Advanced Quantum Mechanics II

PEN425

Dr. H.Ozgur Cildiroglu

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Week 11

Topological Phases

Aharonov-Bohm Phase

Aharonov-Casher Phase

He-Mc-Kellar-Wilkens Phase

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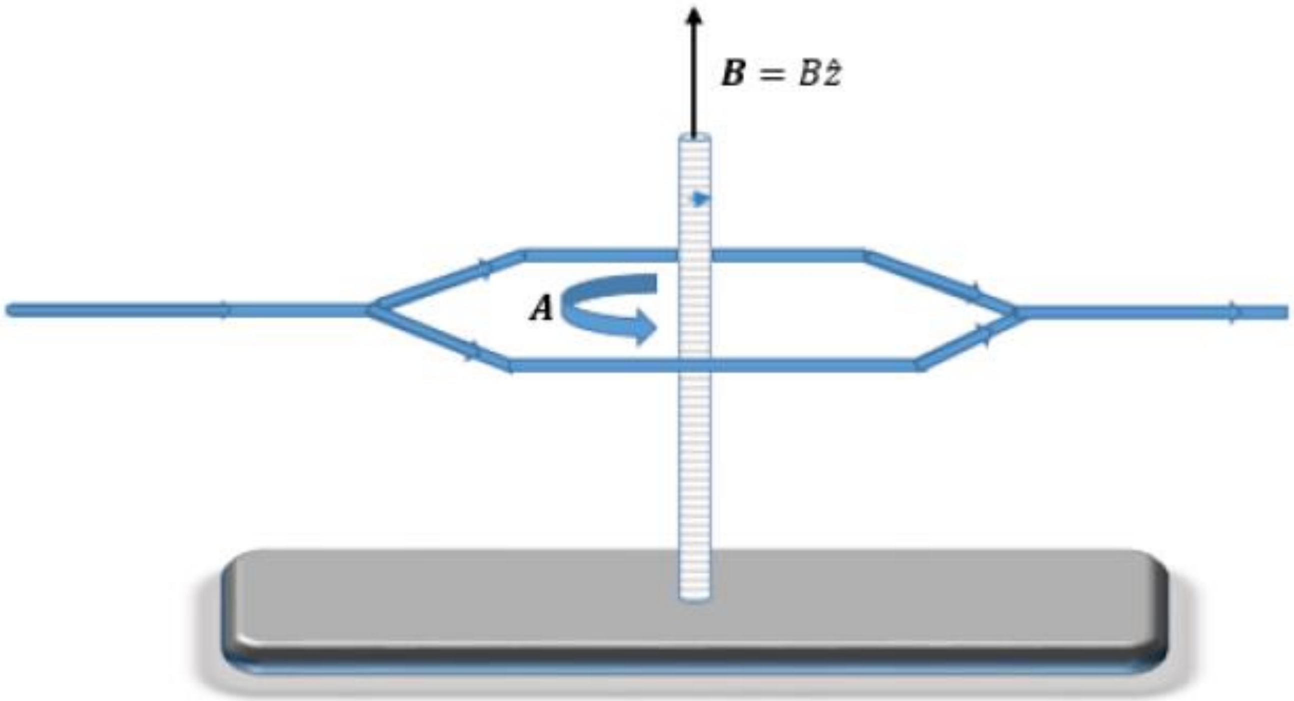
Topological Phases

- Classical Force X
- Multiply-Connected Region
- Vector Potential
- Controlled by Physical Quantity in Singular Region

