NUCLEAR PHYSICS AND APPLICATIONS

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Question 1: Two radioactive sources each have activities of 1.0 μCi at t=0. Their half-lives are, 1.0s and 1.0 d. A) How many radioactive nuclei are present at t=0 in each source? b) How many decay between t=0 and t=1h?

a) Decay constant

$$\lambda_1 = \frac{0.693}{1\,s} = 0.693\,s^{-1} \qquad \lambda_2 = \frac{0.693}{24(60)(60s)} = 8.02(10^{-6})\,s^{-1}$$

Number of radioactive nuclei at t = 0

$$N_{10} = \frac{A_{10}}{\lambda_1} = \frac{1.0\left(10^{-6}\right)\left(3.7x10^{10}\right)}{0.693} = 5.34\left(10^{4}\right)$$

$$N_{20} = \frac{A_{20}}{\lambda_1} = \frac{1.0\left(10^{-6}\right)\left(3.7x10^{10}\right)}{8.02\left(10^{-6}\right)} = 4.61\left(10^{9}\right)$$

b)

$$\Delta N_1 = N_{10}\left(1-e^{-\lambda_1 t}\right) = 5.34\left(10^{4}\right)\left[1-e^{-0.693(3600)}\right] = 5.34\left(10^{4}\right)$$

$$\Delta N_2 = N_{20}\left(1-e^{-\lambda_2 t}\right) = 4.61\left(10^{9}\right)\left[1-e^{-8.02\left(10^{-6}\right)(3600)}\right] = 1.31\left(10^{8}\right)$$

Question 2: The decay chain $^{139}Cs \rightarrow {}^{139}Ba \rightarrow {}^{139}La$ is observed from an initially pure sample of 1 mCi of ^{139}Cs. The half-lives are ^{139}Cs, 9.5 min; , ^{139}Ba, 82.9 min, ^{139}La, stable. What is the maximum ^{139}Ba activity and when does it occur?

Lets denote ^{139}Cs by 1, ^{139}Ba by 2 and ^{139}La by 3.

$$\lambda_1 = \frac{0.693}{9.5\,\text{min}} = 7.30\left(10^{-2}\right)\text{min}^{-1} \qquad \lambda_2 = \frac{0.693}{82.9\,\text{min}} = 8.36\left(10^{-3}\right)\text{min}^{-1}$$

$$A_{10} = 1\,mCi = 3.7x10^{7}\ decay/s$$

$$N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

$$A_2\left(t\right) = \lambda_2 N_2(t) = \frac{\lambda_1 \lambda_2 N_{10}}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) = A_{10}\frac{\lambda_2}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

When A_2(t) is maximum

$$\left.\frac{dA_2(t)}{dt}\right|_{t=t_0} = 0$$

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$$\lambda_1 e^{-\lambda_1 t_0} - \lambda_2 e^{-\lambda_2 t_0} = 0 \quad \Rightarrow \quad t_0 = \frac{1}{(\lambda_2 - \lambda_1)} \ln\left(\frac{\lambda_2}{\lambda_1}\right)$$

$$t_0 = \frac{1}{8.36(10^{-3}) - 7.30(10^{-2})} \ln\left(\frac{8.36\left(10^{-3}\right)}{7.30\left(10^{-2}\right)}\right) = 33.5\,\mathrm{min}$$

$$\left(A_2\right)_{\max} = A_{10}\left(\frac{\lambda_2}{\lambda_2 - \lambda_1}\right)\left(e^{-\lambda_1 t_0} - e^{-\lambda_2 t_0}\right) = 8.66\left(10^{-5}\right)Ci = 86.6\mu Ci$$

Question 3: a) Calculate the Q value and the threshold kinetic energy of the endoergic reaction $p+{}_1H^3\rightarrow n+{}_2He^3$

b) If the energy of the incident proton is 3 MeV and the scattering angle of the exiting neutron is 90^0, find the energies of the neutron and ${}_2\mathrm{He}^3$ (The target nucleus is initially stationary).

