

# NUCLEAR PHYSICS AND APPLICATIONS

Prof. Dr. Turan OLĞAR

Ankara University, Faculty of Engineering  
Department of Physics Engineering

**Question 1:** Two radioactive sources each have activities of  $1.0 \mu\text{Ci}$  at  $t=0$ . Their half-lives are,  $1.0\text{s}$  and  $1.0 \text{ d}$ . A) How many radioactive nuclei are present at  $t=0$  in each source? b) How many decay between  $t=0$  and  $t=1\text{h}$ ?

a) Decay constant

$$\lambda_1 = \frac{0.693}{1 \text{ s}} = 0.693 \text{ s}^{-1} \qquad \lambda_2 = \frac{0.693}{24(60)(60\text{s})} = 8.02(10^{-6}) \text{ s}^{-1}$$

Number of radioactive nuclei at  $t = 0$

$$N_{10} = \frac{A_{10}}{\lambda_1} = \frac{1.0(10^{-6})(3.7 \times 10^{10})}{0.693} = 5.34(10^4)$$

$$N_{20} = \frac{A_{20}}{\lambda_2} = \frac{1.0(10^{-6})(3.7 \times 10^{10})}{8.02(10^{-6})} = 4.61(10^9)$$

b)

$$\Delta N_1 = N_{10} (1 - e^{-\lambda_1 t}) = 5.34(10^4) \left[ 1 - e^{-0.693(3600)} \right] = 5.34(10^4)$$

$$\Delta N_2 = N_{20} (1 - e^{-\lambda_2 t}) = 4.61(10^9) \left[ 1 - e^{-8.02(10^{-6})(3600)} \right] = 1.31(10^8)$$

**Question 2:** The decay chain  $^{139}\text{Cs} \rightarrow ^{139}\text{Ba} \rightarrow ^{139}\text{La}$  is observed from an initially pure sample of 1 mCi of  $^{139}\text{Cs}$ . The half-lives are  $^{139}\text{Cs}$ , 9.5 min; ,  $^{139}\text{Ba}$ , 82.9 min,  $^{139}\text{La}$ , stable. What is the maximum  $^{139}\text{Ba}$  activity and when it does it occur?

Lets denote  $^{139}\text{Cs}$  by 1,  $^{139}\text{Ba}$  by 2 and  $^{139}\text{La}$  by 3.

$$\lambda_1 = \frac{0.693}{9.5 \text{ min}} = 7.30 (10^{-2}) \text{ min}^{-1} \qquad \lambda_2 = \frac{0.693}{82.9 \text{ min}} = 8.36 (10^{-3}) \text{ min}^{-1}$$

$$A_{10} = 1 \text{ mCi} = 3.7 \times 10^7 \text{ decay / s}$$

$$N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10} (e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

$$A_2(t) = \lambda_2 N_2(t) = \frac{\lambda_1 \lambda_2 N_{10}}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) = A_{10} \frac{\lambda_2}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

When  $A_2(t)$  is maximum

$$\left. \frac{dA_2(t)}{dt} \right|_{t=t_0} = 0$$

**Question 2 :** The decay chain  $^{139}\text{Cs} \rightarrow ^{139}\text{Ba} \rightarrow ^{139}\text{La}$  is observed from an initially pure sample of 1 mCi of  $^{139}\text{Cs}$ . The half-lives are  $^{139}\text{Cs}$ , 9.5 min; ,  $^{139}\text{Ba}$ , 82.9 min,  $^{139}\text{La}$ , stable. What is the maximum  $^{139}\text{Ba}$  activity and when it does it occur?

$$\lambda_1 e^{-\lambda_1 t_0} - \lambda_2 e^{-\lambda_2 t_0} = 0 \Rightarrow t_0 = \frac{1}{(\lambda_2 - \lambda_1)} \ln \left( \frac{\lambda_2}{\lambda_1} \right)$$

$$t_0 = \frac{1}{8.36(10^{-3}) - 7.30(10^{-2})} \ln \left( \frac{8.36(10^{-3})}{7.30(10^{-2})} \right) = 33.5 \text{ min}$$

$$(A_2)_{\max} = A_{10} \left( \frac{\lambda_2}{\lambda_2 - \lambda_1} \right) (e^{-\lambda_1 t_0} - e^{-\lambda_2 t_0}) = 8.66(10^{-5}) \text{ Ci} = 86.6 \mu\text{Ci}$$

**Question 3: a)** Calculate the Q value and the threshold kinetic energy of the endoergic reaction  $p + {}_1H^3 \rightarrow n + {}_2He^3$

b) If the energy of the incident proton is 3 MeV and the scattering angle of the exiting neutron is  $90^\circ$ , find the energies of the neutron and  ${}_2He^3$  (The target nucleus is initially stationary).

$$m({}_3H^1) = 3.016049u, \quad m({}_2He^3) = 3.016029u, \quad m_p = 1.007825u, \quad m_n = 1.008665u$$

$$\begin{aligned} Q &= \left[ m_p + m({}_1H^3) - m_n - m({}_2He^3) \right] c^2 \\ &= [1.007825 + 3.016049 - 1.008665 - 3.016029] 931.5 \text{ MeV} \\ &= -0.76383 \text{ MeV} \end{aligned}$$

