Advanced Quantum Mechanics II

PEN425

Dr. H.Ozgur Cildiroglu

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Week 7

Commutation Relations

Wave Functions in Position and Momentum Space

Recitation for MT

Ankara University | Physics Engineering Department

Dr. H. Ozgur Cildiroglu

Commutation Relations

$$[X_i, P_j] = i\hbar\delta_{ij}$$
$$[P_i, P_j] = 0$$
$$[X_i, X_j] = 0$$

- $[A,A] = 0$
- $[A,B] = -[B,A]$
- $[A,cI] = 0$
- $[A+B,C] = [A,C] + [B,C]$
- $[A,BC] = B[A,C] + [A,B]C$
- $[AB,C] = A[B,C] + [A,C]B$
- $[A,[B,C]] + [C,[A,B]] + [B,[C,A]] = 0$

Wave Functions in Position and Momentum Space

- $|\psi\rangle$: Arbitrary State
- $X|x\rangle = x|x\rangle$
  - Complete ; $I = \int dx|x\rangle\langle x|$
  - Orthonormal ; $\langle x|x'\rangle = \delta(x - x')$

$$|\psi\rangle = \left(\int dx|x\rangle\langle x|\right)|\psi\rangle = \int dx|x\rangle\langle x|\psi\rangle$$

$\underbrace{\langle x|\psi\rangle}_{Coordinates\ \psi(x)}$

$$\int dx|\langle x|\psi\rangle|^2 \rightarrow \langle\psi_1|\psi_2\rangle = \int dx\langle\psi_1|x\rangle\langle x|\psi_2\rangle = \int dx\langle x|\psi_1\rangle^*\langle x|\psi_2\rangle$$

$$\langle\psi_1|\psi_2\rangle = \int dx\psi_1(x)^*\psi_2(x)$$

Momentum Space Wave Function

- $P|p\rangle = p|p\rangle$
  - Complete ; $I = \int dp|p\rangle\langle p|$
  - Orthonormal ; $\langle p|p'\rangle = \delta(p - p')$

$$|\psi\rangle = \left(\int dp|p\rangle\langle p|\right)|\psi\rangle = \int dp|p\rangle\underbrace{\langle p|\,\psi\rangle}_{\psi(\mathrm{p})}$$

$$|\psi\rangle = \int dp|p\rangle\psi(\mathrm{p})$$

$\psi(\mathrm{x}) \leftrightarrow \psi(\mathrm{p})$

$\{|x\rangle\} \leftrightarrow \{|p\rangle\}$

$$|\psi\rangle = \int dx|x\rangle\psi(\mathrm{x})$$

$$\langle p| \ / \ |\psi\rangle = \int dx|x\rangle\psi(\mathrm{x})$$

$$\langle p|\psi\rangle = \int dx\langle p|x\rangle\psi(\mathrm{x})$$

$$\psi(\mathrm{p}) = \int dx\langle p|x\rangle\psi(\mathrm{x})$$

$$|\psi\rangle = \int dp|p\rangle\psi(\mathrm{p})$$

$$\langle x| \ / \ |\psi\rangle = \int dp|p\rangle\psi(\mathrm{p})$$

$$\langle x|\psi\rangle = \int dp\langle x|p\rangle\psi(\mathrm{p})$$

$$\psi(\mathrm{x}) = \int dp\langle x|p\rangle\psi(\mathrm{p})$$

$$\langle p|x\rangle = \langle x|p\rangle^*$$

To determine $\langle p|x\rangle$:

$$\langle p| \ / \ P|x\rangle = i\hbar\frac{d}{dx}|x\rangle$$

$$\langle p|P|x\rangle = i\hbar\langle p|\frac{d}{dx}|x\rangle$$

$$p\,\langle p|x\rangle = i\hbar\frac{d}{dx}\langle p|x\rangle$$

$$\frac{d}{dx}\langle p|x\rangle = -\frac{i}{\hbar}p\,\langle p|x\rangle$$

$$\langle p|x\rangle = \mathrm{N}\exp(-\frac{i}{\hbar}px)$$

$$\langle\psi|\psi\rangle = 1$$

$$|\psi\rangle = \int dx|x\rangle\psi(\mathrm{x})$$

$$\langle\psi|\psi\rangle \rightarrow \int dx|\psi(x)|^2 = 1$$

$$1 = \int dx\,\langle p|x\rangle\langle x|p\rangle$$

$$N = \frac{1}{\sqrt{2\pi\hbar}}$$

$$\psi(\mathrm{p}) = \int dx \langle p|x\rangle \psi(\mathrm{x}) = \frac{1}{\sqrt{2\pi\hbar}} \int dx \, \exp(-\frac{i}{\hbar}\boldsymbol{p}.\boldsymbol{x})\, \psi(\mathrm{x})$$

$$\psi(\mathrm{x}) = \int dp \langle x|p\rangle \psi(\mathrm{p}) = \frac{1}{\sqrt{2\pi\hbar}} \int dx \, \exp(\frac{i}{\hbar}\boldsymbol{p}.\boldsymbol{x})\, \psi(\mathrm{p})$$