Ankara University
Department of Geological Engineering

GEO222 STATICS and STRENGTH of MATERIALS

Lecture Notes

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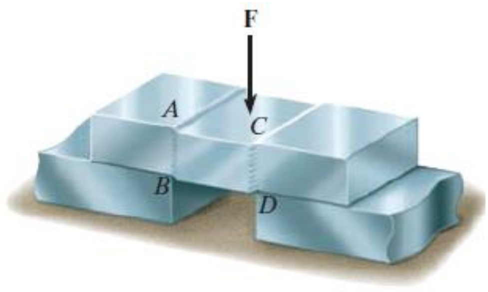
Average Shear Stress

To show how this stress can develop, consider the effect of applying a force **F** to the bar in Fig. *a*. If the supports are considered rigid, and **F** is large enough, it will cause the material of the bar to deform and fail along the planes identified by *AB* and *CD*. A free-body diagram of the unsupported center segment of the bar, Fig. *b*, indicates that the shear force "V=F/2" must be applied at each section to hold the segment in equilibrium. The ***average shear stress*** distributed over each sectioned area that develops this shear force is defined by

$$\boxed{\tau_{\text{avg}} = \frac{V}{A}}$$

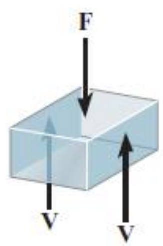


(a)

τ_{avg} = average shear stress at the section, which is assumed to be the *same* at each point located on the section

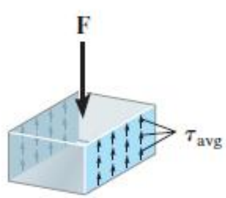
V = internal resultant shear force on the section determined from the equations of equilibrium

A = area at the section



(b)

The distribution of average shear stress acting over the sections is shown in Fig. *c*. Notice that is in the *same direction* as **V**, since the shear stress must create associated forces all of which contribute to the internal resultant force **V** at the section. The loading case discussed here is an example of ***simple or direct shear***, since the shear is caused by the *direct action* of the applied load **F**.

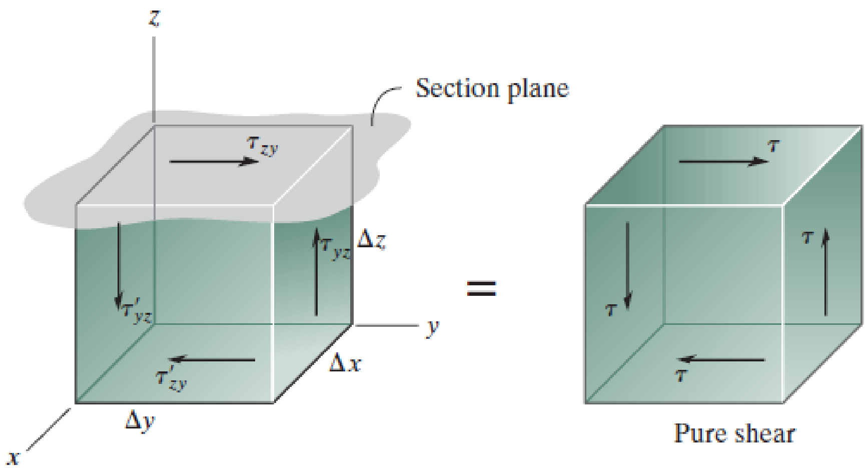


(c)

(Hibbeler, 2010)

Shear Stress Equilibrium

Figure *a* shows a volume element of material taken at a point located on the surface of a sectioned area which is subjected to a shear stress "τ_{zy}". Force and moment equilibrium requires the shear stress acting on this face of the element to be accompanied by shear stress acting on three other faces.To show this we will first consider force equilibrium in the *y* direction. Then;



(b)

$$\Sigma F_y = 0; \qquad \overbrace{\overbrace{\tau_{zy}}^{\text{stress}}\overbrace{(\Delta x\ \Delta y)}^{\text{area}}}^{\text{force}} - \tau'_{zy}\,\Delta x\ \Delta y = 0$$

$$\tau_{zy} = \tau'_{zy}$$

$$\Sigma M_x = 0; \qquad -\overbrace{\overbrace{\overbrace{\tau_{zy}}^{\text{stress}}\overbrace{(\Delta x\ \Delta y)}^{\text{area}}}^{\text{force}}\ \overbrace{\Delta z}^{\text{arm}}}^{\text{moment}} + \tau_{yz}(\Delta x\ \Delta z)\ \Delta y = 0$$

$$\tau_{zy} = \tau_{yz}$$

(Hibbeler, 2010)

Complimentary Shear Stress

Consider an infinitely small rectangular element *ABCD* under shear stress of intensity τ acting on planes *AD* and *BC* as shown in Fig. 1.4a. It is clear from the figure that the shear stress acting on the element will tend to rotate the block in the clockwise direction. As there is no other force acting on the element, static equilibrium of the element can only be attained if another couple of the same magnitude is applied in the counter-clockwise direction. This can be achieved by having shear stress of intensity τ' on the faces *AB* and *CD* (Fig. 1.4b).

