Advanced Quantum Mechanics II

PEN425

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Week 12

Schrödinger Picture

Heisenberg Picture

Dirac Picture

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Early Questions

- Interpretation of Quantum Mechanics
  - Mean?
  - Calculations?
  - State?
  - Measurement?
  - Evolution?
  -
  -

3 Different Answers

Pictures of Quantum Mechanics

Evolution	Picture		
of:	Heisenberg	Interaction	Schrödinger
Ket state	constant	$|\psi_I(t)\rangle = e^{iH_{0,S}\ t/\hbar}|\psi_S(t)\rangle$	$|\psi_S(t)\rangle = e^{-iH_S\ t/\hbar}|\psi_S(0)\rangle$
Observable	$A_H(t) = e^{iH_S\ t/\hbar}A_S e^{-iH_S\ t/\hbar}$	$A_I(t) = e^{iH_{0,S}\ t/\hbar}A_S e^{-iH_{0,S}\ t/\hbar}$	constant
Density matrix	constant	$\rho_I(t) = e^{iH_{0,S}\ t/\hbar}\rho_S(t)e^{-iH_{0,S}\ t/\hbar}$	$\rho_S(t) = e^{-iH_S\ t/\hbar}\rho_S(0)e^{iH_S\ t/\hbar}$

3 Different Answers

Pictures of Quantum Mechanics

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Table: https://i.stack.imgur.com/F6TZz.png

Measurement: Physical quantities

Classical mechanics: Time dependent variables (Vectors, Real Space)

Quantum mechanics: Operators, State functions (Linear Complex Vector Space)

Schrödinger Picture: Time-independent operators,
Time-dependent States

$$i\frac{\partial}{\partial t}\psi_S(\mathbf{x},t) = H^{op}\psi_S(\mathbf{x},t)$$

Expectation value: $\langle A(t)\rangle = \langle \psi_s(t)|O_s|\psi_s(t)\rangle$

Heisenberg Picture: Time dependent operators,
Time-independent States

$$i\frac{\partial}{\partial t}\psi_H(\mathbf{x},t) = 0$$

Time evolution operator $HU = i\frac{\partial}{\partial t}U$

Operators in Heisenberg Picture

$$\langle \hat{O}(t) \rangle = \langle \Psi_S^+(t) | \hat{O}_S | \Psi_S(t) \rangle$$

$$\Psi_S(t) \rightarrow \hat{U}(t,0)\Psi_H(0) \quad \Psi_S^+(t) \rightarrow \left(\hat{U}(t,0)\Psi_H(0)\right)^+ = \Psi_H^+(0)\hat{U}^+(t)$$

$$\langle \hat{O}(t) \rangle = \langle \Psi_H^+(0)\hat{U}^+(t,0) | \hat{O}_S | \hat{U}(t)\Psi_H(0) \rangle = \langle \Psi_H^+(0) | \hat{U}^+(t)\hat{O}_S\hat{U}(t) | \Psi_H(0) \rangle$$

$$= \langle \Psi_H^+(0) | \hat{O}_H(t) | \Psi_H(0) \rangle$$

$$\hat{O}_H(t) = \hat{U}^+(t)\hat{O}_S\hat{U}(t) = e^{i\hat{H}t}\hat{O}_S e^{-i\hat{H}t}$$

https://www.slideserve.com/avel/schr-dinger-heisenberg-interaction-pictures

Heisenberg EoM

$$\frac{\partial}{\partial t}\hat{O}_H(t) = \frac{\partial}{\partial t}\left(\hat{U}^+(t)\hat{O}_S\hat{U}(t)\right) = \frac{\partial}{\partial t}\hat{U}^+(t)\hat{O}_S\hat{U}(t) + \hat{U}^+(t)\hat{O}_S\frac{\partial}{\partial t}\hat{U}(t)$$

$$\frac{\partial}{\partial t}\hat{U} = -i\hat{H}\hat{U} \qquad \frac{\partial}{\partial t}\hat{U}^+ = +i\left(\hat{H}\hat{U}\right)^+ = +i\hat{U}^+\hat{H}$$

$$\frac{\partial}{\partial t}\hat{O}_H(t) = i\left(\hat{U}^+\hat{H}\hat{O}_S\hat{U} - \hat{U}^+\hat{O}_S\hat{H}\hat{U}\right) = i\left(\hat{H}\hat{U}^+\hat{O}_S\hat{U} - \hat{U}^+\hat{O}_S\hat{U}\hat{H}\right) = i\left(\hat{H}\hat{O}_H - \hat{O}_H\hat{H}\right)$$

$$= i\left[\hat{H}, \hat{O}_H\right]$$

$$i\frac{\partial}{\partial t}\hat{O}_H(t) = \left[\hat{O}_H, \hat{H}\right]$$

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Operators in Dirac Picture

$$\Psi_I(t) = e^{+i\hat{H}_o t}\Psi_S(t)$$

$$\Psi_S(t) = e^{-i(\hat{H}_1 + \hat{H}_o)t}\Psi_S(0)$$

EoM in Dirac Picture

$$i\frac{\partial}{\partial t}\Psi_I(t) = i(i\hat{H}_o)\Psi_I(t) + e^{+i\hat{H}_o t} i\frac{\partial}{\partial t}\Psi_S(t)$$

$$= -\hat{H}_o\Psi_I(t) + e^{+i\hat{H}_o t}(\hat{H}_o + \hat{H}_1)\Psi_S(t)$$

$$= -\hat{H}_o\Psi_I(t) + e^{+i\hat{H}_o t}(\hat{H}_o + \hat{H}_1)e^{-i\hat{H}_o t}\Psi_I(t)$$

$$= e^{+i\hat{H}_o t}(-\hat{H}_o + \hat{H}_o + \hat{H}_1)e^{-i\hat{H}_o t}\Psi_I(t)$$

$$= e^{+i\hat{H}_o t}\hat{H}_1 e^{-i\hat{H}_o t}\Psi_I(t)$$

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- <u>Operators in interaction picture</u>

$$i\frac{\partial}{\partial t}\Psi_I(t)=\hat{H}_1(t)\Psi_I(t) \qquad \hat{H}_1(t)=e^{+i\hat{H}_o t}\hat{H}_1 e^{-i\hat{H}_o t}$$

<u>Recitation:</u> Prove that $i\frac{\partial}{\partial t}\hat{O}_I(t)=\left[\hat{O}_I,\hat{H}_o\right]$

- <u>Schrödinger picture</u>

$$i\frac{\partial}{\partial t}\Psi_S(t)=\left(\hat{H}_o+\hat{H}_1\right)\Psi_S(t) \qquad \hat{O}_S=\hat{O}_S \qquad i\frac{\partial}{\partial t}\hat{O}_S=0$$

- <u>Interaction picture</u>

$$i\frac{\partial}{\partial t}\Psi_I(t)=\hat{H}_1(t)\Psi_I(t) \qquad \hat{O}_I(t)=e^{+i\hat{H}_o t}\hat{O}_S e^{-i\hat{H}_o t} \qquad i\frac{\partial}{\partial t}\hat{O}_I(t)=\left[\hat{O}_I,\hat{H}_o\right]$$

- <u>Heisenberg picture</u>

$$i\frac{\partial}{\partial t}\Psi_H(t)=0 \qquad \hat{O}_H(t)=e^{+i\hat{H}t}\hat{O}_S e^{-i\hat{H}t} \qquad i\frac{\partial}{\partial t}\hat{O}_H(t)=\left[\hat{O}_H,\hat{H}_H\right]$$

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