Aharonov - Bohm Phase



$$\psi' = e^{-ie\oint \mathbf{A}.d\mathbf{l}}\psi$$

$$\mathbf{A}' = \mathbf{A} + \nabla g(\mathbf{x}, t).$$

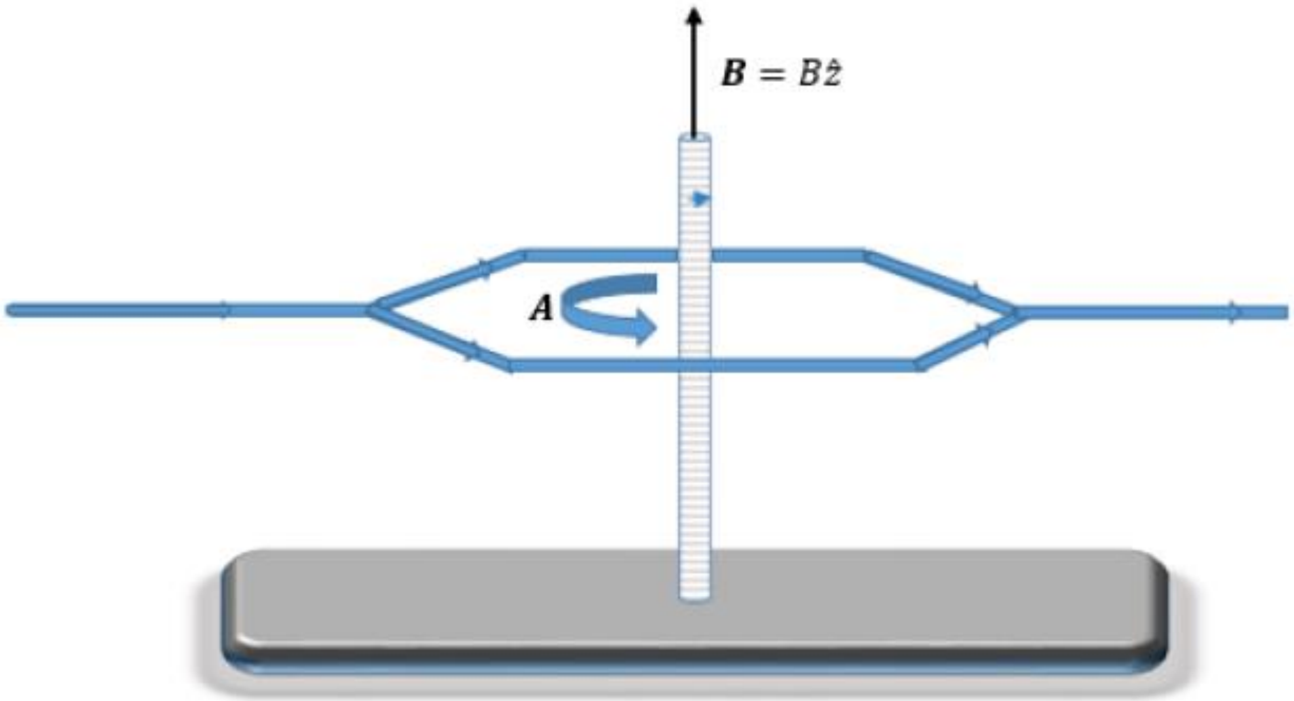
$$\mathbf{A}' = 0$$

$$\mathbf{A} = -\nabla g(\mathbf{x}, t)$$

$$g(\mathbf{x}, t) = -\int^{\mathbf{x}} \mathbf{A}\left(\mathbf{x}'\right) . d\mathbf{x}'$$

$$\psi \to \psi' = U\psi \qquad U = e^{ieg(\mathbf{x},t)}$$

Aharonov - Bohm Phase



$$\mathbf{F}_{Lorentz} = q\left[\mathbf{E} + \mathbf{v} \times \mathbf{B}\right]$$

$$\mathbf{A}' = \mathbf{A} + \nabla g(\mathbf{x}, t).$$

$$\mathbf{A}' = 0$$

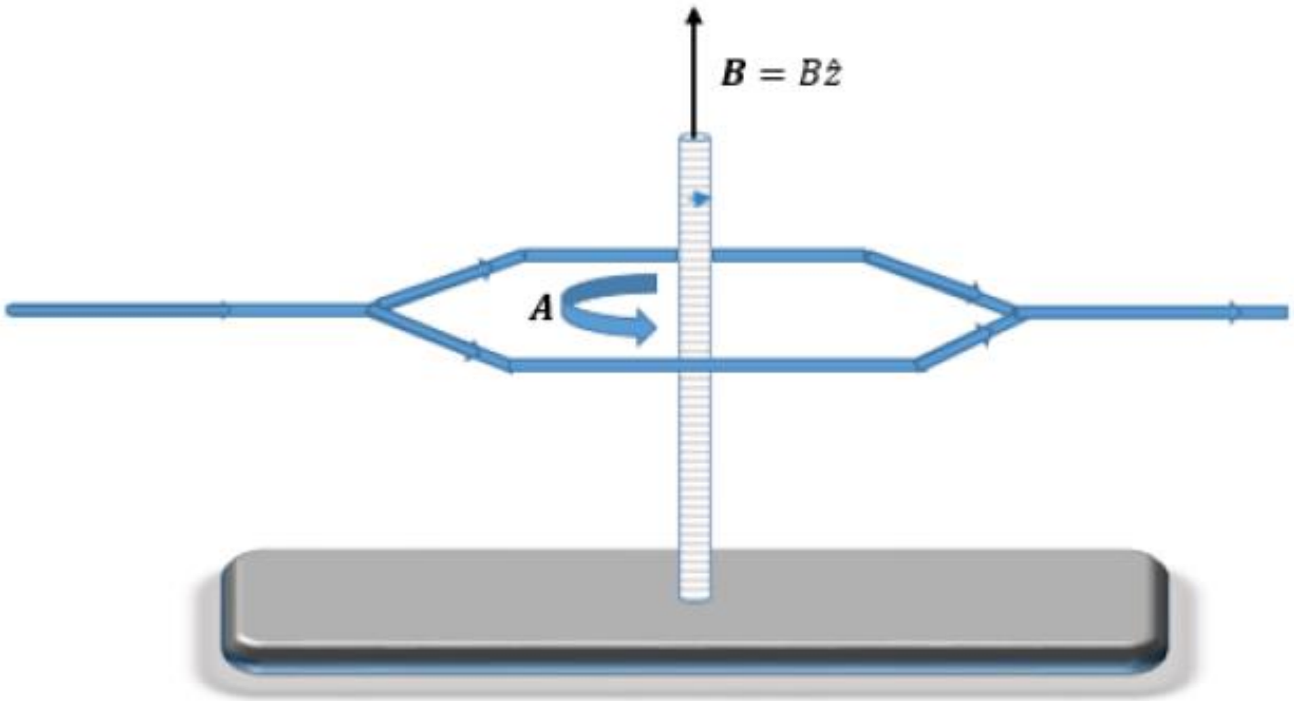
$$\mathbf{A} = -\nabla g(\mathbf{x}, t)$$