$$m\left({}_3H^1\right)=3.016049u\ ,\quad m\left({}_2He^3\right)=3.016029u,\ \ m_p=1.007825u,\ \ m_n=1.008665u$$

$$\begin{aligned}Q&=\left[m_p+m\left({}_1H^3\right)-m_n-m\left({}_2He^3\right)\right]c^2\\&=\left[1.007825+3.016049-1.008665-3.016029\right]931.5\ MeV\\&=-0.76383\ MeV\end{aligned}$$

Since Q <0, it is an endothermic reaction and an externally energized reaction. Threshold energy is

$$E_{Threshold}=\left|-Q\right|\left(1+\frac{m_x}{M_X}\right)=\left(0.76383\right)\left(1+\frac{1.007825u}{3.016049u}\right)=1.019069\ MeV$$

Question 3: a) Calculate the Q value and the threshold kinetic energy of the endoergic reaction $p + {}_1H^3 \rightarrow n + {}_2He^3$

b) If the energy of the incident proton is 3 MeV and the scattering angle of the exiting neutron is 90^0, find the energies of the neutron and ${}_2He^3$ (The target nucleus is initially stationary).

$$m\left({}_3H^1\right) = 3.016049u\,,\quad m\left({}_2He^3\right) = 3.016029u,\ m_p = 1.007825u,\ m_n = 1.008665u$$

$$\sqrt{K_y} = \frac{\sqrt{m_x m_y K_x}\cos\theta \pm \left\{\left(m_x m_y K_x \cos^2\theta\right) + \left(M_Y + m_y\right)\left[K_x\left(M_Y - m_x\right) + M_Y Q\right]\right\}^{1/2}}{\left(M_Y + m_y\right)}$$

$$K_y = \frac{\left(m_{He} + m_n\right)\left[m_{He}Q + \left(m_{He} - m_p\right)K_p\right]}{\left(m_{He} + m_n\right)^2} = \frac{m_{He}Q + \left(m_{He} - m_p\right)K_p}{\left(m_{He} + m_n\right)}$$

$$K_y = \frac{-0.764x3.016029 + \left(3.016029 - 1.007825\right)3}{\left(3.016029 + 1.008665\right)} = 0.924\,MeV$$

If θ=90^0 , cos 90^0=0 and K_p=3 MeV

$$Q = K_n + K_{He} - K_p - K_H \qquad -0.764 = 0.924 + K_{He} - 3 \quad ise \qquad K_{He} = 1.312\,MeV$$

Question 4: For the following endoergic reactions, find the Q value and the threshold kinetic energy, assuming in each case that the lighter particle is incident on the heavier particle at rest.

$$^{7}Li + p \rightarrow {}^{7}Be + n \qquad {}^{12}C + p \rightarrow n + {}^{12}N \qquad {}^{35}Cl + \alpha \rightarrow n + {}^{38}K$$

$$Q = \left[m\left({}^{7}Li\right) + m\left({}^{1}H\right) - m\left({}^{7}Be\right) - m_n \right]c^2$$

$$= \left[7.016003u + 1.007825u - 7.016928u - 1.008665u\right]931.5\, MeV / u = -1.644\, MeV$$

$$E_{eşik} = \left|-Q\right|\left(1 + \frac{m_x}{M_X}\right) = \left(1.644\, MeV\right)\left(1 + \frac{1.007825u}{7.016003u}\right) = 1.880\, MeV$$

$$Q = \left[m\left({}^{12}C\right) + m\left({}^{1}H\right) - m_n - m\left({}^{12}N\right) \right]c^2$$

$$= \left[12.000000u + 1.007825u - 1.008665u - 12.018613u\right]931.5\, MeV / u = -18.121\, MeV$$

$$E_{eşik} = \left|-Q\right|\left(1 + \frac{m_x}{M_X}\right) = \left(18.121\, MeV\right)\left(1 + \frac{1.007825u}{12.000000u}\right) = 19.640\, MeV$$

$$Q = \left[m\left({}^{35}Cl\right) + m\left({}^{4}He\right) - m_n - m\left({}^{38}K\right) - \right]c^2$$

$$= \left[34.968853u + 4.002603u - 1.008665u - 37.969080u\right]931.5\, MeV / u = -5.858\, MeV$$

$$E_{eşik} = \left|-Q\right|\left(1 + \frac{m_x}{M_X}\right) = \left(5.858\right)\left(1 + \frac{4.002603u}{34.968853u}\right) = 6.529\, MeV$$