Since  $Q < 0$ , it is an endothermic reaction and an externally energized reaction. Threshold energy is

$$E_{\text{Threshold}} = |-Q| \left( 1 + \frac{m_x}{M_x} \right) = (0.76383) \left( 1 + \frac{1.007825u}{3.016049u} \right) = 1.019069 \text{ MeV}$$

**Question 3:** a) Calculate the Q value and the threshold kinetic energy of the endoergic reaction  $p + {}_1H^3 \rightarrow n + {}_2He^3$

b) If the energy of the incident proton is 3 MeV and the scattering angle of the exiting neutron is  $90^\circ$ , find the energies of the neutron and  ${}_2He^3$  (The target nucleus is initially stationary).

$$m({}_3H^1) = 3.016049u, \quad m({}_2He^3) = 3.016029u, \quad m_p = 1.007825u, \quad m_n = 1.008665u$$

$$\sqrt{K_y} = \frac{\sqrt{m_x m_y K_x} \cos \theta \pm \left\{ (m_x m_y K_x \cos^2 \theta) + (M_Y + m_y) [K_x (M_Y - m_x) + M_Y Q] \right\}^{1/2}}{(M_Y + m_y)}$$

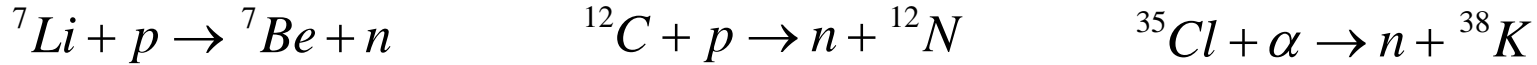
$$K_y = \frac{(m_{He} + m_n) \left[ m_{He} Q + (m_{He} - m_p) K_p \right]}{(m_{He} + m_n)^2} = \frac{m_{He} Q + (m_{He} - m_p) K_p}{(m_{He} + m_n)}$$

$$K_y = \frac{-0.764 \times 3.016029 + (3.016029 - 1.007825) \times 3}{(3.016029 + 1.008665)} = 0.924 \text{ MeV}$$

If  $\theta = 90^\circ$ ,  $\cos 90^\circ = 0$  and  $K_p = 3 \text{ MeV}$

$$Q = K_n + K_{He} - K_p - K_H \quad -0.764 = 0.924 + K_{He} - 3 \quad \text{ise} \quad K_{He} = 1.312 \text{ MeV}$$

**Question 4:** For the following endoergic reactions, find the Q value and the threshold kinetic energy, assuming in each case that the lighter particle is incident on the heavier particle at rest.



$$Q = [m({}^7\text{Li}) + m({}^1\text{H}) - m({}^7\text{Be}) - m_n]c^2$$

$$= [7.016003u + 1.007825u - 7.016928u - 1.008665u]931.5 \text{ MeV} / u = -1.644 \text{ MeV}$$

$$E_{\text{eşik}} = |-Q| \left( 1 + \frac{m_x}{M_X} \right) = (1.644 \text{ MeV}) \left( 1 + \frac{1.007825u}{7.016003u} \right) = 1.880 \text{ MeV}$$

$$Q = [m({}^{12}\text{C}) + m({}^1\text{H}) - m_n - m({}^{12}\text{N})]c^2$$

$$= [12.000000u + 1.007825u - 1.008665u - 12.018613u]931.5 \text{ MeV} / u = -18.121 \text{ MeV}$$

$$E_{\text{eşik}} = |-Q| \left( 1 + \frac{m_x}{M_X} \right) = (18.121 \text{ MeV}) \left( 1 + \frac{1.007825u}{12.000000u} \right) = 19.640 \text{ MeV}$$

$$Q = [m({}^{35}\text{Cl}) + m({}^4\text{He}) - m_n - m({}^{38}\text{K})]c^2$$

$$= [34.968853u + 4.002603u - 1.008665u - 37.969080u]931.5 \text{ MeV} / u = -5.858 \text{ MeV}$$

$$E_{\text{eşik}} = |-Q| \left( 1 + \frac{m_x}{M_X} \right) = (5.858) \left( 1 + \frac{4.002603u}{34.968853u} \right) = 6.529 \text{ MeV}$$

**Question 5:** Using the semi-empirical mass formula, calculate the total binding energies of  ${}_{29}\text{Cu}^{64}$  and  ${}_{30}\text{Zn}^{64}$  nuclei.

$$B(Z, A) = a_1 A - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} \pm a_5 A^{-3/4}$$

$a_1=15.5$  MeV,  $a_2=16.8$  MeV,  $a_3=0.72$  MeV,  $a_4= 23$  MeV ve  $a_5=34$  MeV,  $m_p=1.007825$  u,  $m_n= 1.008665$ u  $c^2= 931.5$  MeV

For  ${}_{29}\text{Cu}^{64}$   $A=64$  ,  $Z=29$  (Single) ,  $N= 35$  (64-29) (Single)

$$B = 15.5(64) - 16.8(64)^{2/3} - 0.72 \frac{29 \times 28}{(64)^{1/3}} - 24 \frac{(64-58)^2}{64} - 34(64)^{-3/4} = 562.04 \text{ MeV}$$

For  ${}_{30}\text{Zn}^{64}$   $A=64$  ,  $Z=30$  (Double ) ,  $N= 34$  (64-30) (Double)

$$B = 15.5(64) - 16.8(64)^{2/3} - 0.72 \frac{30 \times 29}{(64)^{1/3}} - 24 \frac{(64-60)^2}{64} + 34(64)^{-3/4} = 562.10 \text{ MeV}$$