$$g(\mathbf{x}, t) = -\int^{\mathbf{x}} \mathbf{A}\left(\mathbf{x}'\right) . d\mathbf{x}'$$

$$\psi \to \psi' = U\psi \qquad U = e^{ieg(\mathbf{x},t)}$$

$$\psi' = e^{-ie\int^{x} \mathbf{A}.d\mathbf{l}}\psi$$

Aharonov - Bohm Phase



$$\psi' = e^{-ie\int^{x}\mathbf{A}.d\mathbf{l}}\psi$$

$$\begin{aligned}\psi &= \psi_1 + \psi_2 \\ &= U_{(1)}\psi_1^{'} + U_{(2)}\psi_2^{'}.\end{aligned}$$

$$\psi = e^{-ie\int_{l_1}\mathbf{A}.d\mathbf{l}}\psi_1^{'} + e^{-ie\int_{l_2}\mathbf{A}.d\mathbf{l}}\psi_2^{'}$$

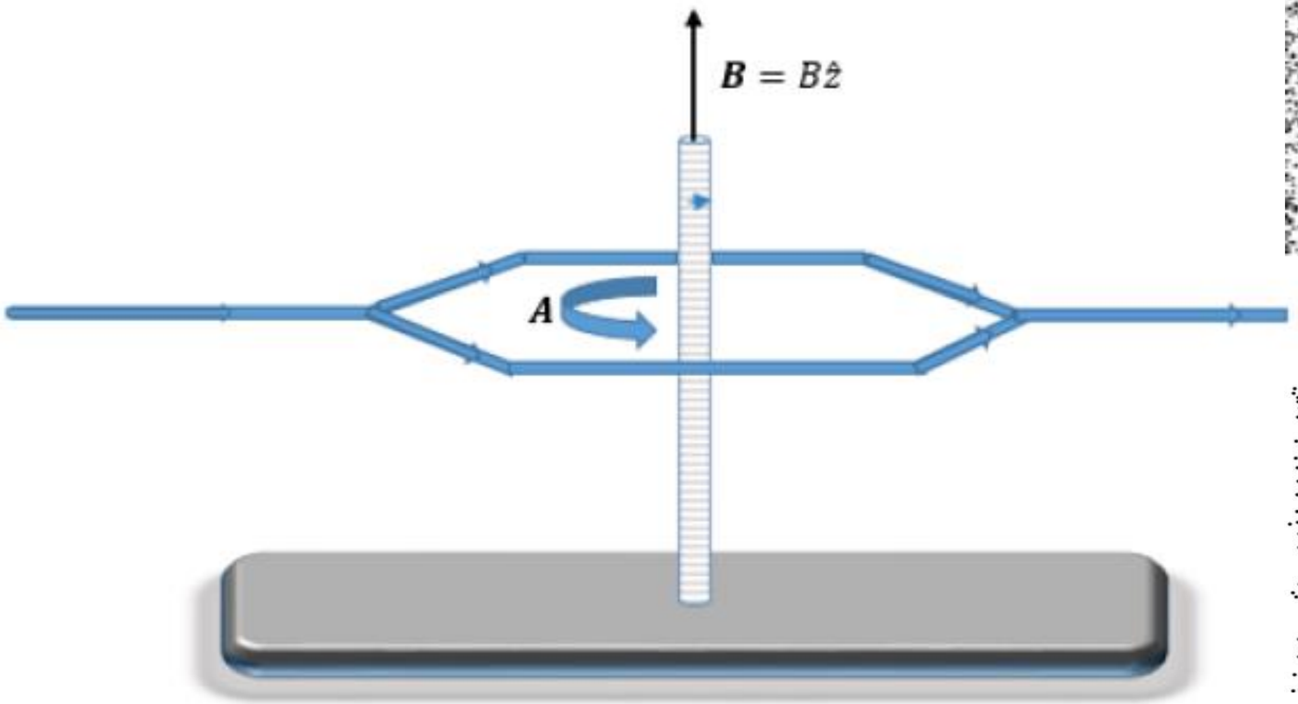
$$\psi = e^{-ie\int_{l_1}\mathbf{A}.d\mathbf{l}}\left[\psi_1^{'} + e^{ie\left(\int_{l_1}\mathbf{A}.d\mathbf{l} - \int_{l_2}\mathbf{A}.d\mathbf{l}\right)}\psi_2^{'}\right]$$

$$\int_{l_1}\mathbf{A}.d\mathbf{l} - \int_{l_2}\mathbf{A}.d\mathbf{l} = \int_{l_1}\mathbf{A}.d\mathbf{l} + \int_{l_2^{'}}\mathbf{A}.d\mathbf{l} = \oint\mathbf{A}.d\mathbf{l}$$

