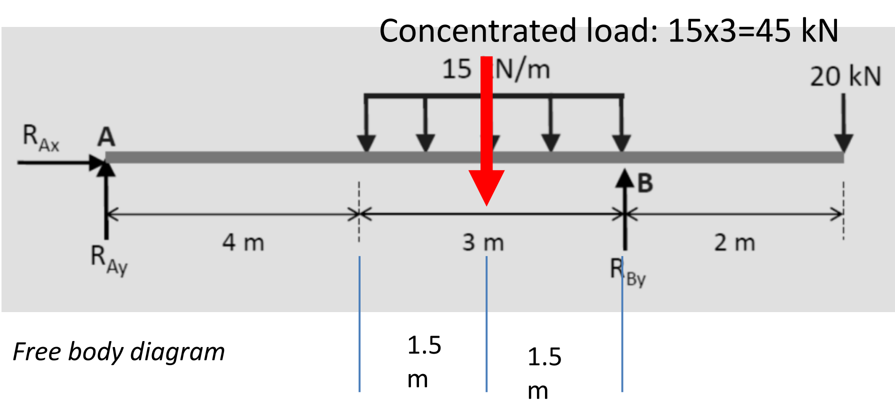
$\sum F_Y=0$; $R_A+R_B=20+(15x3)=$**65 kN**

A noktasına göre moment alınırsa;

$\sum M_A=0$

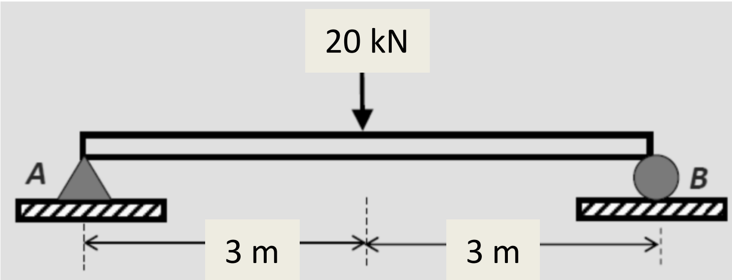
$(R_B x7)-(45x5.5)-(20x9)=0$, $R_B=61.07$ kN;

$R_A=3.93$ kN



Free body diagram

Example 18. Please construct the shear force-bending moment diagram.



Point A, (x=0)

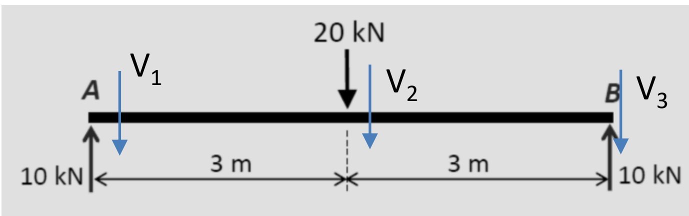
V_1-10=0, V_1=10 kN; M_1=0

X=3m

V_2+20-10=0, V_2=-10 kN; M_2=-10x3=-30 kN.m

Point B (x=6 m)