**Question 6:** a) Calculate the total binding energy of  ${}_{92}\text{U}^{236}$  nuclei using the semi-empirical mass formula. b) Calculate the neutron separation energy.

$$B(Z, A) = a_1 A - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} \pm a_5 A^{-3/4}$$

$$m({}_{92}\text{U}^{236}) = 236.045563 \text{u}, \quad m({}_{92}\text{U}^{235}) = 235.043924 \text{u}, \quad m_n = 1.008665 \text{u}$$

$$a_1 = 15.5 \text{ MeV}, \quad a_2 = 16.8 \text{ MeV}, \quad a_3 = 0.72 \text{ MeV}, \quad a_4 = 23 \text{ MeV} \quad \text{and} \quad a_5 = 34 \text{ MeV}, \quad c^2 = 931.5 \text{ MeV}$$

a) For  ${}_{92}\text{U}^{236}$   $A=236$ ,  $Z=92$  (Double),  $N=144$  ( $236-92$ ) (Double)

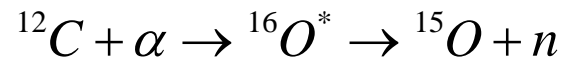
$$B = 15.5(236) - 16.8(236)^{2/3} - 0.72 \frac{92 \times 91}{(236)^{1/3}} - 23 \frac{(236-184)^2}{236} + 34(236)^{-3/4} = 1778.040907 \text{ MeV}$$

b)  ${}_{92}\text{U}^{236} \rightarrow {}_{92}\text{U}^{235} + n$

$$S_n = [m({}_{92}\text{U}^{235}) - m({}_{92}\text{U}^{236}) + m_n] c^2$$

$$= [235.043924 - 236.045563 + 1.008665] 931.5 \text{ MeV} = 6.544719 \text{ MeV}$$

**Question 7:** The  $^{15}\text{O}$  radioisotope with important medical applications can be produced by the reaction  $^{12}\text{C}(\alpha, n)$ . a) When the energy of the incoming  $\alpha$  particles in the laboratory system is 14.6 MeV, the cross section reaches a peak. What is the excitation energy of the compound nuclear state? b) The reaction cross section for the above incidence energy is 25 mb. For another target of  $0.10 \text{ mg} / \text{cm}^2$  and an  $\alpha$  current of 20 nA, calculate the  $^{15}\text{O}$  activity that occurs after 4 minutes of irradiation.



a) First energy =  $m(^{12}\text{C})c^2 + m(^4\text{He})c^2 + K_\alpha$

The energy of the composite nucleus =  $m(^{16}\text{O})c^2 + E_{\text{ex}}$

$$E_{\text{ex}} = [m(^{12}\text{C}) + m(^4\text{He}) - m(^{16}\text{O})]c^2 + K_\alpha$$

$$= [12.000000u + 4.002603u - 15.994915u]931.502\text{MeV} / u + 14.6 \text{ MeV} = 21.8 \text{ MeV}$$

b) 
$$R_b = \sigma I_\alpha N = 25(10^{-27} \text{ cm}^2) \left[ \frac{20(10^{-9} \text{ C} / \text{s})}{2(1.6 \times 10^{-19} \text{ C} / \text{particle})} \right] \frac{(10^{-4} \text{ g} / \text{cm}^2)}{(12 \text{ g} / \text{mol})} 6.02(10^{23}) \text{ atom} / \text{mol} = 7.84(10^3) \text{ s}^{-1}$$

In 4 minutes the reaction produces  $2.04(10^6)$   $^{15}\text{O}$  nuclei.

$$A = \lambda N = \left( \frac{0.693}{122\text{s}} \right) 2.04(10^6) = 1.16(104) \text{ s}^{-1} = 0.31 \mu\text{Ci}$$

**Question 8:** Find the initial activity of 2 milligrams of  $I^{131}$  with a half-life  $t_{1/2} = 8.04$  days and the remaining activity after 4 days.

Starting activity

$$A = \lambda N = \left( \frac{0.693}{t_{1/2}} \right) \frac{mN_A}{M_A} = \frac{0.693(2 \times 10^{-3}) 6.02 \times 10^{23}}{8.04(86400) 131} = 9.17 \times 10^{12} \text{ atom / s}$$

Remaining activity after 4 days

$$A = A_0 e^{-\frac{0.693}{t_{1/2}} t} = 9.17 \times 10^{12} \cdot e^{-\frac{0.693}{8.04} 4} \cong 6.5 \times 10^{12} \text{ Bq}$$

**Question 9:** Calculate the weight of  $C^{14}$  of 1 milliCurie in grams. The half-life of  $C^{14}$  is 5730 years?

$$A = 3.7 \times 10^{10} \cdot 10^{-3} = 3.7 \times 10^7 \text{ decay / s}$$

$$A = \lambda N = \left( \frac{0.693}{t_{1/2}} \right) \frac{m N_A}{M_A} \quad \Rightarrow \quad m = A \frac{t_{1/2} M_A}{0.693 N_A}$$

$$m = \frac{3.7 \times 10^7 (5730)(365)(86400)14}{0.693 \times 6.02 \times 10^{23}} = 0.224 \times 10^{-3} \text{ g} = 0.224 \text{ kg}$$