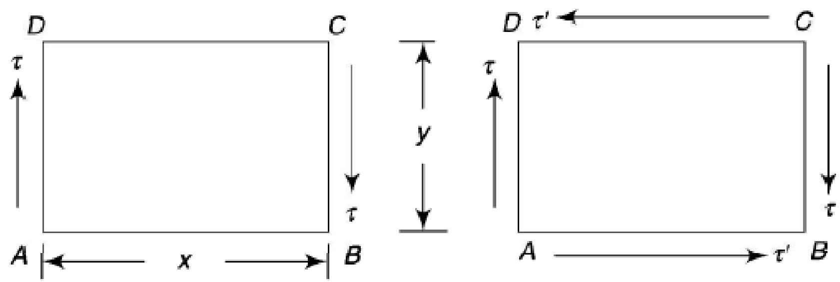


Fig. 1.4

Assuming x and y to be the lengths of the sides *AB* and *BC* of the rectangular element and a unit thickness perpendicular to the figure,

The force of the given couple $= \tau \cdot (y \cdot 1)$

The moment of the given couple $= (\tau \cdot y) \cdot x$

Similarly,

The force of balancing couple $= \tau' \cdot (x \cdot 1)$

The moment of balancing couple $= (\tau' \cdot x) \cdot y$

For equilibrium, equating the two,

$$(\tau \cdot y) \cdot x = (\tau' \cdot x) \cdot y$$

or

$$\tau = \tau'$$

which shows that the magnitude of the balancing shear stresses is the same as of the applied stresses. The shear stresses on the transverse pair of faces are known as *complimentary shear stresses*. Thus every shear stress is always accompanied by an equal complimentary shear stress on perpendicular planes.

The presence of complimentary shear stress may cause an early failure of *anisotropic materials* such as timber which is weaker in shear along the grain than normal to the grain.

- *Owing to the characteristic of complimentary shear stresses for the equilibrium of members subjected to shear stresses, near a free boundary on which no external force acts, the shear stress must follow a direction parallel to the boundary. This is because any component of the shear force perpendicular to the surface will find no complimentary shear stress on the boundary plane.*

(Rattan, 2011)

Allowable Stress

To properly *design* a structural or mechanical element it is necessary to restrict the stress to a level that will be safe. To ensure this safety, it is necessary to choose an allowable stress that restricts the applied load to one that is *less* than the load the member can fully support. There are many reasons for doing this. For example, the load for which the member is designed may be different from actual loadings placed on it. The intended measurements of a structure or machine may not be exact, due to errors in fabrication or in the assembly of its component parts. Unknown vibrations, impact, or accidental loadings can occur that may not be accounted for in the design. Atmospheric corrosion, decay, or weathering tend to cause materials to deteriorate during service. And lastly, some materials, such as wood, concrete, or fiber-reinforced composites, can show high variability in mechanical properties. One method of specifying the allowable load for a member is to use a number called the factor of safety. The ***factor of safety*** (F.S.) is a ratio of the failure load "F_{fail}" to the allowable load "F_{allow}" Here is found from experimental testing of the material, and the f ' is selected based on experience so that the above mentioned uncertainties are accountec member is used under similar conditions of loading and geometry.

$$\text{F.S.} = \frac{\sigma_{\text{fail}}}{\sigma_{\text{allow}}}$$

If the load applied to the member is *linearly related* to the stress developed within the member, as in the case of using $\sigma = P/A$ and $\tau_{\text{avg}} = V/A$, then we can also express the factor of safety as a ratio of the failure stress σ_{fail} (or τ_{fail}) to the allowable stress σ_{allow} (or τ_{allow});* that is,

$$\boxed{\text{F.S.} = \frac{F_{\text{fail}}}{F_{\text{allow}}}}$$

$$\text{F.S.} = \frac{\tau_{\text{fail}}}{\tau_{\text{allow}}}$$