



Ankara University  
Department of Geological Engineering



# GEO222 STATICS and STRENGTH of MATERIALS

Lecture Notes

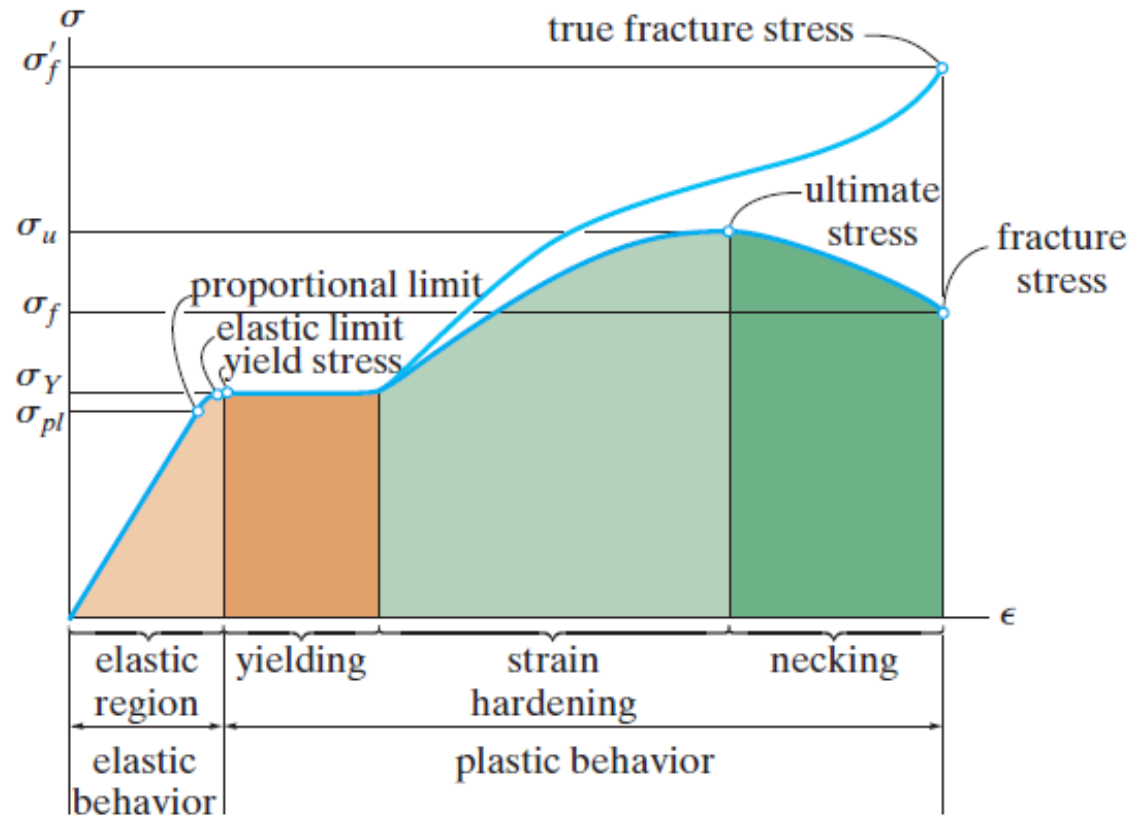
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# Stress-Strain Diagram

The “Engineering Stress” can be determined by tension or compression tests by applying axial load to a specimen until fracturing takes place. Since the load is applied throughout the sample’s cross-section area, the stress is the ratio of load to the cross section. Similarly, “Engineering Strain” is determined by the change in length over original length of the sample either by recording the data or using strain gages.

$$\sigma = \frac{P}{A_0}$$

$$\epsilon = \frac{\delta}{L_0}$$



Engineering and True stress-strain diagram (Hibbeler, 2010)

## Proportional Limit

The curve is a *straight line* throughout most of this region, so that the stress is *proportional* to the strain. The material in this region is *linear elastic*. The upper stress limit to this linear relationship is called the ***proportional limit***, if the stress slightly exceeds the proportional limit, the curve tends to bend and flatten

## Elastic Limit

In case loading continues until the stress reaches the ***elastic limit***. Upon reaching this point, if the load is removed the specimen will still return back to its original shape.

## Yielding

A slight increase in stress above the elastic limit will result in a breakdown of the material and cause it to *deform permanently*. This behavior is called ***yielding***, and the stress that causes yielding is called the ***yield stress*** or ***yield point***, and the deformation that occurs is called ***plastic deformation***. Material is in this state, it is often referred to as being ***perfectly plastic***.