**Question 10:** Radioactive decay of  $\text{Th}^{232}$  results in stable  $\text{Po}^{210}$  atoms and three  $\alpha$  particles. A rock is determined to contain 4.2 g of  $\text{Th}^{232}$  and 0.9 g of  $\text{Po}^{210}$ . What is the age of the rock inferred from the Th / Po ratio?  $m(\text{Th}^{232}) = 232.038051\text{u}$ ,  $m(\text{Po}^{210}) = 209.982848\text{u}$ ,  $t_{1/2}(\text{Th}^{232}) = 1.41 \times 10^{10}$  years

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{1.41 \times 10^{10}} = 4.92 \times 10^{-11} \text{ year}^{-1}$$

$$\frac{N_{\text{daughter}}}{N_{\text{parent}}} = \frac{\frac{m_{\text{daughter}}}{M_{A,\text{daughter}}} N_A}{\frac{m_{\text{parent}}}{M_{A,\text{parent}}} N_A} = \left( \frac{0.9 \text{ g}}{209.982848 \text{ g / mol}} \right) \left( \frac{232.038051 \text{ g / mol}}{4.2 \text{ g}} \right) = 0.237$$

$$\Delta t = \frac{1}{\lambda} \ln \left( 1 + \frac{N_{\text{daughter}}}{N_{\text{parent}}} \right) = \frac{1}{4.92 \times 10^{-11}} \ln(1 + 0.237) = 4.33 \times 10^9 \text{ year}$$

**Question 11:** The half-life of  $\text{Rn}^{223}$  is 3.82 days. a) What percentage of Rn would be decayed after 10 days? b) What is the activity of the Rn sample whose initial amount is 2  $\mu\text{g}$ ? c) What is the Rn amount remaining after 10 days?

a)

$$N = N_0 e^{-\lambda t_1} = N_0 e^{-\frac{0.693}{t_{1/2}} t_1} = N_0 e^{-\frac{0.693}{3.82} \cdot 10}$$

$$N / N_0 = 0.1630$$

is the percentage of the remaining amount. The amount decayed is

$$\frac{N_0 - N}{N_0} = 1 - \frac{N}{N_0} = 1 - 0.1630 = 0.837 = \% 83.7$$

b)

$$A_0 = \lambda N_0 = \frac{0.693}{t_{1/2}} \frac{N_A m}{M_A} = \frac{0.693}{3.82 \times 86400} \frac{6.02 \times 10^{23} \cdot 2 \times 10^{-6}}{223} = 1.1336 \times 10^{10} \text{ decay} / \text{s} = 0.3063 \text{ Ci}$$

c) The remaining amount of 2  $\mu\text{g}$  Rn after 10 days;  $2 \times 0.1630 = 0.3260 \mu\text{g}$ .

**Question 12:** The Q value of the  ${}^9\text{Be}$  (p, d)  ${}^8\text{Be}$  reaction is  $559.5 \pm 0.4$  keV. Using this value and the precisely known masses of  ${}^9\text{Be}$ ,  ${}^2\text{H}$ , and  ${}^1\text{H}$ , calculate the mass of  ${}^8\text{Be}$ .



$$Q = \left[ m({}^9\text{Be}) + m({}^1\text{H}) - m({}^2\text{H}) - m({}^8\text{Be}) \right] c^2$$

$$m({}^8\text{Be}) = m({}^9\text{Be}) + m({}^1\text{H}) - m({}^2\text{H}) - \left( \frac{Q}{c^2} \right)$$

$$= 9.012182u + 1.007825u - 2.014102u - \left( \frac{0.5595 \pm 0.0004 \text{ MeV}}{931.502 \text{ MeV} / u} \right)$$

$$= (8.0053044 \pm 0.0000004)u$$

**Question 13:** Calculate the binding energy of the  ${}_{26}\text{Fe}^{56}$  nucleus and the binding energy per nucleon

$$m({}_{26}\text{Fe}^{56})=55.934939\text{u}, m_n=1.008665\text{u}, m({}_1\text{H}^1)=1.007825\text{u}$$

$$\begin{aligned} B &= \left[ Zm({}_1\text{H}^1) + Nm_n - m({}_{26}\text{Fe}^{56}) \right] 931.5 \text{ MeV} / u \\ &= \left[ 26(1.007825\text{u}) + 30(1.008665\text{u}) - 55.934939\text{u} \right] 931.5 \text{ MeV} / u \\ &= 492.26 \text{ MeV} \end{aligned}$$

Binding energy per nucleon;  $B/A=(492.26/56)=8.79 \text{ MeV/nucleon}$



**Question 14:** Compute the total binding energy and the binding energy per nucleon for

(a)  ${}^7\text{Li}$ ;

(b)  ${}^{56}\text{Fe}$ ;

