REACTION CROSS SECTIONS

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CROSS SECTION

In the previous sections, we have been considering the energetics of nuclear reactions without considering what fraction of the beam of the incident particles will participate in a reaction.

The decay of a radioactive atom was defined in terms of the probability λ. Similarly, we need to find some way of expressing the probability of something happening to the particles of an incident beam when they strike the target nuclei.

The term of the cross section, σ, has been introduced for the purpose of calculating the attenuation of the incident beam.

CROSS SECTION

Consider a beam of particles of intensity I incident on a thin sheet of material of thickness dt and face area A. Assume that σ is the effective area surrounding an atom, such that if the incident particle falls within this area, the nuclear reaction will take place (See Fig. 4.8 in the Fundamentals of Nuclear Physics by Atam P. Arya)

Let there be n target nuclei per unit volume of the sheet. The fractional effective area, f, is given by

$$f = \frac{Total\ effective\ area}{Total\ \ face\ area} = \frac{An\sigma dt}{A} = n\sigma dt$$

CROSS SECTION

The fractional effective area represents the fractional change in the intensity *I* of the beam as it passes through the foil. Thus the change in the intensity *dI* is given by,

$$dI = -f\ I$$

Actually σ is proportional to the probability for a nuclear reaction to take place.

$$-\frac{dI}{I} = n\sigma dt \qquad (1)$$

Assuming $I=I_0$ at t=0 and integrating Eq. 1 we get

$$I = I_0 e^{-n\sigma t} \qquad (2)$$

CROSS SECTION

Because the number of particles N in the beam is proportional to the intensity of the beam, Eq. 2 in terms of the number of particles, can be written as

$$N = N_0 e^{-n\sigma t} \qquad (3)$$

Where N_0 is the number of the particles incident on the foil, and N is the number of particles left after traversing a thickness t of the foil. The unit of the cross section is the barn, where

$$1\,b = 10^{-24}\,cm^2$$

If we are dealing with absorption only, then the term absorption coefficient η, is given by

$$\eta = n\sigma$$

CROSS SECTION

Eq. 3 can be written as,

$$N = N_0 e^{-\eta t} \qquad (4)$$

In the case of $\eta t \ll 1$ (if the foil is sufficiently geometrically thin or if the cross section is sufficiently small)

$$e^{-\eta t} = 1 - \eta t$$

$$N = N_0 (1 - \eta t)$$

Thus the number of particles absorbed while traversing a thickness t, is given by

$$dN = N_0 - N_0 (1 - \eta t) = N_0 \eta t = N_0 n \sigma t \qquad (5)$$

MEAN FREE PATH

To derive an expression for the average distance traveled by the particles, i.e., the mean free path, before they are absorbed or scattered, similar concept that used in calculating the average life can be used.

Multiply the distance x by the number of particles dN absorbed in distance dx at x, integrate it over all x, and divide by the total number of particles. The mean free path is,

$$\overline{x} = \int_0^{N_0} x\, dN \,/ \int_0^{N_0} dN = \int_0^{N_0} x\, dN \,/ N_0 \qquad (6)$$

From Eq. 3, and using x instead of t, we get

$$dN = -n\sigma N_0 e^{-n\sigma x} dx$$

MEAN FREE PATH

Substituting the value dN in Eq. 6 we get,

$$\bar{x} = \int_0^\infty xn\sigma N_0 e^{-\eta x} dx / N_0 = \int_0^\infty xn\sigma e^{-n\sigma x} dx \qquad (7)$$

$$\bar{x} = \frac{1}{\eta} \qquad (8)$$

The absorption mean free path, therefore, is the reciprocal of the absorption coefficient

REACTION RATE

In most cases one would like to know, if a beam of particles is incident on a certain material, what is the *reaction rate*, that is, the number of nuclear reactions that take place in unit time.

Let *v* be the velocity of the particles in a beam having a number density q particles per cm^3. This beam is incident on a foil of thickness *t*, face area *A*, and having n atoms per unit volume.

The reaction rate is given by,

$$\text{Re}\,action\ rate\left(R.R.\right) = qv\left(n\sigma t\right)A\left({}^{1}\!/_{\text{sec}}\right)$$

REACTION RATE

The flux, ϕ, is defined as the number of particles crossing a unit area in a unit time. In this case $\phi = qv$. Also $tA = V$, the volume of the material of the foil. Therefore,

$$R.R. = \phi n \sigma V = \phi \sigma N \qquad (9)$$

Eq. 9 represents the number of events or reactions per second.

DIFFERENTIAL CROSS-SECTION

When the incoming particles interact with the target nuclei, it is not always necessary that only one kind of nuclear reaction take place. One might be interested in knowing the number of particles scattered per second into a solid angle dΩ making an angle θ with the direction of incidence. To make such calculations angular dependent differential cross-section is introduced. The differential cross-section is denoted as σ(θ,ϕ) and defined as the cross-section per unit solid angle.

$$\sigma(\theta,\phi) = \frac{d\sigma}{d\Omega}\left(cross-\mathrm{sec}tion\,/\,steradian\right) \qquad (10)$$

and the total cross-section σ_T become,

$$\sigma_T = \int_\Omega \frac{d\sigma}{d\Omega} d\Omega \qquad (11)$$

DIFFERENTIAL CROSS-SECTION

The value of the solid angle dΩ can be calculated with the help of Fig. 4.9 in the Fundamentals of Nuclear Physics by Atam P. Arya.

$$d\Omega = \frac{area}{(dis\tan ce)^2} = \frac{dA}{r^2} = \frac{(rd\theta)(r\sin\theta d\phi)}{r^2} = \sin\theta d\theta d\phi \qquad (12)$$

The total solid angle is

$$\Omega = \int_{\Omega} d\Omega = \int_0^{2\pi}\int_0^{\pi} \sin\theta d\theta d\phi = 4\pi$$

The fractional solid angle is

$$\frac{d\Omega}{\Omega} = \frac{A}{r^2}\frac{1}{4\pi} = \frac{A}{4\pi r^2}$$

DIFFERENTIAL CROSS-SECTION

The total cross-section σ_T can be found by combining Eq. 11 and Eq. 12.

$$\sigma_T = \int \frac{d\sigma}{d\Omega} d\Omega = \int \frac{d\sigma}{d\Omega} \sin\theta d\theta d\phi \qquad (13)$$

If the differential cross-section has no dependency to ϕ, then the total cross-section is given by

$$\sigma_T = 2\pi \int \frac{d\sigma}{d\Omega} \sin\theta d\theta \qquad (14)$$

where $d\sigma/d\Omega = \sigma(\theta)$, the differential cross section.

REFERENCES

1. Introductory Nuclear Physics. Kenneth S. Krane

2. Fundamentals of Nuclear Physics. Atam. P. Arya