## Strain Hardening

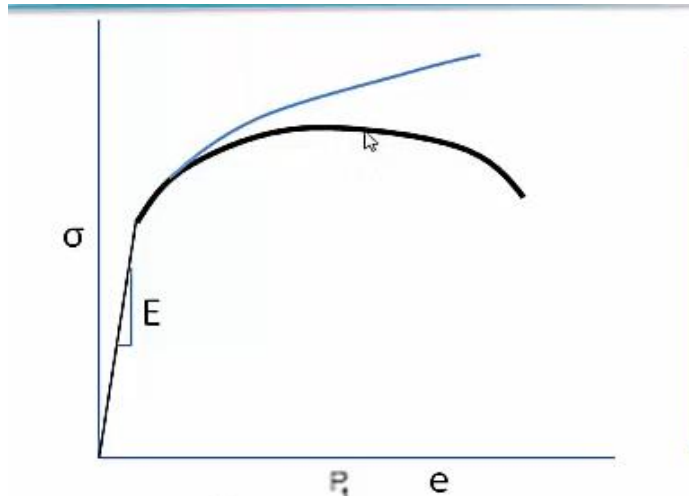
When yielding has ended, an increase in load can be supported by the specimen, resulting in a curve that rises continuously but becomes flatter until it reaches a maximum stress referred to as the ***ultimate stress***, The rise in the curve in this manner is called ***strain hardening***.

## Necking

Up to the ultimate (maximum) stress, as the specimen elongates, its cross-sectional area will decrease. This decrease is *uniform* over the specimen's entire length; however, just after, at the ultimate stress, the cross-sectional area will begin to decrease in a *localized* region of the specimen. As a result, a constriction or "neck" tends to form in this region as the specimen elongates further.

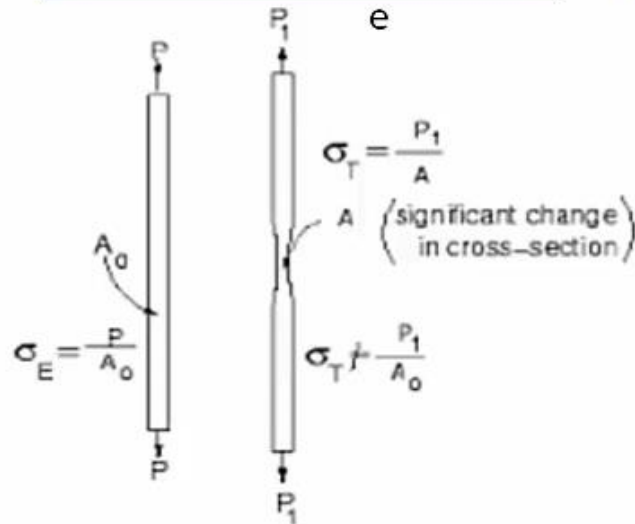
If the load is continuously applied, finally the "**Fracture Stress**" is reached. At this point, the sample is physically split in two pieces.

## Engineering vs True Stress-Strain Diagram



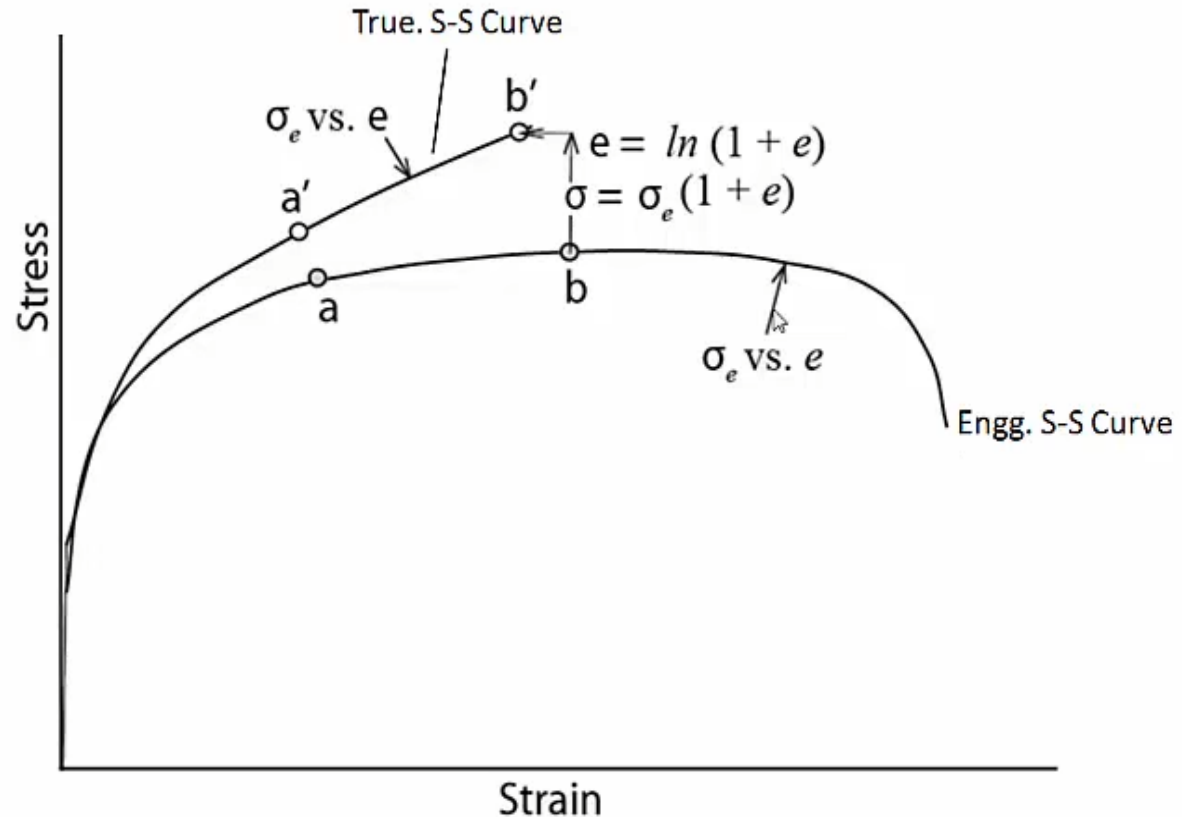
Engineering stress-strain curve is based on original sample cross-section area.