Question 5: Using the semi-empirical mass formula, calculate the total binding energies of ${}_{29}Cu^{64}$ and ${}_{30}Zn^{64}$ nuclei.

$$B(Z,A) = a_1 A - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} \pm a_5 A^{-3/4}$$

a_1=15.5 MeV, a_2=16.8 MeV, a_3=0.72 MeV, a_4= 23 MeV ve a_5=34 MeV, m_p=1.007825 u, m_n= 1.008665u c^2= 931.5 MeV

For ${}_{29}Cu^{64}$ A=64 , Z=29 (Single) , N= 35 (64-29) (Single)

$$B = 15.5(64) - 16.8(64)^{2/3} - 0.72\frac{29\ x\ 28}{(64)^{1/3}} - 24\frac{(64-58)^2}{64} - 34(64)^{-3/4} = 562.04\ MeV$$

For ${}_{30}Zn^{64}$ A=64 , Z=30 (Double) , N= 34 (64-30) (Double)

$$B = 15.5(64) - 16.8(64)^{2/3} - 0.72\frac{30\ x\ 29}{(64)^{1/3}} - 24\frac{(64-60)^2}{64} + 34(64)^{-3/4} = 562.10\ MeV$$

Question 6: a) Calculate the total binding energy of ${}_{92}U^{236}$ nuclei using the semi-empirical mass formula. b) Calculate the neutron separation energy.

$$B\left(Z,A\right)=a_1A-a_2A^{2/3}-a_3\frac{Z\left(Z-1\right)}{A^{1/3}}-a_4\frac{\left(A-2Z\right)^2}{A}\pm a_5A^{-3/4}$$

m(${}_{92}U^{236}$)=236.045563u, m(${}_{92}U^{235}$)=235.043924u , m_n=1.008665u

a_1=15.5 MeV, a_2=16.8 MeV, a_3=0.72 MeV, a_4= 23 MeV and a_5=34 MeV, c^2= 931.5 MeV

a) For ${}_{92}U^{236}$ A=236 , Z=92 (Double) , N= 144 (236-92) (Double)

$$B=15.5(236)-16.8\left(236\right)^{2/3}-0.72\frac{92\ x\ 91}{\left(236\right)^{1/3}}-23\frac{\left(236-184\right)^2}{236}+34\left(236\right)^{-3/4}=1778.040907\ MeV$$

b)

$$ {}_{92}U^{236}\rightarrow {}_{92}U^{235}+n$$

$$S_n=\left[m({}_{92}U^{235}-{}_{92}U^{236}+m_n\right]c^2$$

$$=\left[235.043924-236.045563+1.008665\right]931.5MeV=6.544719\ MeV$$

Question 7: The ^{15}O radioisotope with important medical applications can be produced by the reaction ^{12}C (α, n). a) When the energy of the incoming α particles in the laboratory system is 14.6 MeV, the cross section reaches a peak. What is the excitation energy of the compound nuclear state? b) The reaction cross section for the above incidence energy is 25 mb. For another target of 0.10 mg / cm^2 and an α current of 20 nA, calculate the ^{15}O activity that occurs after 4 minutes of irradiation.

$$^{12}C + \alpha \rightarrow {}^{16}O^{*} \rightarrow {}^{15}O + n$$

a) First energy = m(^{12}C) c^2 + m(^{4}He) c^2 + K_α

The energy of the composite nucleus = m (^{16}O) c^2 + E_{ex}

$$E_{uy} = \left[m\left({}^{12}C\right) + m\left({}^{4}He\right) - m\left({}^{16}O\right) \right] c^2 + K_\alpha$$

$$= \left[12.000000u + 4.002603u - 15.994915u\right] 931.502 MeV / u + 14.6\ MeV = 21.8\ MeV$$

b) $$R_b = \sigma I_a N = 25\left(10^{-27} cm^2\right)\left[\frac{20\left(10^{-9} C/s\right)}{2\left(1.6x10^{-19} C/\,particle\right)}\right]\frac{\left(10^{-4} g/cm^2\right)}{\left(12\ g/mol\right)} 6.02\left(10^{23}\right) atom/mol = 7.84\left(10^{3}\right) s^{-1}$$

