Ankara University
Department of Geological Engineering

GEO222 STATICS and STRENGTH of MATERIALS

Lecture Notes

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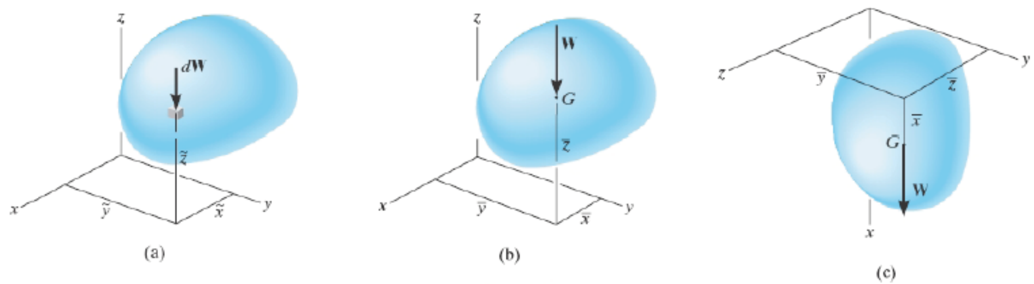
Center of Gravity

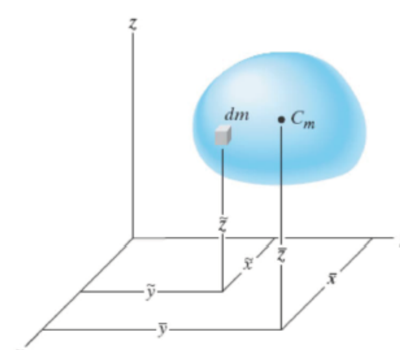
- Locates the resultant weight of a system of particles
- Consider system of n particles fixed within a region of space
- The weights of the particles can be replaced by a single (equivalent) resultant weight having defined point G of application



Center Mass

- A rigid body is composed of an infinite number of particles
- Consider arbitrary particle having a weight of dW

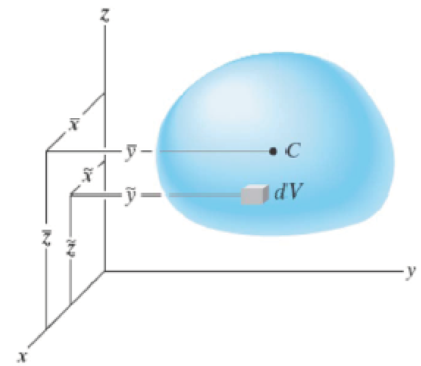
$$\bar{x}=\frac{\int \tilde{x}\,dW}{\int dW};\bar{y}=\frac{\int \tilde{y}\,dW}{\int dW};\bar{z}=\frac{\int \tilde{z}\,dW}{\int dW}$$



Centroid of a Volume

- Consider an object subdivided into volume elements dV, for location of the centroid,

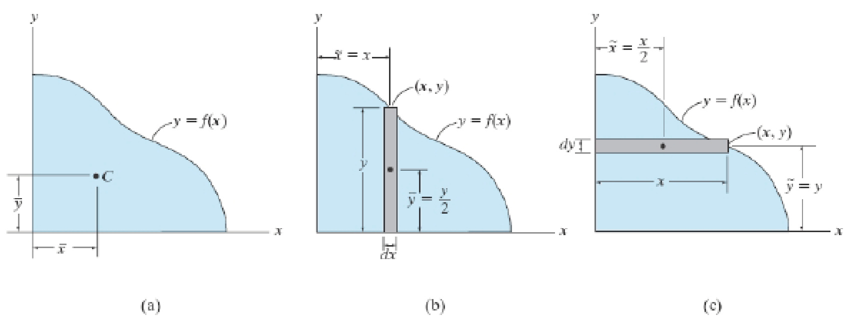
$$\bar{x} = \frac{\int_V \tilde{x}\,dV}{\int_V dV};\quad \bar{y} = \frac{\int_V \tilde{y}\,dV}{\int_V dV};\quad \bar{z} = \frac{\int_V \tilde{z}\,dV}{\int_V dV}$$