True stress-strain diagram is based on the actual cross section area. The sample's cross section area changes during testing



Uniaxial Test

# Conversion Engg. To True Stress-Strain curve



- True Strain =  $\ln(1 + e)$
- True Stress = Engg. Stress  $\times$   $\exp(\text{True Strain})$
- Eff. Plastic Strain = Total strain – Elastic Strain

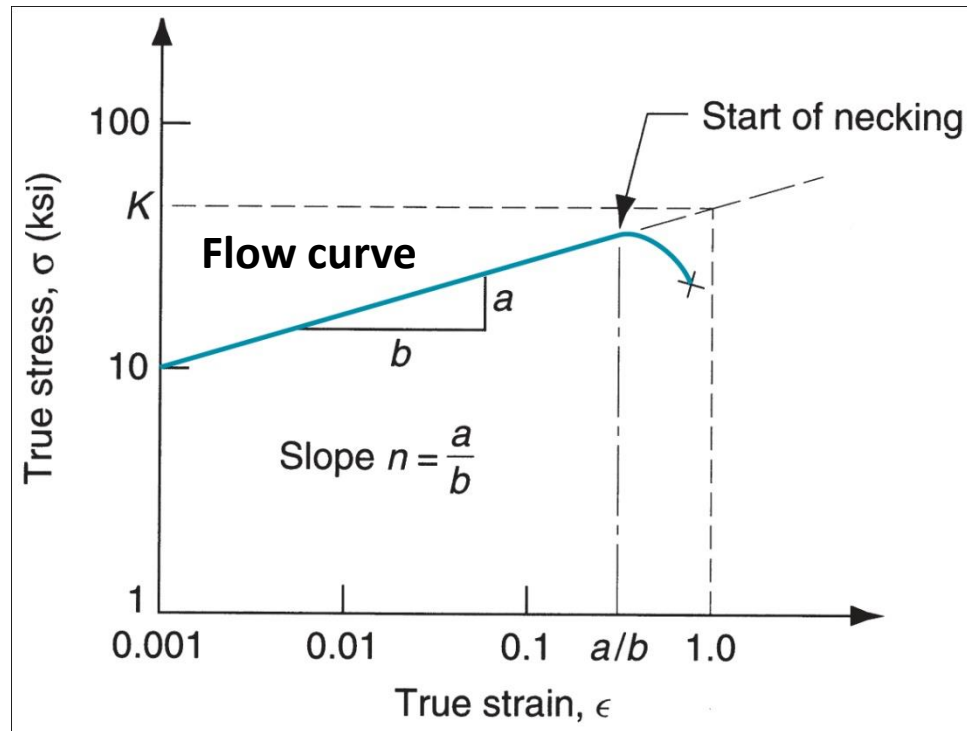
True stress

$$\sigma_t = \sigma_e(1 + \epsilon_e)$$

True strain

$$\epsilon_t = \ln(1 + \epsilon_e)$$

## True Stress-Strain Diagram in Log-Log Plot



- Because it is a straight line in a log-log plot, the relationship between true stress and true strain in the plastic region is

$$\sigma = K\epsilon^n$$

where  $K$  = strength coefficient; and  $n$  = strain hardening exponent