(c)  ${}^{235}\text{U}$

a)

$$\begin{aligned}\text{BE}({}^7\text{Li}) &= \left[ 3m({}^1\text{H}) + 4(m_n) - m({}^7\text{Li}) \right] c^2 \\ &= \left[ 3(1.00782504u) + 4(1.008665u) - (7.016003u) \right] \left( 931.502 \frac{\text{MeV}}{u} \right) \\ &= 39.246 \text{ MeV}\end{aligned}$$

$$\text{BE} / A = 39.246 \text{ MeV} / 7 = 5.607 \text{ MeV}$$

b)  $\text{BE}({}^{56}\text{Fe}) = \left[ 26m({}^1\text{H}) + 30(m_n) - m({}^{56}\text{Fe}) \right] c^2$

$$\begin{aligned}&= \left[ 26(1.00782504u) + 30(1.008665u) - (55.934939u) \right] \left( 931.502 \frac{\text{MeV}}{u} \right) \\ &= 492.262 \text{ MeV}\end{aligned}$$

$$\text{BE} / A = 492.262 \text{ MeV} / 56 = 8.79 \text{ MeV}$$

**Question 14:** Compute the total binding energy and the binding energy per nucleon for

(a)  ${}^7\text{Li}$ ;

(c)  ${}^{56}\text{Fe}$ ;

(c)  ${}^{235}\text{U}$

c)

$$\begin{aligned} \text{BE}({}^{235}\text{U}) &= \left[ 92m({}^1\text{H}) + 143(m_n) - m({}^{235}\text{U}) \right] c^2 \\ &= \left[ 92(1.00782504u) + 143(1.008665u) - (235.043924u) \right] \left( 931.502 \frac{\text{MeV}}{u} \right) \\ &= 1783.892 \text{ MeV} \end{aligned}$$

$$\text{BE} / A = 1783.892 \text{ MeV} / 235 = 7.591 \text{ MeV}$$

**Question 15:** For each of the following nuclei, use the semiempirical mass formula to compute the total binding energy:

(a)  $^{21}\text{Ne}$ ;      (b)  $^{57}\text{Fe}$ ;      (c)  $^{209}\text{Bi}$ ;      (d)  $^{256}\text{Fm}$

$$\text{a) } BE(Z, A) = a_h A - a_y A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{sim} \frac{(A-2Z)^2}{A} + \delta$$

$$a_h = 15.5 \text{ MeV} \quad a_y = 16.8 \text{ MeV} \quad a_c = 0.72 \text{ MeV} \quad a_{sim} = 23 \text{ MeV} \quad a_p = 34 \text{ MeV}$$

$$\delta = \begin{cases} 34 A^{-3/4}, & Z \text{ and } N \text{ even} \\ -34 A^{-3/4}, & Z \text{ and } N \text{ odd} \\ 0, & \text{for } A \text{ odd} \end{cases}$$

For  $^{21}\text{Ne}$   $Z=10$  and  $N=11$

$$\begin{aligned} BE(^{21}\text{Ne}) &= 15.5(21) - 16.8(21)^{2/3} - 0.72(10)(9)(21)^{-1/3} - 23 \frac{(21-20)^2}{21} + 0 \\ &= 173.04 \text{ MeV} \end{aligned}$$

**Question 15:** For each of the following nuclei, use the semiempirical mass formula to compute the total binding energy and the Coulomb energy:

(a)  $^{21}\text{Ne}$ ;      (b)  $^{57}\text{Fe}$ ;      (c)  $^{209}\text{Bi}$ ;      (d)  $^{256}\text{Fm}$

b) For  $^{57}\text{Fe}$   $Z=26$  and  $N=31$

$$\begin{aligned} BE(^{57}\text{Fe}) &= 15.5(57) - 16.8(57)^{2/3} - 0.72(26)(25)(57)^{-1/3} - 23 \frac{(57-26)^2}{57} + 0 \\ &= 502.98 \text{ MeV} \end{aligned}$$

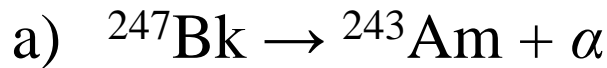
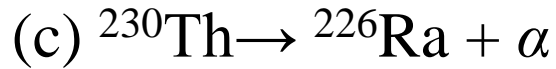
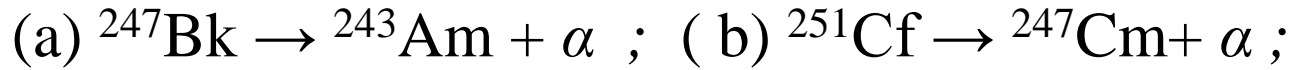
c) For  $^{209}\text{Bi}$   $Z=83$  and  $N=126$

$$\begin{aligned} BE(^{209}\text{Bi}) &= 15.5(209) - 16.8(209)^{2/3} - 0.72(83)(126)(209)^{-1/3} - 23 \frac{(209-83)^2}{209} + 0 \\ &= 1618.62 \text{ MeV} \end{aligned}$$

d) For  $^{256}\text{Fm}$   $Z=100$  and  $N=156$

$$\begin{aligned} BE(^{256}\text{Fm}) &= 15.5(256) - 16.8(256)^{2/3} - 0.72(100)(156)(256)^{-1/3} - 23 \frac{(256-100)^2}{256} + 34(256)^{-3/4} \\ &= 1886.86 \text{ MeV} \end{aligned}$$

**Question 16:** Find the Q values of the following decays: For each decay, calculate the kinetic energy and velocity of the daughter nucleus after the decay.