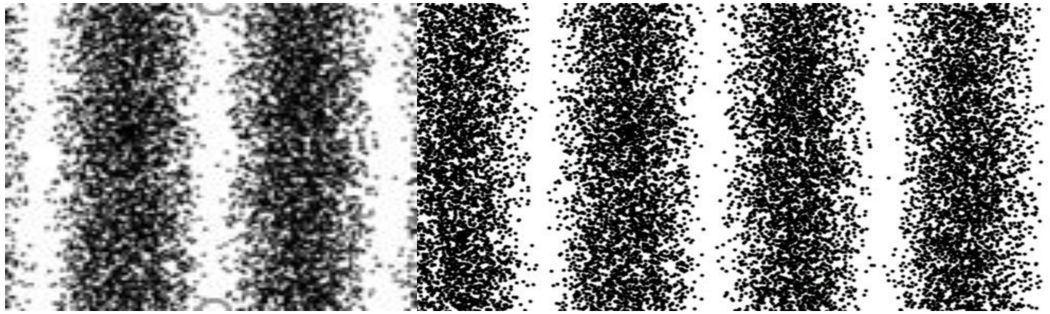
$$\psi = e^{-ie\int_{l_1}\mathbf{A}.d\mathbf{l}}\left[\psi_1^{'} + e^{ie\oint\mathbf{A}.d\mathbf{l}}\psi_2^{'}\right]$$