In 4 minutes the reaction produces 2.04(10^6) ^{15}O nuclei.

$$A = \lambda N = \left(\frac{0.693}{122s}\right) 2.04\left(10^{6}\right) = 1.16\left(104\right) s^{-1} = 0.31\ \mu Ci$$

Question 8: Find the initial activity of 2 milligrams of I^{131} with a half-life $t_{1/2}$ = 8.04 days and the remaining activity after 4 days.

Starting activity

$$A = \lambda N = \left(\frac{0.693}{t_{1/2}}\right)\frac{mN_A}{M_A} = \frac{0.693\left(2x10^{-3}\right)6.02x10^{23}}{8.04\left(86400\right)131} = 9.17x10^{12}\ atom/s$$

Remaining activity after 4 days

$$A = A_0\ e^{-\frac{0.693}{t_{1/2}}t} = 9.17x10^{12}.\ e^{-\frac{0.693}{8.04}4} \cong 6.5x10^{12}\ Bq$$

Question 9: Calculate the weight of C^{14} of 1 milliCurie in grams. The half-life of C^{14} is 5730 years?

$$A = 3.7x10^{10}.10^{-3} = 3.7x10^{7}\ decay/s$$

$$A = \lambda N = \left(\frac{0.693}{t_{1/2}}\right)\frac{mN_A}{M_A} \quad \Rightarrow \quad m = A\frac{t_{1/2}M_A}{0.693\,N_A}$$

$$m = \frac{3.7x10^{7}(5730)(365)(86400)14}{0.693x6.02x10^{23}} = 0.224x10^{-3}\,g = 0.224\,kg$$

Question 10: Radioactive decay of Th^{232} results in stable Po^{210} atoms and three α particles. A rock is determined to contain 4.2 g of Th^{232} and 0.9 g of Po^{210}. What is the age of the rock inferred from the Th / Po ratio? m (Th^{232}) = 232.038051u, m (Po^{210}) = 209.982848u, $t_{1/2}(Th^{232})=1.41x10^{10}$ years

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{1.41x10^{10}} = 4.92x10^{-11}\ year^{-1}$$

$$\frac{N_{daughter}}{N_{parent}} = \frac{\frac{m_{daughter}}{M_{A,daughter}} N_A}{\frac{m_{parent}}{M_{A,parent}} N_A} = \left(\frac{0.9g}{209.982848\ g/mol}\right)\left(\frac{232.038051\,g/mol}{4.2\ g}\right) = 0.237$$

$$\Delta t = \frac{1}{\lambda}\ln\left(1+\frac{N_{daughter}}{N_{parent}}\right) = \frac{1}{4.92x10^{-11}}\ln\left(1+0.237\right) = 4.33x10^{9}\ year$$

Question 11: The half-life of Rn^{223} is 3.82 days. a) What percentage of Rn would be decayed after 10 days? b) What is the activity of the Rn sample whose initial amount is 2 μg? c) What is the Rn amount remaining after 10 days?

a)

$$N = N_0 e^{-\lambda t_1} = N_0 e^{-\frac{0.693}{t_{1/2}} t_1} = N_0 e^{-\frac{0.693}{3.82}.10}$$

$$N / N_0 = 0.1630$$

is the percentage of the remaining amount. The amount decayed is

$$\frac{N_0 - N}{N_0} = 1 - \frac{N}{N_0} = 1 - 0.1630 = 0.837 = \%\,83.7$$

b)

$$A_0 = \lambda N_0 = \frac{0.693}{t_{1/2}} \frac{N_A m}{M_A} = \frac{0.693}{3.82x86400} \frac{6.02x10^{23}.2x10^{-6}}{223} = 1.1336x10^{10}\ decay / s = 0.3063\,Ci$$

c) The remaining amount of 2 μg Rn after 10 days; 2x0.1630 = 0.3260 μg.