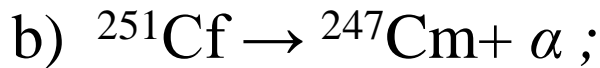
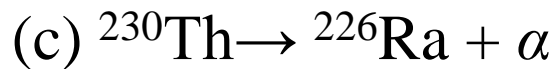
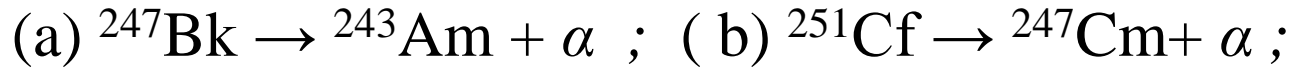


$$\begin{aligned} Q &= \left[ m(^{247}\text{Bk}) - m(^{243}\text{Am}) - m(^4\text{He}) \right] c^2 \\ &= [247.070300u - 243.061375u - 4.002603u](931.502 \text{ MeV} / u) \\ &= 5.889 \text{ MeV} \end{aligned}$$

$$K(^{243}\text{Am}) = \frac{Q}{\left[ 1 + \frac{m(^{243}\text{Am})}{m_\alpha} \right]} = 95.4 \text{ keV}$$

$$v(^{243}\text{Am}) = \sqrt{\frac{2K(^{243}\text{Am})}{m(^{243}\text{Am})}} = 2.75(10^5) \text{ m} / \text{s}$$

**Question 16:** Find the Q values of the following decays: For each decay, calculate the kinetic energy and velocity of the daughter nucleus after the decay.

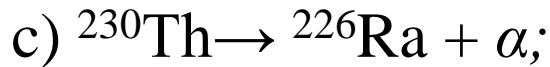
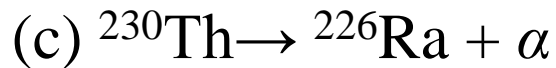
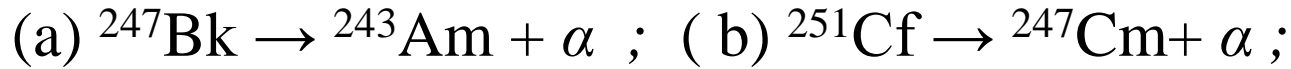


$$\begin{aligned}
 Q &= \left[ m(^{251}\text{Cf}) - m(^{247}\text{Cm}) - m(^4\text{He}) \right] c^2 \\
 &= \left[ 251.079580u - 247.070347u - 4.002603u \right] (931.502 \text{ MeV} / u) \\
 &= 6.176 \text{ MeV}
 \end{aligned}$$

$$K(^{247}\text{Cm}) = \frac{Q}{\left[ 1 + \frac{m(^{247}\text{Cm})}{m_\alpha} \right]} = 98.5 \text{ keV}$$

$$v(^{247}\text{Cm}) = \sqrt{\frac{2K(^{247}\text{Cm})}{m(^{247}\text{Cm})}} = 2.77(10^5) \text{ m} / \text{s}$$

**Question 16:** Find the Q values of the following decays: For each decay, calculate the kinetic energy and velocity of the daughter nucleus after the decay.

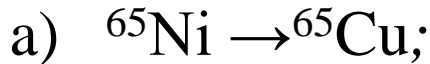
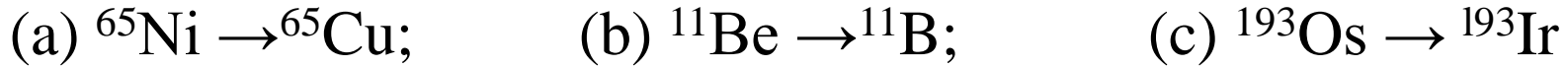


$$\begin{aligned}
 Q &= \left[ m(^{230}\text{Th}) - m(^{226}\text{Ra}) - m(^4\text{He}) \right] c^2 \\
 &= \left[ 230.033128u - 226.025403u - 4.002603u \right] (931.502 \text{ MeV} / u) \\
 &= 4.771 \text{ MeV}
 \end{aligned}$$

$$K(^{226}\text{Ra}) = \frac{Q}{\left[ 1 + \frac{m(^{226}\text{Ra})}{m_\alpha} \right]} = 83 \text{ keV}$$

$$v(^{226}\text{Ra}) = \sqrt{\frac{2K(^{226}\text{Ra})}{m(^{226}\text{Ra})}} = 2.66(10^5) \text{ m} / \text{s}$$

**Question 17:** Compute the Q values for the following  $\beta^-$  decays:



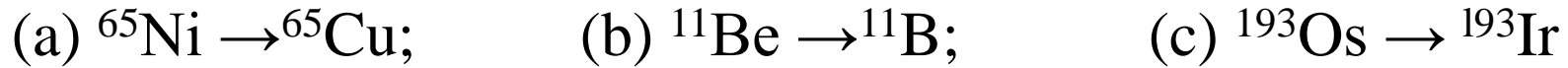
$$\begin{aligned} Q_{\beta^-} &= \left[ m({}^{65}\text{Ni}) - m({}^{65}\text{Cu}) \right] c^2 \\ &= [64.930086u - 64.927793u](931.502 \text{ MeV} / u) \\ &= 2.136 \text{ MeV} \end{aligned}$$



$$\begin{aligned} Q_{\beta^-} &= \left[ m({}^{11}\text{Be}) - m({}^{11}\text{B}) \right] c^2 \\ &= [11.021658u - 11.009305u](931.502 \text{ MeV} / u) \\ &= 11.507 \text{ MeV} \end{aligned}$$



