Ankara University
Department of Geological Engineering

GEO222 STATICS and STRENGTH of MATERIALS

Lecture Notes

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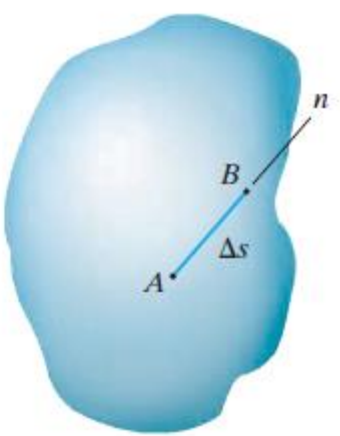
Normal Strain

If we define the normal strain as the change in length of a line per unit length, then we will not have to specify the *actual length* of any particular line segment. Consider, for example, the line *AB*, which is contained within the undeformed body shown in Fig. *a*. This line lies along the *n* axis and has an original length of "Δs" After deformation, points *A* and *B* are displaced to A' and B' , and the line becomes a curve having a length of "$\Delta s'$ " (Fig.*b*). The change in length of the line is therefore "$\Delta s' - \Delta s$". If we define the *average normal strain* using the symbol (ε_{avg}), then;

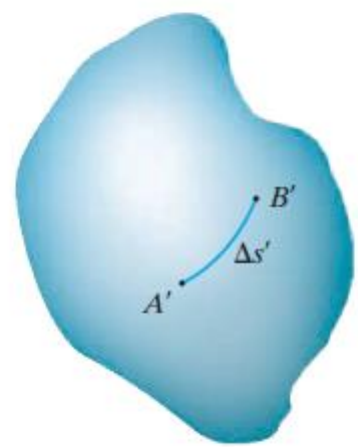
$$\epsilon_{avg} = \frac{\Delta s' - \Delta s}{\Delta s}$$

As point *B* is chosen closer and closer to point *A*, the length of the line will become shorter and shorter, such that Δs approaches zero , this causes B' to approach A' such that $\Delta s' \sim 0$. Consequently, in the limit the normal strain at point *A* and in the direction of *n* is

$$\epsilon = \lim_{B \to A \text{ along } n} \frac{\Delta s' - \Delta s}{\Delta s}$$



Undeformed body
(a)



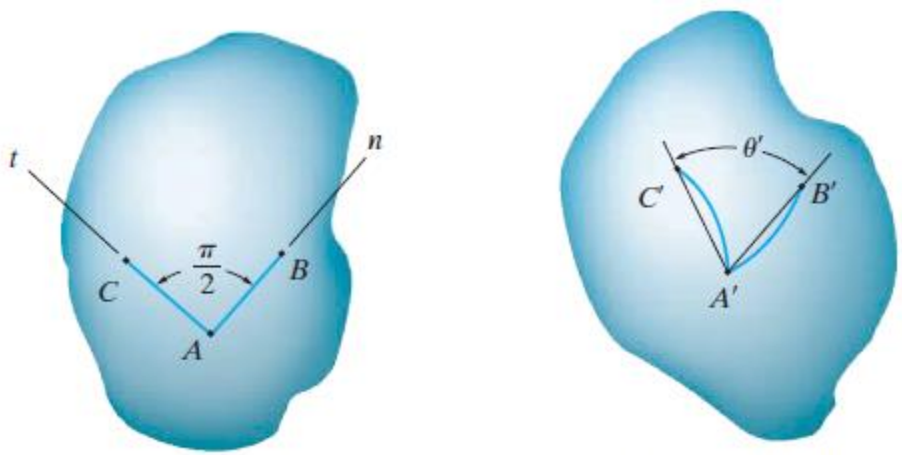
Deformed body
(b)

(Hibbeler, 2010)

Shear Strain

Deformations not only cause line segments to elongate or contract, but they also cause them to change direction. If we select two line segments that are originally perpendicular to one another, then the change in angle that occurs between these two line segments is referred to as ***shear strain***. This angle is denoted by (γ) and is always measured in radians (rad), which are dimensionless. For example, consider the line segments *AB* and *AC* originating from the same point *A* in a body, and directed along the perpendicular *n* and *t* axes, Fig. 2–2*a*. After deformation, the ends of both lines are displaced, and the lines themselves become curves, such that the angle between them at *A* is Fig. 2–2*b*. Hence the shear strain at point *A* associated with the *n* and *t* axes becomes

$$\gamma_{nt} = \frac{\pi}{2} - \lim_{\substack{B \to A \text{ along } n \\ C \to A \text{ along } t}} \theta'$$



Undeformed body

(a)

Deformed body

(b)

Notice that if θ' is smaller than $\pi/2$ the shear strain is positive, whereas if θ' is larger than $\pi/2$ the shear strain is negative.