Centroid of an Area

- For centroid for surface area of an object, such as plate and shell, subdivide the area into differential elements dA

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA};\quad \bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA};\quad \bar{z} = \frac{\int_A \tilde{z}\,dA}{\int_A dA}$$

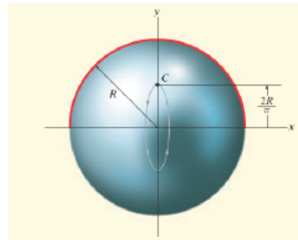


Example 21. Please prove that the surface area of a sphere is **$A=4\pi R^2$** and the volume is **$V=4/3\pi R^3$**

Surface Area

Generated by rotating semi-arc about the x axis

For centroid, $\bar{r} = 2R/\pi$



For surface area, $A = \theta \tilde{r} L;$

$$A = 2\pi\left(\frac{2R}{\pi}\right)\pi R = 4\pi R^2$$

Volume

Generated by rotating semicircular area about the x axis

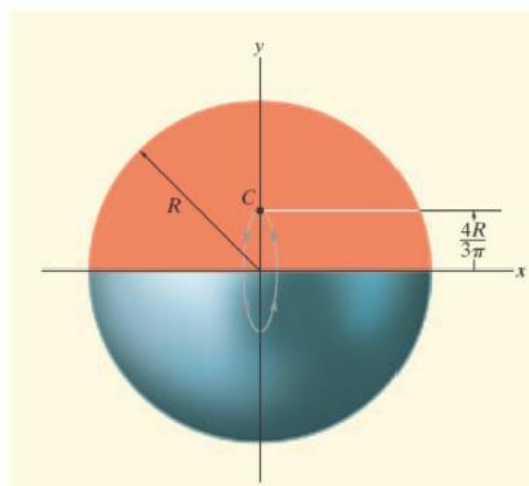
For centroid,

$\bar{r} = 4R/3\pi$

For volume,

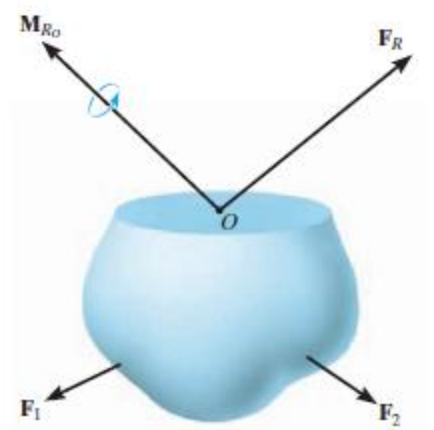
$V = \theta \tilde{r} A;$

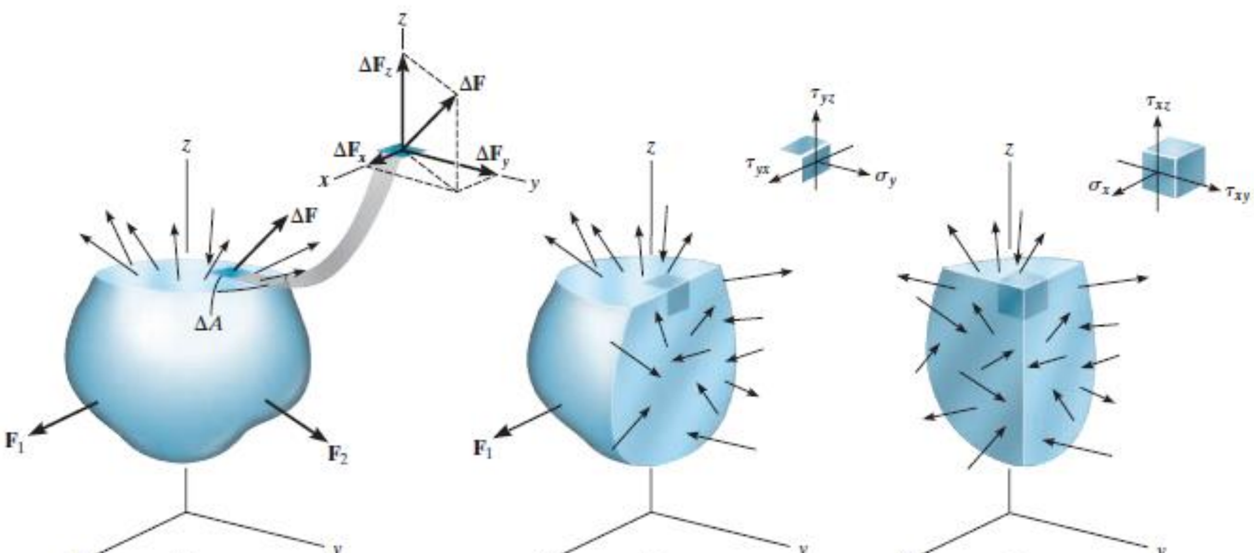
$$V = 2\pi\left(\frac{4R}{3\pi}\right)\left(\frac{1}{2}\pi R^2\right) = \frac{4}{3}\pi R^3$$