**Question 17:** Compute the Q values for the following  $\beta^-$  decays:

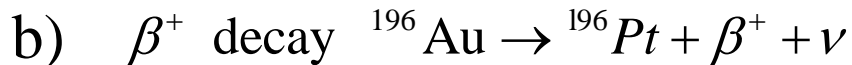


$$\begin{aligned} Q_{\beta^-} &= \left[ m({}^{193}\text{Os}) - m({}^{193}\text{Ir}) \right] c^2 \\ &= [192.964138u - 192.962917u](931.502 \text{ MeV} / u) \\ &= 1.137 \text{ MeV} \end{aligned}$$

**Question 18:**  $^{196}\text{Au}$  can decay by  $\beta^-$ ,  $\beta^+$ , and  $\varepsilon$ . Find the Q values for the three decay modes.



$$\begin{aligned} Q_{\beta^-} &= \left[ m(^{196}\text{Au}) - m(^{196}\text{Hg}) \right] c^2 \\ &= [195.966544u - 195.965807u](931.502 \text{ MeV} / u) \\ &= 0.686 \text{ MeV} \end{aligned}$$



$$\begin{aligned} Q_{\beta^+} &= \left[ m(^{196}\text{Au}) - m(^{196}\text{Pt}) \right] c^2 - 2m_e c^2 \\ &= [195.966544u - 195.964926u](931.502 \text{ MeV} / u) - 2(0.511 \text{ MeV}) \\ &= 0.485 \text{ MeV} \end{aligned}$$

**Question 18:**  $^{196}\text{Au}$  can decay by  $\beta^-$ ,  $\beta^+$ , and  $\varepsilon$ . Find the Q values for the three decay modes.



Here we assume that the electron capture event occurs in the K shell.

$$\begin{aligned} Q_\varepsilon &= \left[ m(^{196}\text{Au}) - m(^{196}\text{Pt}) \right] c^2 - B_K \\ &= [195.966544u - 195.964926u](931.502 \text{ MeV} / u) - 0.078395 \text{ MeV} = 1.429 \text{ MeV} \end{aligned}$$

**Question 19:** The maximum kinetic energy of the positron spectrum emitted in the decay  $^{11}\text{C} \rightarrow ^{11}\text{B}$  is 1.983 MeV. Use this information and the known mass of  $^{11}\text{B}$  to calculate the mass of  $^{11}\text{C}$ .

$$m_e = 5.485803(10^{-4})u, \quad m(^{11}\text{B}) = 11.009305u$$

$$Q_{\beta^+} = [m(^{11}\text{C}) - m(^{11}\text{B}) - 2m_e]c^2$$

$$m(^{11}\text{C}) = \left( \frac{Q_{\beta^+}}{c^2} \right) + m(^{11}\text{B}) + 2m_e = \left[ \left( \frac{1.983}{931.502} \right) + 11.009305u + 2(5.485803)(10^{-4})u \right]$$
$$= 11.012531u$$

**Question 20:** The isotope  $^{113}\text{Cd}$  captures a very low-energy neutron, leading to an excited state of  $^{114}\text{Cd}$ , which emits a  $\gamma$  ray leading directly to the  $^{114}\text{Cd}$  ground state. Find the energy of the  $\gamma$  ray, neglecting the nuclear recoil.



$\gamma$ -ray energy

$$\begin{aligned} E_\gamma &= \left[ m(^{113}\text{Cd}) + m_n - m(^{114}\text{Cd}) \right] c^2 \\ &= \left[ 112.904400u + 1.008665u - 113.903357u \right] (931.502 \text{ MeV} / u) \\ &= 9.043 \text{ MeV} \end{aligned}$$

**Question 21:** The isotope  $^{254}\text{Cf}$  decays almost exclusively by spontaneous fission, with a half-life of 60.5 days. The energy released is about 225 MeV per fission. Calculate the total fission power produced by 1.0  $\mu\text{g}$  of  $^{254}\text{Cf}$ .

$$N = \left( \frac{10^{-6} \text{ g}}{254 \text{ g / mol}} \right) (6.02) (10^{23} \text{ atom / mol}) = 2.37 (10^{15})$$

$$A = \lambda N = 2.37 (10^{15}) \left[ \frac{0.693}{(60.5 \text{ day})(86400 \text{ s / day})} \right] = 3.14 (10^8) \text{ s}^{-1}$$

$$P = (225 \text{ MeV / fission}) (3.14) (10^8 \text{ fission / s}) (1.6) (10^{-13} \text{ J / MeV}) = 0.0113 \text{ W}$$

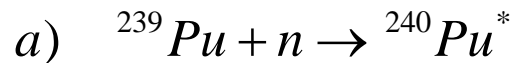
**Question 22:** Given that the activation energy of  $^{236}\text{U}$  is 6.2 MeV, what is the minimum-energy  $\alpha$  particle that can produce fission following bombardment of a  $^{232}\text{Th}$  target?