Question 12: The Q value of the ^{9}Be (p, d) ^{8}Be reaction is 559.5±0.4 keV. Using this value and the precisely known masses of ^{9}Be, ^{2}H, and ^{1}H, calculate the mass of ^{8}Be.

$$^9Be + {}^1H \rightarrow {}^2H + {}^8Be \qquad Q = 559.5 \pm 0.4\, keV$$

$$Q = \left[m\left({}^9Be\right) + m\left({}^1H\right) - m\left({}^2H\right) - m\left({}^8Be\right) \right] c^2$$

$$m\left({}^8Be\right) = m\left({}^9Be\right) + m\left({}^1H\right) - m\left({}^2H\right) - \left(\frac{Q}{c^2} \right)$$

$$= 9.012182u + 1.007825u - 2.014102u - \left(\frac{0.5595 \pm 0.0004\, MeV}{931.502\, MeV / u} \right)$$

$$= \left(8.0053044 \pm 0.0000004\right) u$$

Question 13: Calculate the binding energy of the ${}_{26}Fe^{56}$ nucleus and the binding energy per nucleon

$m({}_{26}Fe^{56})$=55.934939u, m_n=1.008665u, $m({}_{1}H^{1})$=1.007825u

$$B=\left[Zm\left({}_{1}H^{1}\right)+Nm_{n}-m\left({}_{26}Fe^{56}\right)\right]931.5\,MeV/u$$

$$=\left[26(1.007825u)+30(1.008665u)-55.934939u\right]931.5\,MeV/u$$

$$=492.26\,MeV$$

Binding energy per nucleon; B/A=(492.26/56)=8.79 MeV/nucleon

Question 14: Compute the total binding energy and the binding energy per nucleon for

(a) ^{7}Li; (b) ^{56}Fe; (c) ^{235}U

a)

$$\mathrm{BE}(^{7}\mathrm{Li}) = \left[3m\left(^{1}H\right) + 4\left(m_n\right) - m\left(^{7}Li\right)\right]c^2$$

$$= \left[3\left(1.00782504u\right) + 4\left(1.008665u\right) - \left(7.016003u\right)\right]\left(931.502\ \frac{MeV}{u}\right)$$

$$= 39.246\ MeV$$

$$BE / A = 39.246\ MeV / 7 = 5.607\ MeV$$

b)

$$\mathrm{BE}(^{56}Fe) = \left[26m\left(^{1}H\right) + 30\left(m_n\right) - m\left(^{56}Fe\right)\right]c^2$$

$$= \left[26\left(1.00782504u\right) + 30\left(1.008665u\right) - \left(55.934939u\right)\right]\left(931.502\ \frac{MeV}{u}\right)$$

$$= 492.262\ MeV$$

$$BE / A = 492.262\ MeV / 56 = 8.79\ MeV$$

Question 14: Compute the total binding energy and the binding energy per nucleon for

(a) ^{7}Li; (c) ^{56}Fe; (c) ^{235}U

c)

$$\mathrm{BE}(^{235}U) = \left[92m\left(^{1}H\right) + 143\left(m_n\right) - m\left(^{235}U\right)\right]c^2$$

$$= \left[92\left(1.00782504u\right) + 143\left(1.008665u\right) - \left(235.043924u\right)\right]\left(931.502\,\frac{MeV}{u}\right)$$

$$= 1783.892\ MeV$$

$$BE\,/\,A = 1783.892\ MeV\ /\ 235 = 7.591\,MeV$$

Question 15: For each of the following nuclei, use the semiempirical mass formula to compute the total binding energy:

(a) ^{21}Ne; (b) ^{57}Fe; (c) ^{209}Bi; (d) ^{256}Fm

a)

$$BE(Z,A) = a_h A - a_y A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{sim}\frac{\left(A-2Z\right)^2}{A} + \delta$$

$$a_h = 15.5\,MeV \quad a_y = 16.8\,MeV \quad a_c = 0.72\,MeV \quad a_{sim} = 23\,MeV \quad a_p = 34\,MeV$$

$$\delta = \begin{cases} 34\,A^{-3/4}, Z\ and\ N\ even \\ -34\,A^{-3/4}, Z\ and\ N\ odd \\ 0 \quad , \quad for\ A\ odd \end{cases}$$