V_3+20-10-10=0, V_3=0 kN; M_3=(-10x6)+(20x3)=0 kN.m



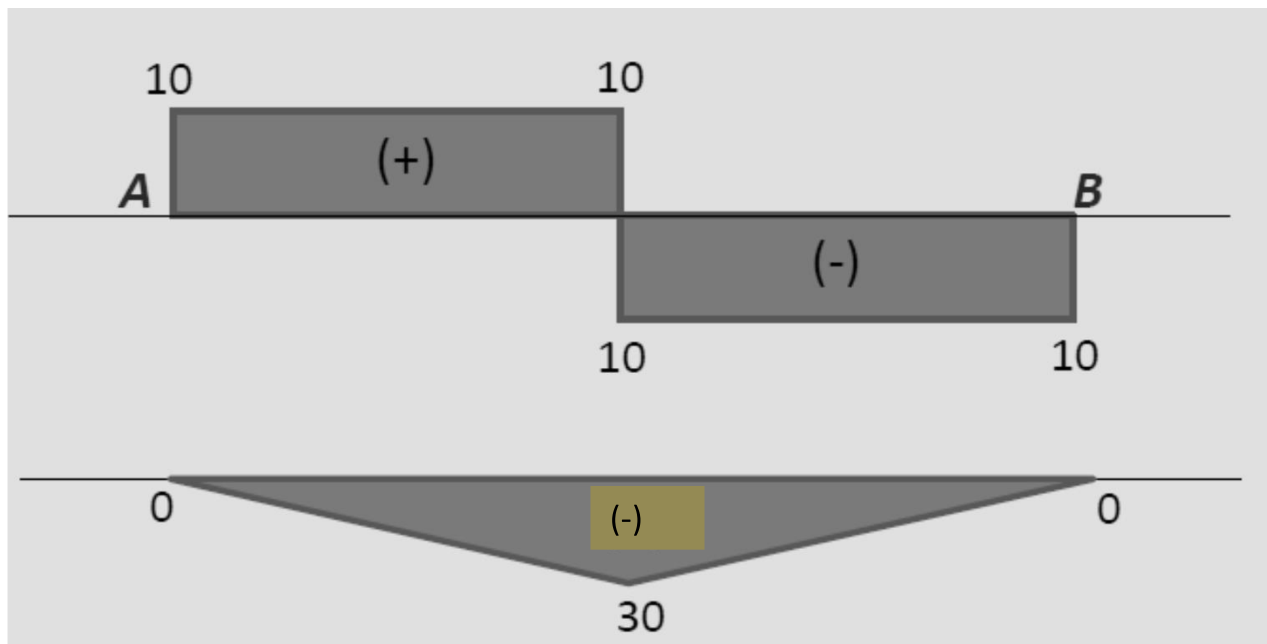
Free body diagram

$\sum F_Y$=0; R_A+R_B=20 kN

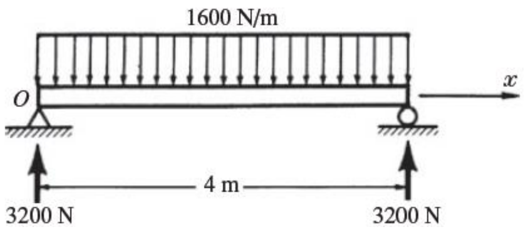
Moment at "A";

$\sum M_A$=0

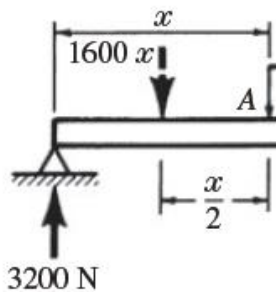
(R_Bx6)-(20x3)=0, R_B= R_A=10 kN

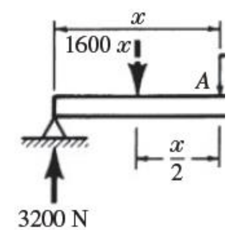


Example 19. Please construct the shear force-bending moment diagram.



Concentrated load= 1600x4=6400 N
3200 N is distributed to each support





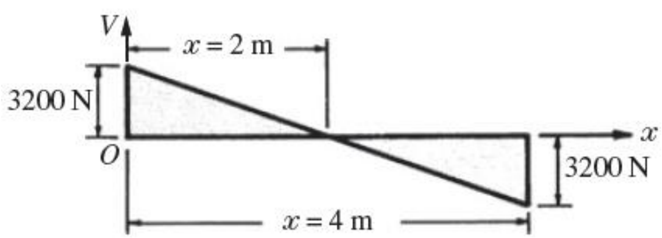
$$M = 3200x - 1600x\left(\frac{x}{2}\right)$$
$$= 3200x - 800x^2$$

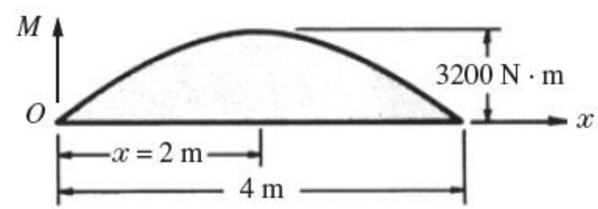
X=0, M_1=0 N
X=1 m, M_2=2400 N
X=2 m, M_3=3200 N
X=4 m, M_4=0 N

Variation of shear force from "O"
At "x"; V+1600x-3200=0 (applies at mid point)

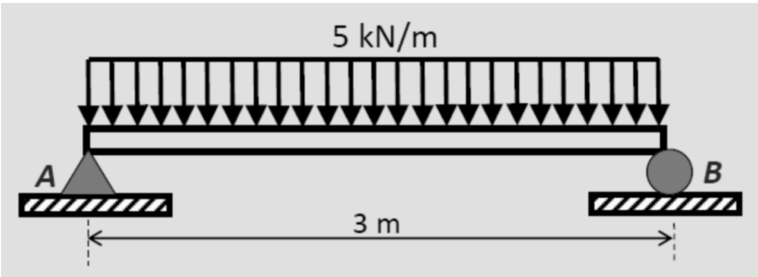
$$V = 3200 - 1600x$$

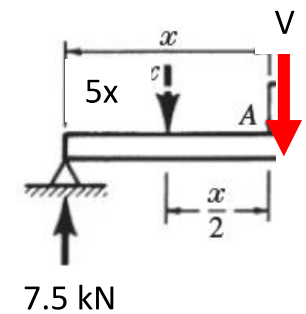
X=0, V_1=3200 N
X=1 m, V_2=1600 N
X=2 m, V_3=0 N
X=4 m, V_4=-3200 N





Example 20. Please construct the shear force-bending moment diagram.