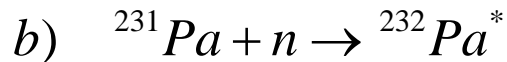
$$\begin{aligned} E_{ex} &= \left[ m(^{236}\text{U}^*) - m(^{236}\text{U}) \right] c^2 + K_{\alpha} = \left[ m(^{232}\text{Th}) + m(^4\text{He}) - m(^{236}\text{U}) \right] c^2 + K_{\alpha} \\ &= (232.038051u + 4.002603u - 236.045563u)(931.502 \text{ MeV} / u) + K_{\alpha} \\ &= -4.51 \text{ MeV} + K_{\alpha} > 6.2 \text{ MeV} \quad \Rightarrow \quad K_{\alpha} > 10.8 \text{ MeV} \end{aligned}$$

**Question 23:** Compare the excitation and activation energies for thermal neutron-induced fission of (a)  $^{239}\text{Pu}$  (b)  $^{231}\text{Pa}$

Activation energies of the  $^{239}\text{Pu}$  and  $^{231}\text{Pa}$  are 6.0 and 7.6 MeV, respectively.



$$\begin{aligned} E_{ex} &= \left[ m(^{240}\text{Pu}^*) - m(^{240}\text{Pu}) \right] c^2 = \left[ m(^{239}\text{Pu}) + m_n - m(^{240}\text{Pu}) \right] c^2 \\ &= (239.052158u + 1.008665u - 240.053808u)(931.502 \text{ MeV} / u) \\ &= 6.5 \text{ MeV} \end{aligned}$$

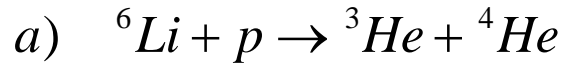
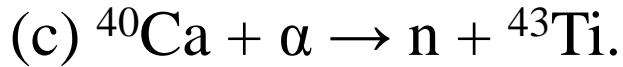


$$\begin{aligned} E_{ex} &= \left[ m(^{232}\text{Pa}^*) - m(^{232}\text{Pa}) \right] c^2 = \left[ m(^{231}\text{Pa}) + m_n - m(^{232}\text{Pa}) \right] c^2 \\ &= (231.035880u + 1.008665u - 232.038565u)(931.502 \text{ MeV} / u) \\ &= 5.6 \text{ MeV} \end{aligned}$$

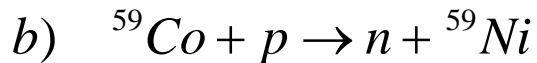
According to these results, while Pu is fissioned with thermal neutrons, Pa must be bombarded with MeV-energized neutrons to undergo fission.



**Question 24:** Compute the Q values for the reactions



$$\begin{aligned} Q &= \left[ m({}^6\text{Li}) + m({}^1\text{H}) - m({}^3\text{He}) - m({}^4\text{He}) \right] c^2 \\ &= (6.015121u + 1.007825u - 3.016029u - 4.002603u)(931.502 \text{ MeV} / u) \\ &= 4.018 \text{ MeV} \end{aligned}$$

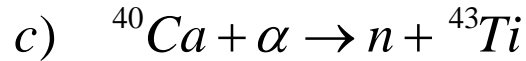
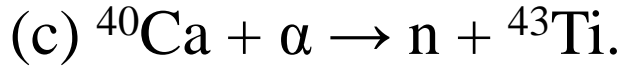


$$\begin{aligned} Q &= \left[ m({}^{59}\text{Co}) + m({}^1\text{H}) - m_n - m({}^{59}\text{Ni}) \right] c^2 \\ &= (58.933198u + 1.007825u - 1.008665u - 58.934349u)(931.502 \text{ MeV} / u) \\ &= -1.855 \text{ MeV} \end{aligned}$$

The threshold kinetic energy for this endoergic reaction

$$E_{th} = |-Q| \left( 1 + \frac{m_x}{M_X} \right) = 1.855 \left( 1 + \frac{1.007825u}{59.933198u} \right) = 1.886 \text{ MeV}$$

**Question 24:** Compute the Q values for the reactions



$$Q = \left[ m({}^{40}\text{Ca}) + m_{\alpha} - m_{\text{n}} - m({}^{43}\text{Ti}) \right] c^2$$

$$= (39.962591u + 4.002603u - 1.008665u - 42.968523u)(931.502 \text{ MeV} / u)$$

$$= -11.172 \text{ MeV}$$

The threshold kinetic energy for this endoergic reaction

$$E_{th} = |-Q| \left( 1 + \frac{m_x}{M_X} \right) = 11.172 \left( 1 + \frac{4.002603u}{39.96259u} \right) = 12.291 \text{ MeV}$$

**Question 25:** How many  $C^{14}$  nuclei are required to achieve 1 Ci of activity.  $T_{1/2} = 5730$  year

Activity

$$A = \lambda N = \frac{0.693}{5730 (365)(86400s)} \frac{m(6.02 \times 10^{23} \text{ atom / mol})}{14 \text{ g / mol}} = 1 \times 3.7 \left( 10^{10} \frac{\text{atom}}{s} \right)$$

$$m = 0.224 \text{ g}$$

**Question 26:** It is desired to determine the age of a wood timber used to construct an ancient shelter. A sample of the wood is analyzed for its  $^{14}C$  content and gives 2.1 decays per minute. Another sample of the same size from a recently cut tree of the same type gives 5.3 decays per minute. What is the age of the sample.

$$\begin{aligned} A(t_1) = A_0 e^{-\lambda t_1} &\Rightarrow t_1 = \left( \frac{1}{\lambda} \right) \ln \left( \frac{A_0}{A(t_1)} \right) = \left( \frac{t_{1/2}}{0.693} \right) \ln \left( \frac{A_0}{A(t_1)} \right) \\ &= \left( \frac{5730 \text{ y}}{0.693} \right) \ln \left( \frac{5.3}{2.1} \right) = 7.65 (10^3) \text{ y} \end{aligned}$$