$$\delta = e\oint\mathbf{A}.d\mathbf{l}$$

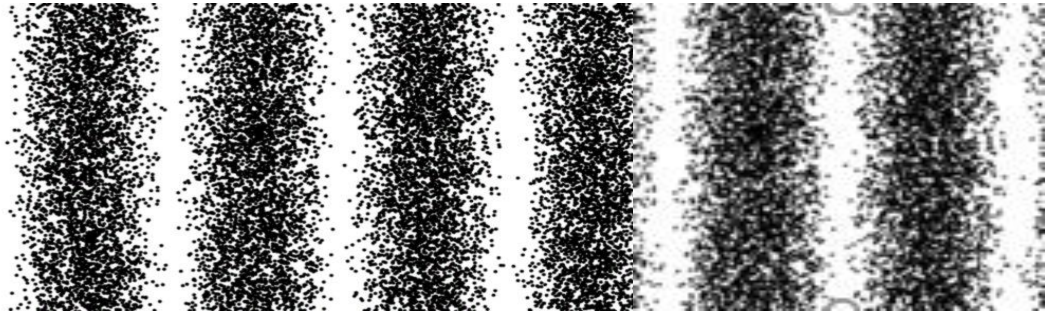
Aharonov - Bohm Phase



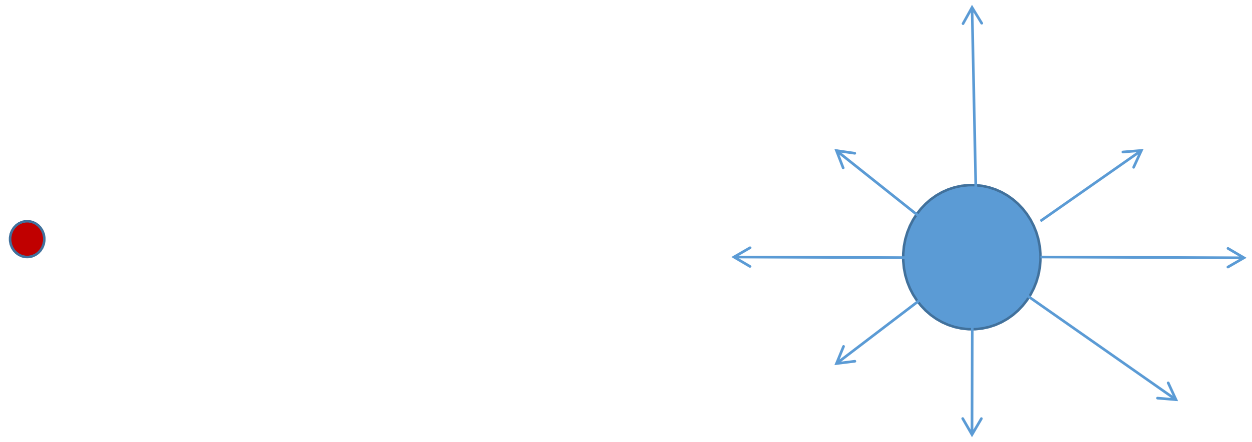
$B = 0$



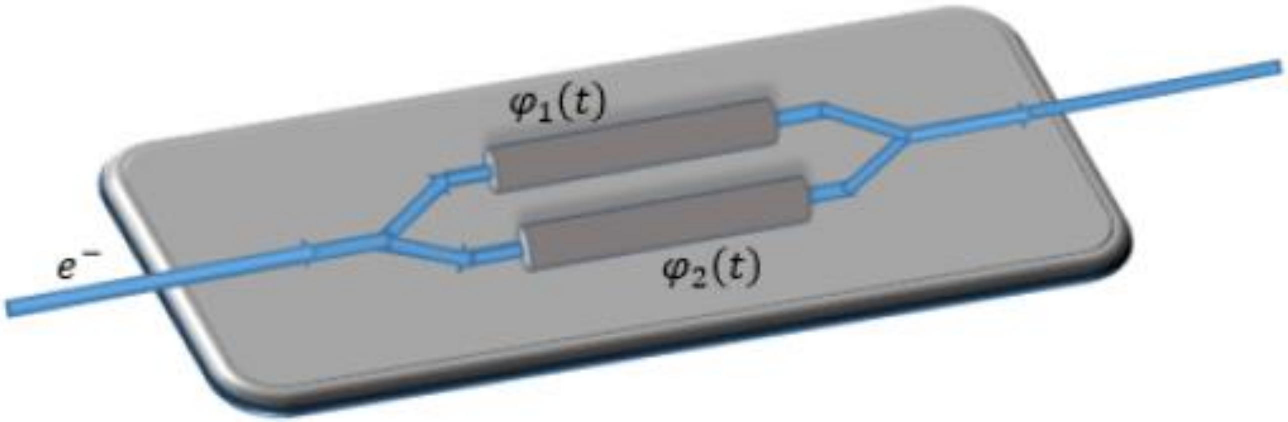
$B \neq 0$



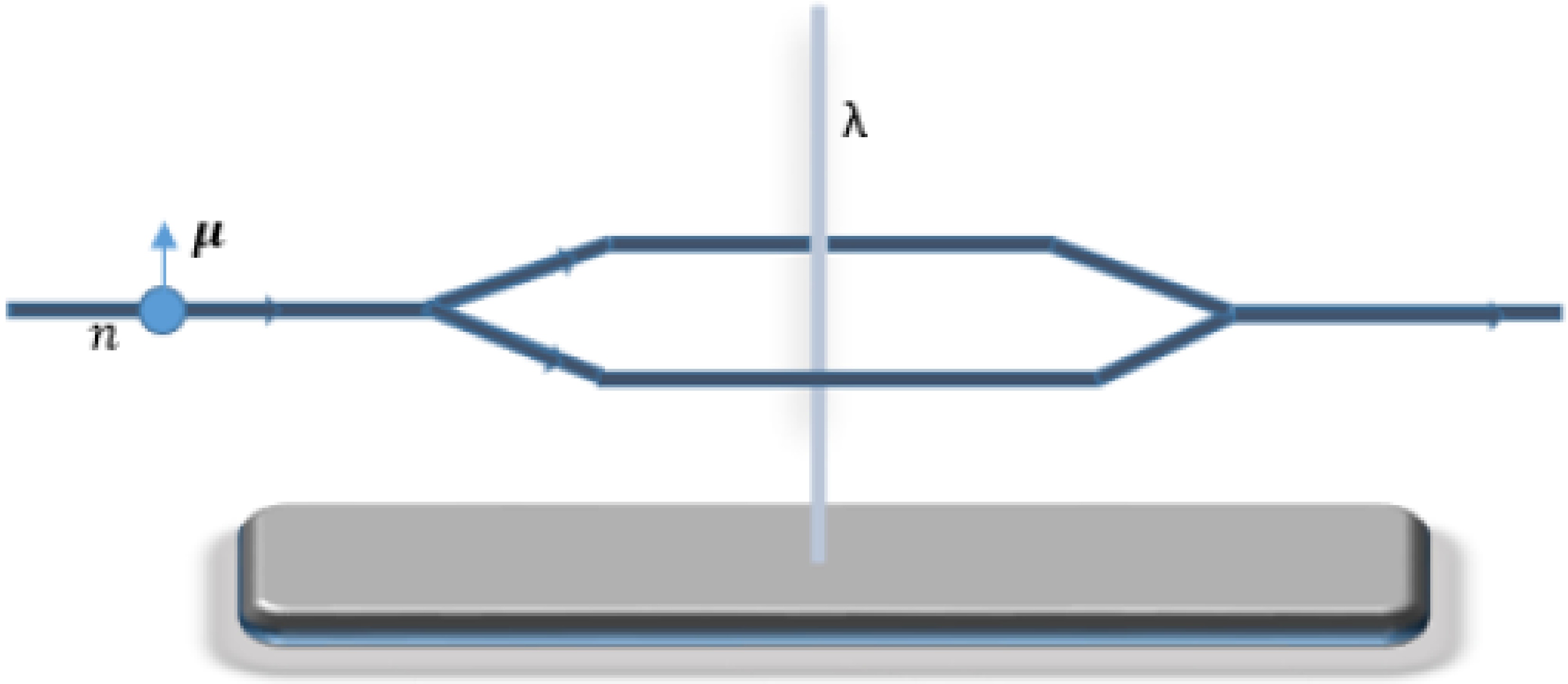
$$\psi' = e^{-ie\Phi}\psi.$$



Scalar AB Effect

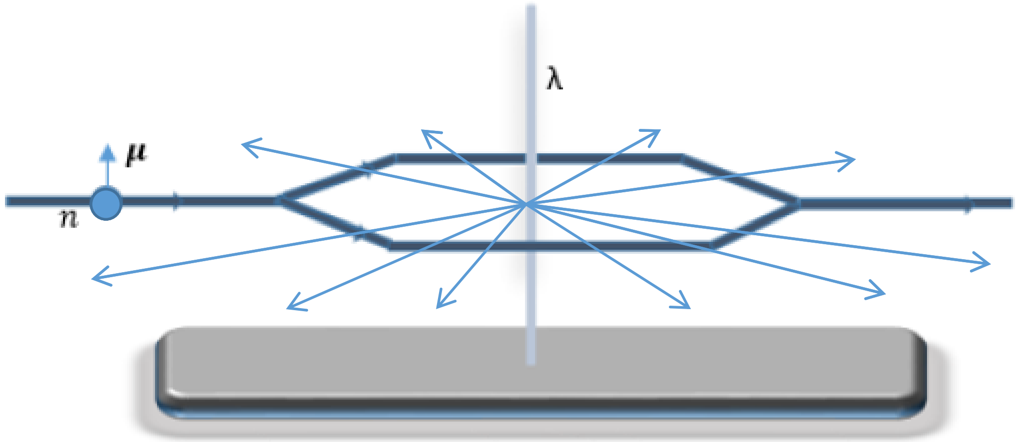


Recall: Dynamical Phases



Aharonov-Casher Effect

Classical Force?