(Hibbeler, 2010)

Cartesian Strain Components

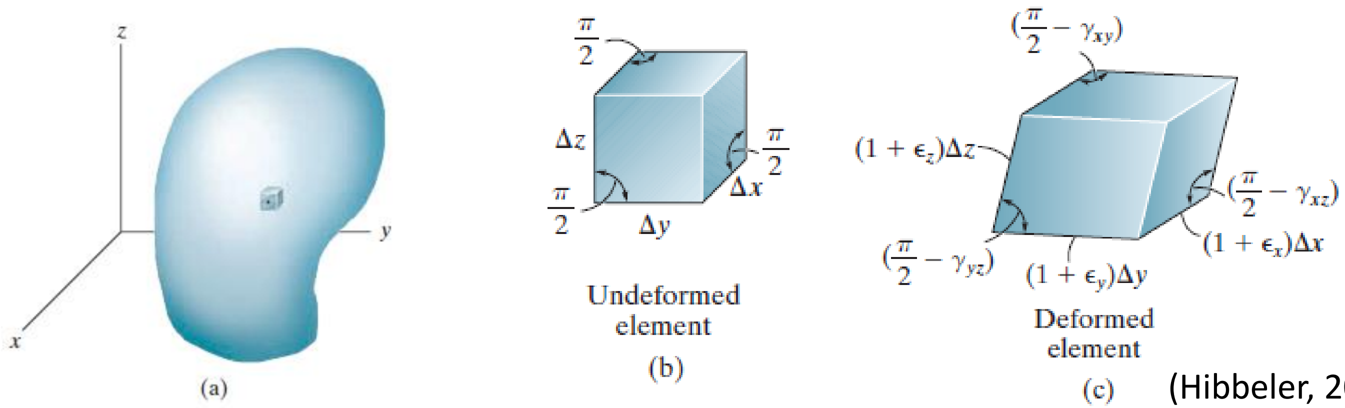
If the rigid body in Fig.*a* is subdivided into small rectangular elements (Fig.*b*) which have undeformed dimensions of Δx, Δy, Δz. If the element's dimensions are very small, then its deformed shape will be a parallelepiped (Fig.*c*) since very small line segments will remain approximately straight after the body is deformed. In order to achieve this deformed shape, consider how normal strain changes the lengths of the sides of the rectangular element, and then how the shear strain changes the angles of each side. For example, Δx elongates to "$\varepsilon_x \Delta x$", so its new length is "$\Delta x + \varepsilon_x \Delta x$" . Each length of three sides of the parallelepiped are

$$(1 + \epsilon_x)\,\Delta x \qquad (1 + \epsilon_y)\,\Delta y \qquad (1 + \epsilon_z)\,\Delta z$$

And the approximate angles between these sides are

$$\frac{\pi}{2} - \gamma_{xy} \qquad \frac{\pi}{2} - \gamma_{yz} \qquad \frac{\pi}{2} - \gamma_{xz}$$

Normal strains cause a change in volume of the element, while ***shear strains cause a change in its shape***. Both of these effects occur simultaneously during the deformation. In summary, the *state of strain* at a point requires specifying three normal strains, and three shear strains. These strains completely describe the deformation of a rectangular volume element of material located at the point and oriented so that its sides are originally parallel to the *x*, *y*, *z* axes.