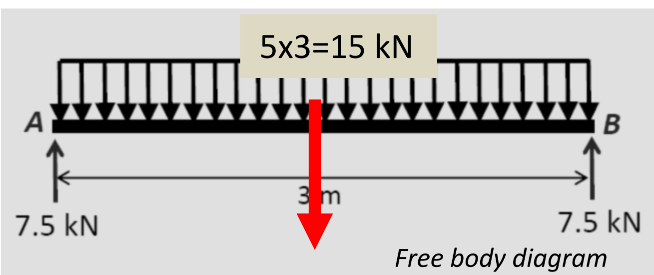


$\sum F_Y=0$; $R_A+R_B=15$ kN

Moment at "A"

$\sum M_A=0$

$(R_B x3)-(15x1.5)=0$, $R_B=7.5$ kN $R_A=7.5$ kN



Free body diagram

At point "A" (x=0)

$V_1-7.5=0$, $V_1=7.5$ kN

V+5x-7.5=0; V=7.5-5x

X=0.5.m

$V_2+(\mathbf{5x0.5})-7.5=0$, $V_2= 5$ kN

X=1.0.m

$V_3+(\mathbf{5x1})-7.5=0$, $V_3= 2.5$ kN

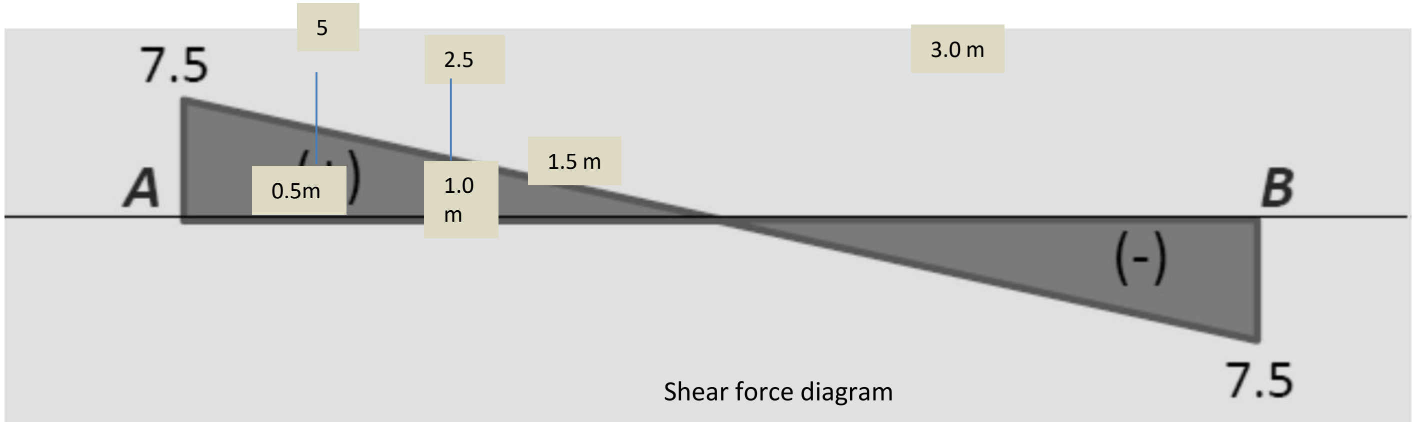
Distributed load, 5 kN per meter

X=1.5.m

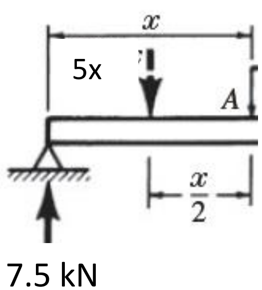
$V_4+(5x1.5)-7.5=0$, $V_3= 0$ kN

At point "B" (x=3 m)

$V_4+(5x3)-7.5=0$, $V_4=-7.5$ kN



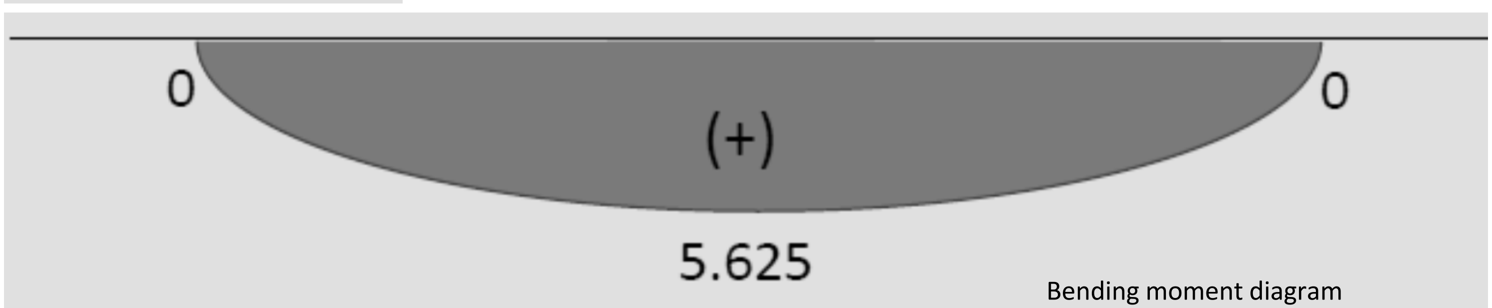
Shear force diagram



$-M + 7.5x - 5x\,(x/2) = 0$

$M = 7.5x - 5x^2/2$

X=0, M_1=0 kN
X=0.5 m, M_2=3.125 kN
X=1 m, M_3=5.0 kN
X=1.5 m, M_4=5.625 kN
X=3 m, M_5=0 kN



Bending moment diagram