$$L = \frac{mv^2}{2} + \frac{MV^2}{2} + e\mathbf{A}.\mathbf{v}$$

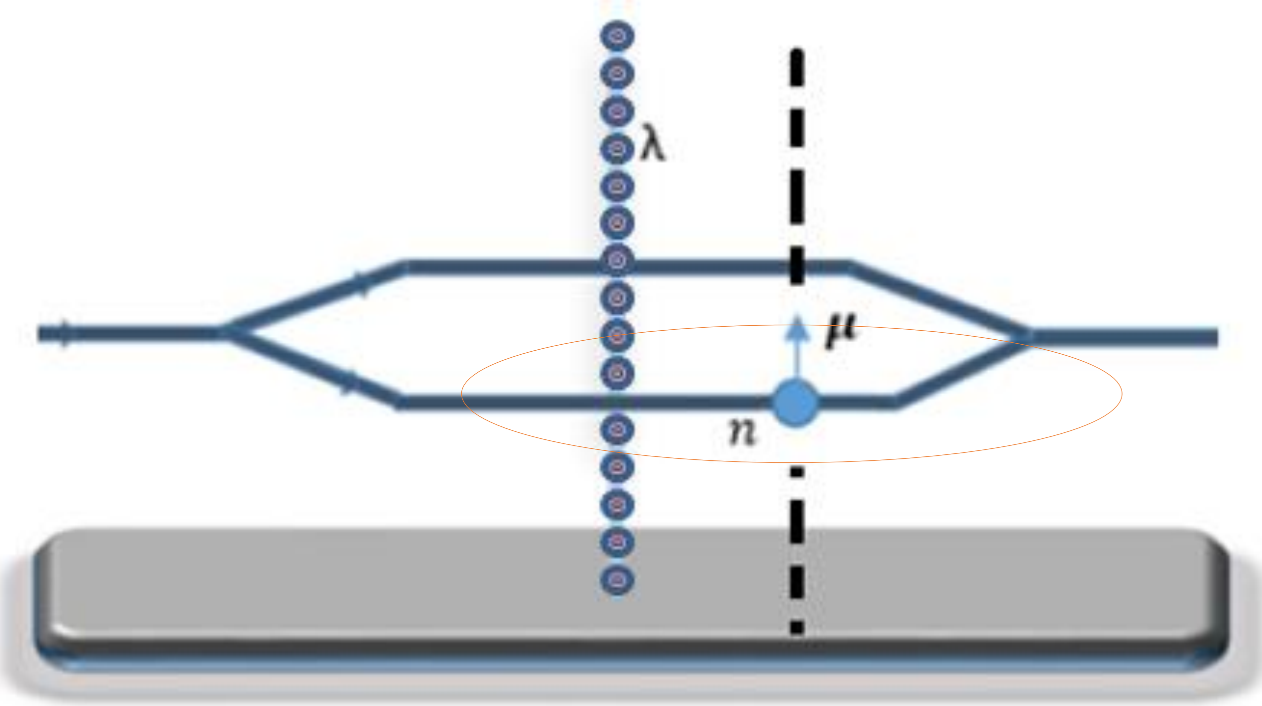
$$\frac{d}{dt}\left(\frac{\partial L}{\partial v_i}\right) = \frac{\partial L}{\partial r_i}$$

$$m\dot{v}_i = e\varepsilon_{ijk}v_jB_k + e\left[\partial_j A_i\right]V_j$$

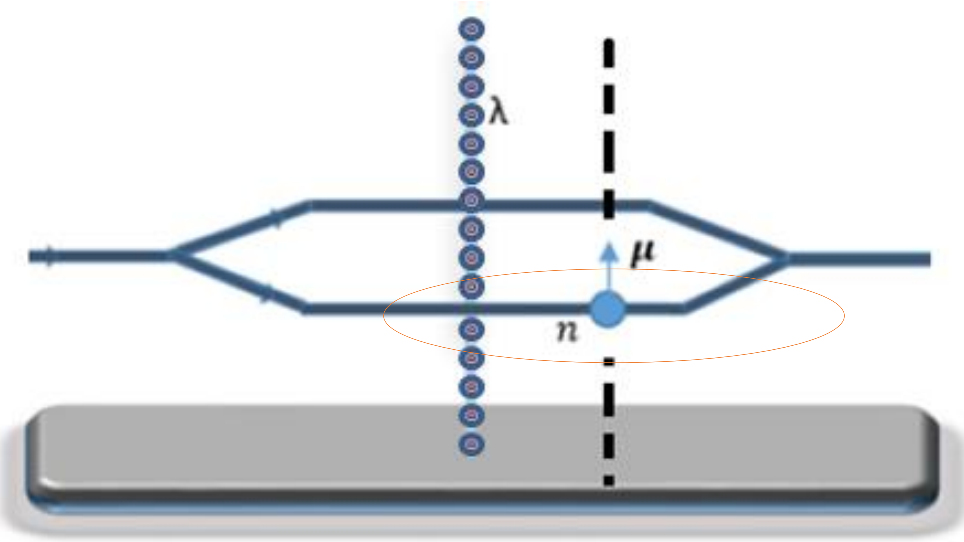
$$m\dot{v}_i = e\left[\partial_j A_i\right]V_j \neq 0.$$