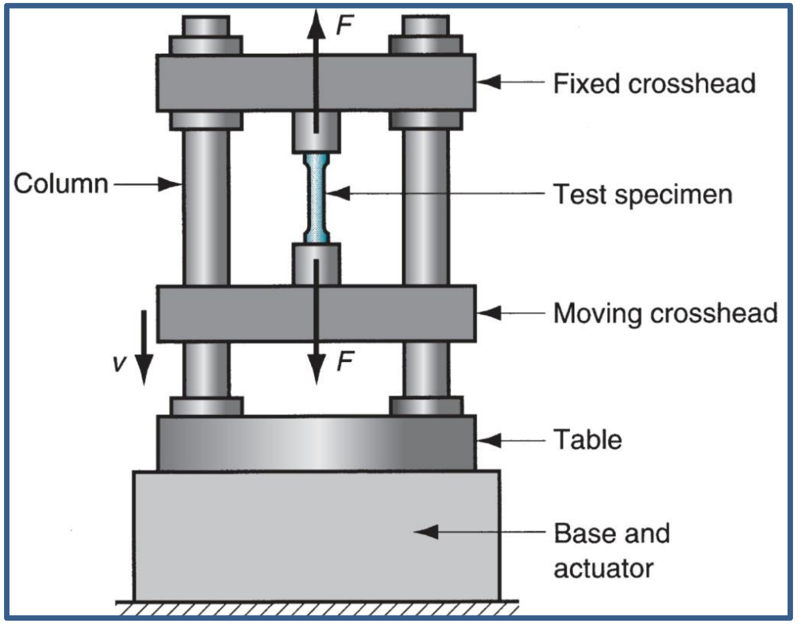
(Hibbeler, 2010)

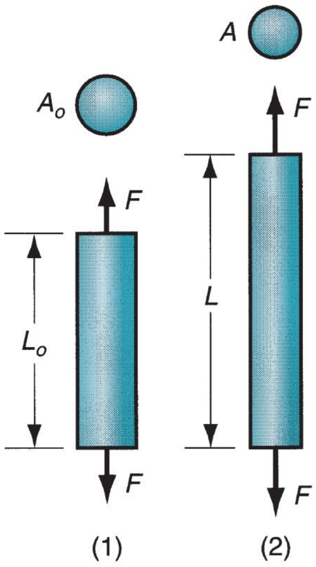
Stress - Strain Relation

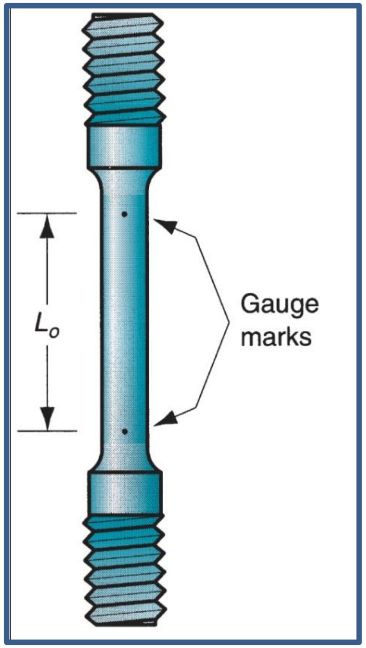
- Main types of stresses to be applied to materials are;
  1. Tensile - stretching the material
  2. Compressive - squeezing the material
  3. Shear - causing adjacent portions of the material to slide against each other