Stress Concept

Based on the force and moment acting at a specified point *O* on the sectioned area of the body, (Fig.a) represents the resultant effects of the actual *distribution of loading* acting over the sectioned area, Fig.b. Considering the sectioned area to be subdivided into small areas, such as ΔA (Fig.*b).* As we reduce ΔA to a smaller and smaller size, we must make two assumptions regarding the properties of the material. We will consider the material to be ***continuous***, that is, to consist of a *continuum* or uniform distribution of matter having no voids. A typical finite yet very small force ΔF acting on ΔA is shown in Fig. *a*. This force, like all the others, will have a unique direction, but for further discussion we will replace it by its *three components*, namely, ΔF_x ΔF_y and ΔF_z . As ΔA approaches zero, so do ΔF and its components; however, the quotient of the force and area will, in general, approach a finite limit. **This quotient is called *stress*, and as noted, it describes the *intensity of the internal force* acting on a *specific plane* (area) passing through a point** (Hibbeler, 2010)**.**





(a) (b) (c)

Normal Stress

The *intensity* of the force acting normal to ΔA is defined as the ***normal stress***, (σ). Since DF_z is normal to the area then;

$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

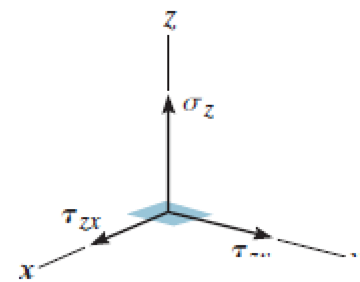
If the normal force or stress “pulls”, it is referred to as *tensile stress*, whereas if it “pushes” on it is called *compressive stress*.

Shear Stress

The intensity of force acting tangent to ΔA is called the ***shear stress***, (τ). Here we have shear stress components,

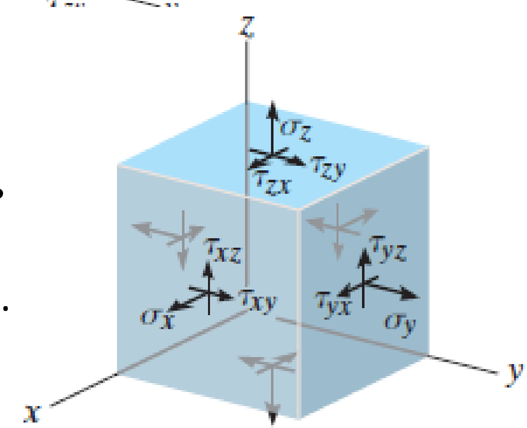
$$\tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