For ^{21}Ne Z=10 and N=11

$$BE(^{21}Ne) = 15.5\left(21\right) - 16.8\left(21\right)^{2/3} - 0.72\left(10\right)(9)\left(21\right)^{-1/3} - 23\frac{\left(21-20\right)^2}{21} + 0$$

$$= 173.04\,MeV$$

Question 15: For each of the following nuclei, use the semiempirical mass formula to compute the total binding energy and the Coulomb energy:

(a) ^{21}Ne; (b) ^{57}Fe; (c) ^{209}Bi; (d) ^{256}Fm

b) For ^{57}Fe Z=26 and N=31

$$BE(^{57}Fe)=15.5(57)-16.8(57)^{2/3}-0.72(26)(25)(57)^{-1/3}-23\frac{(57-52)^2}{57}+0$$

$$=502.98\ MeV$$

c) For ^{209}Bi Z=83 and N=126

$$BE(^{209}Bi)=15.5(209)-16.8(209)^{2/3}-0.72(83)(82)(209)^{-1/3}-23\frac{(209-166)^2}{209}+0$$

$$=1618.62\ MeV$$

d) For ^{256}Fm Z=100 and N=156

$$BE(^{256}Fm)=15.5(256)-16.8(256)^{2/3}-0.72(100)(99)(256)^{-1/3}-23\frac{(256-200)^2}{256}+34(256)^{-3/4}$$

$$=1886.86\ MeV$$

Question 16: Find the Q values of the following decays: For each decay, calculate the kinetic energy and velocity of the daughter nucleus after the decay.

(a) $^{247}\mathrm{Bk} \rightarrow {}^{243}\mathrm{Am} + \alpha$; (b) $^{251}\mathrm{Cf} \rightarrow {}^{247}\mathrm{Cm} + \alpha$;

(c) $^{230}\mathrm{Th} \rightarrow {}^{226}\mathrm{Ra} + \alpha$

a) $^{247}\mathrm{Bk} \rightarrow {}^{243}\mathrm{Am} + \alpha$

$$
\begin{aligned}
Q &= \left[m\left(^{247}Bk\right) - m\left(^{243}Am\right) - m\left(^{4}He\right) \right] c^2 \\
&= \left[247.070300u - 243.061375u - 4.002603u \right] \left(931.502\ MeV/u \right) \\
&= 5.889\ MeV
\end{aligned}
$$

$$
K\left(^{243}Am\right) = \frac{Q}{\left[1 + \dfrac{m\left(^{243}Am\right)}{m_\alpha} \right]} = 95.4\ keV
$$

$$
v\left(^{243}Am\right) = \sqrt{\frac{2K\left(^{243}Am\right)}{m\left(^{243}Am\right)}} = 2.75\left(10^5\right) m/s
$$

Question 16: Find the Q values of the following decays: For each decay, calculate the kinetic energy and velocity of the daughter nucleus after the decay.

(a) $^{247}\text{Bk} \rightarrow {}^{243}\text{Am} + \alpha$; (b) $^{251}\text{Cf} \rightarrow {}^{247}\text{Cm} + \alpha$;

(c) $^{230}\text{Th} \rightarrow {}^{226}\text{Ra} + \alpha$

b) $^{251}\text{Cf} \rightarrow {}^{247}\text{Cm} + \alpha$;

$$Q = \left[m\left({}^{251}Cf \right) - m\left({}^{247}Cm \right) - m\left({}^{4}He \right) \right] c^2$$

$$= \left[251.079580u - 247.070347u - 4.002603u \right] \left(931.502\ MeV / u \right)$$

$$= 6.176\ MeV$$

$$K\left({}^{247}Cm \right) = \frac{Q}{\left[1 + \frac{m\left({}^{247}Cm \right)}{m_\alpha} \right]} = 98.5\, keV$$

$$v\left({}^{247}Cm \right) = \sqrt{\frac{2K\left({}^{247}Cm \right)}{m\left({}^{247}Cm \right)}} = 2.77\left(10^5 \right) m / s$$

Question 16: Find the Q values of the following decays: For each decay, calculate the kinetic energy and velocity of the daughter nucleus after the decay.