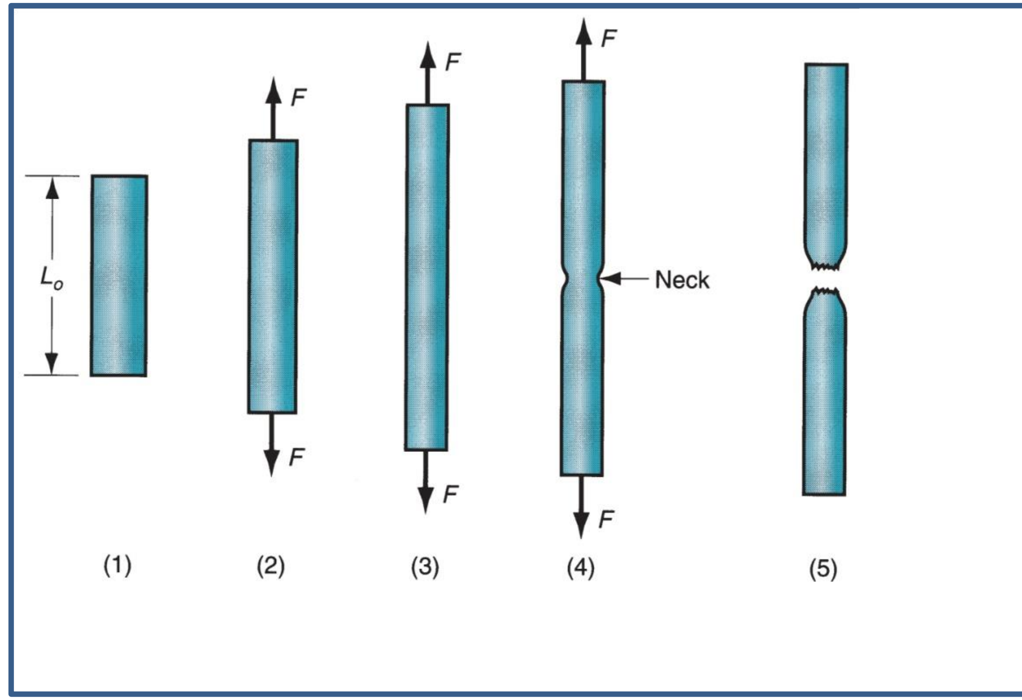
Tensile Test

- Most common test for studying stress-strain relationship, especially metals
- In the test, a force pulls the material, elongating it and reducing its diameter



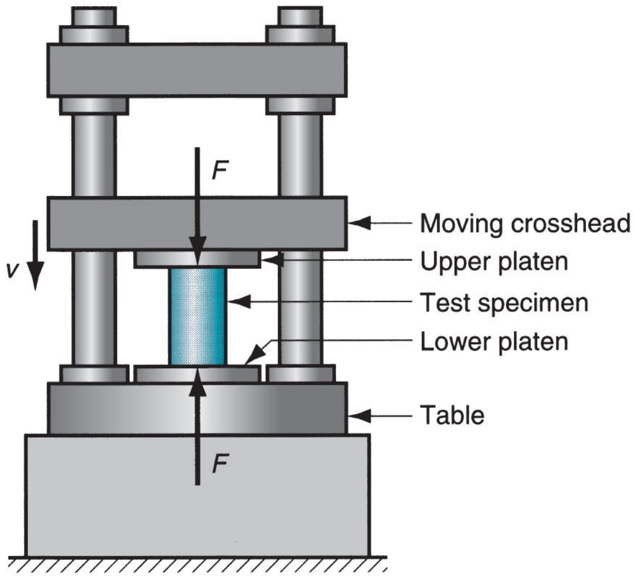






(1) Original state
(2) Elongation and cross section area reduction
(3) Maximum load
(4) Necking
(5) Fracture (Original material has two parts)

Compression Test



(b)

Bulge deformation

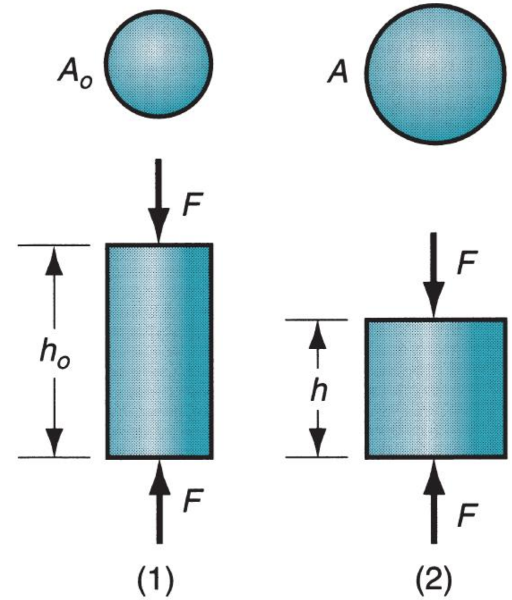


Fig. 2.3 Porous Al_2O_3, Non-Porous Al_2O_3, aluminum, brass, steel, copper. Each pair of materials contain samples before (left) and after (right) compression testing.

Khlystov, et al. (2013)