$$\tau_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}$$



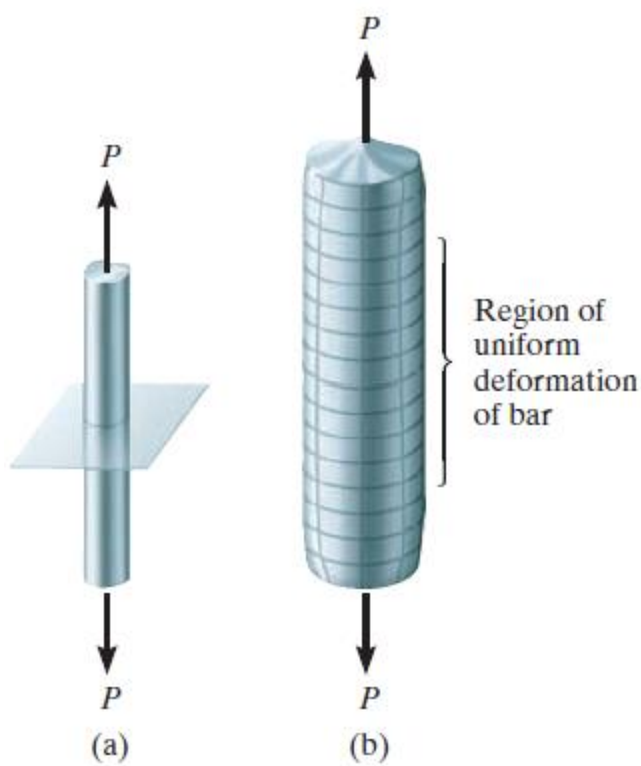
General State of Stress

If we “cut out” a cubic volume element of material that represents the ***state of stress*** acting around the chosen point in the body. This state of stress is then characterized by three components acting on each face of the element.



Average Normal Stress in an Axially Loaded Bar

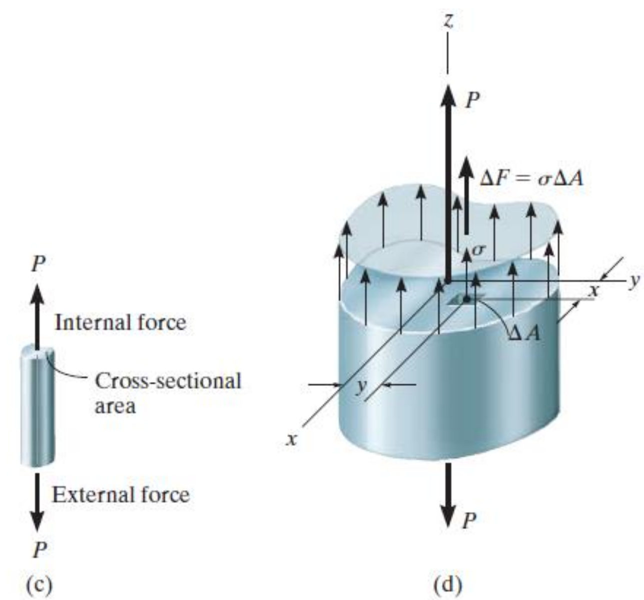
In this section we will determine the average stress distribution acting on the cross-sectional area of an axially loaded bar (Fig.*a*). This bar is **prismatic** since all cross sections are the same throughout its length. When the load "*P*" is applied to the bar through the centroid of its cross-sectional area, then the bar will deform uniformly throughout the central region of its length (Fig. *b*) provided the material of the bar is both homogeneous and isotropic. ***Homogeneous material*** has the same physical and mechanical properties throughout its volume, and ***isotropic material*** has these same properties in all directions. Many engineering materials may be approximated as being both homogeneous and isotropic as assumed here.



(Hibbeler, 2010)

Average Normal Stress Distribution

If we pass a section through the bar, and separate it into two parts, then equilibrium requires the resultant normal force at the section to be *P*, (Fig. *c*.) Due to the *uniform* deformation of the material, it is necessary that the cross section be subjected to a *constant normal stress distribution* (Fig. *d*)



As a result, each small area ΔA on the cross section is subjected to a force ,and the *sum* of these forces "F=σΔA" acting over the entire cross-sectional area must be equivalent to the internal resultant force **P** at the section.

$$+\uparrow F_{Rz} = \Sigma F_z; \qquad \int dF = \int_A \sigma \, dA$$

$$P = \sigma A$$

$$\sigma = \frac{P}{A}$$

σ = average normal stress at any point on the cross-sectional area

P = *internal resultant normal force*, which acts through the *centroid* of the cross-sectional area. P is determined using the method of sections and the equations of equilibrium

A = cross-sectional area of the bar where σ is determined

(Hibbeler, 2010)