$$L = \frac{m\mathbf{v}^2}{2} + \frac{M\mathbf{V}^2}{2} - e\mathbf{A}.(\mathbf{v} - \mathbf{V})$$

$$\begin{aligned} m\dot{v}_i &= e\frac{\partial A_i}{\partial r_j}(v_j - V_j) - e\frac{\partial A_j}{\partial r_i}(v_j - V_j) \\ &= e\left[\frac{\partial A_i}{\partial r_j} - \frac{\partial A_j}{\partial r_i}\right](v_j - V_j) \\ &= -e\left[\partial_i A_j - \partial_j A_i\right](v - V)_j \\ &= -e\varepsilon_{ijk}(v - V)_j B_k = 0. \end{aligned}$$



λ
μ
n



$$L = \frac{MV^2}{2} + e\mathbf{A}.\mathbf{V}$$

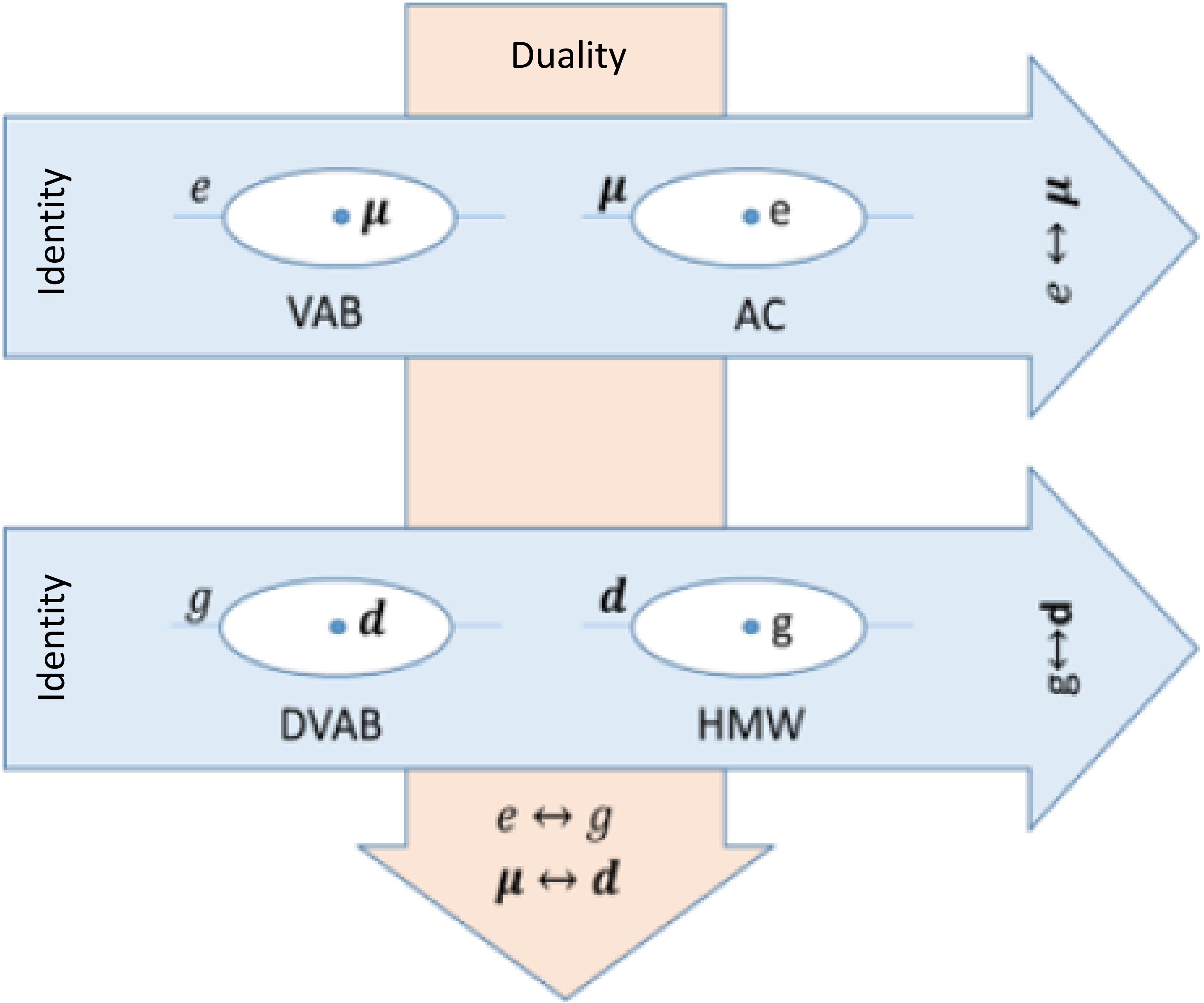
$$H = \dot{\mathbf{x}}.\mathbf{p} - L$$

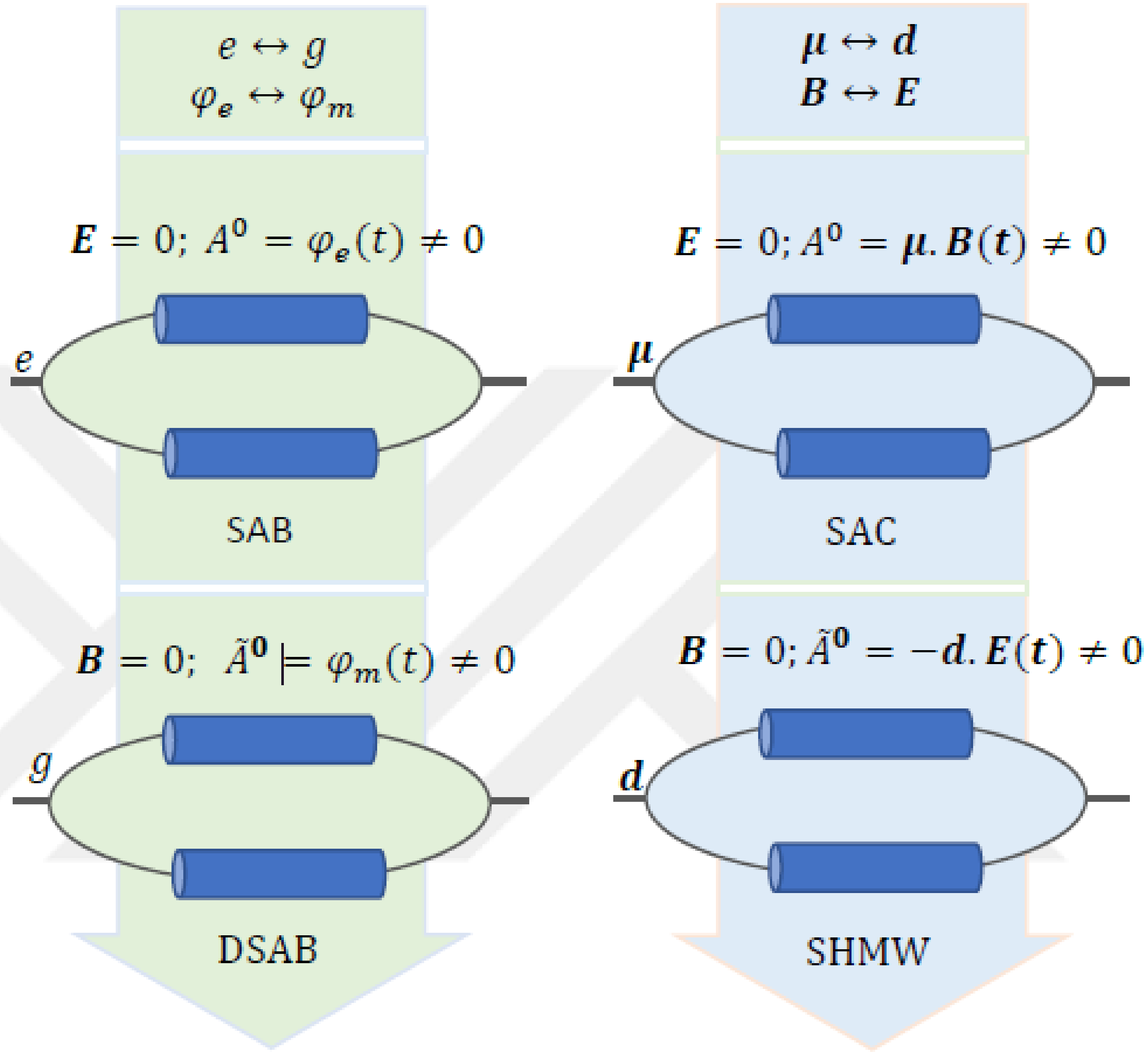
$$H = \frac{[\mathbf{p} - e\mathbf{A}]^2}{2m}$$

$$\mathbf{p} = \frac{\partial L}{\partial \dot{x}} = M\mathbf{V} + e\mathbf{A}$$

$$\delta = \oint \mathbf{A}.d\mathbf{R}$$

$$\psi' = e^{-ie\oint \mathbf{A}.d\mathbf{R}}\psi$$


Duality
Identity
e
• μ
VAB
μ
• e
AC
e ↔ μ
Identity
g
• d
DVAB
d
• g
HMW
g↔d
e ↔ g
μ ↔ d


$e \leftrightarrow g$
$\varphi_e \leftrightarrow \varphi_m$
$\boldsymbol{\mu} \leftrightarrow \boldsymbol{d}$
$\boldsymbol{B} \leftrightarrow \boldsymbol{E}$
$\boldsymbol{E} = 0;\ A^0 = \varphi_e(t) \neq 0$
e
SAB
$\boldsymbol{E} = 0;\ A^0 = \boldsymbol{\mu}.\boldsymbol{B}(\boldsymbol{t}) \neq 0$
$\boldsymbol{\mu}$
SAC
$\boldsymbol{B} = 0;\ \tilde{A}^0 = \varphi_m(t) \neq 0$
g
DSAB
$\boldsymbol{B} = 0;\ \tilde{A}^0 = -\boldsymbol{d}.\boldsymbol{E}(\boldsymbol{t}) \neq 0$
$\boldsymbol{d}$
SHMW