(a) ^{247}Bk ⟶ ^{243}Am + α ; (b) ^{251}Cf ⟶ ^{247}Cm+ α ;

(c) ^{230}Th⟶ ^{226}Ra + α

c) ^{230}Th⟶ ^{226}Ra + α;

$$Q = \left[m\left({}^{230}Th \right) - m\left({}^{226}Ra \right) - m\left({}^{4}He \right) \right] c^2$$

$$= \left[230.033128u - 226.025403u - 4.002603u \right] \left(931.502\ MeV/u \right)$$

$$= 4.771\, MeV$$

$$K\left({}^{226}Ra \right) = \frac{Q}{\left[1 + \frac{m\left({}^{226}Ra \right)}{m_\alpha} \right]} = 83\, keV$$

$$v\left({}^{226}Ra \right) = \sqrt{\frac{2K\left({}^{226}Ra \right)}{m\left({}^{226}Ra \right)}} = 2.66\left(10^5 \right) m/s$$

Question 17: Compute the Q values for the following β^- decays:

(a) $^{65}\text{Ni} \rightarrow {}^{65}\text{Cu}$; (b) $^{11}\text{Be} \rightarrow {}^{11}\text{B}$; (c) $^{193}\text{Os} \rightarrow {}^{193}\text{Ir}$

a) $^{65}\text{Ni} \rightarrow {}^{65}\text{Cu}$;

$$
\begin{aligned}
Q_{\beta^-} &= \left[m\left({}^{65}Ni\right) - m\left({}^{65}Cu\right) \right] c^2 \\
&= \left[64.930086u - 64.927793u \right] \left(931.502\ MeV/u \right) \\
&= 2.136 MeV
\end{aligned}
$$

b) $^{11}\text{Be} \rightarrow {}^{11}\text{B}$

$$
\begin{aligned}
Q_{\beta^-} &= \left[m\left({}^{11}Be\right) - m\left({}^{11}B\right) \right] c^2 \\
&= \left[11.021658u - 11.009305u \right] \left(931.502\ MeV/u \right) \\
&= 11.507 MeV
\end{aligned}
$$

Question 17: Compute the Q values for the following β^- decays:

(a) ^{65}Ni →^{65}Cu; (b) ^{11}Be →^{11}B; (c) ^{193}Os → ^{193}Ir

c) ^{193}Os → ^{193}Ir

$$Q_{\beta^-} = \left[m\left({}^{193}Os\right) - m\left({}^{193}Ir\right) \right] c^2$$

$$= \left[192.964138u - 192.962917u\right]\left(931.502\ MeV/u\right)$$

$$= 1.137 MeV$$

Question 18: ^{196}Au can decay by β^-, β^+, and ε. Find the Q values for the three decay modes.

a) β^- decay $\ {}^{196}\mathrm{Au} \rightarrow {}^{196}\mathrm{Hg} + \beta^- + \bar{\nu}$

$$Q_{\beta^-} = \left[m\left({}^{196}Au\right) - m\left({}^{196}Hg\right)\right]c^2$$

$$= \left[195.966544u - 195.965807u\right]\left(931.502\ MeV/u\right)$$

$$= 0.686 MeV$$

b) β^+ decay $\ {}^{196}\mathrm{Au} \rightarrow {}^{196}Pt + \beta^+ + \nu$

$$Q_{\beta^+} = \left[m\left({}^{196}Au\right) - m\left({}^{196}Pt\right)\right]c^2 - 2m_e c^2$$

$$= \left[195.966544u - 195.964926u\right]\left(931.502\ MeV/u\right) - 2\left(0.511\, MeV\right)$$

$$= 0.485 MeV$$

Question 18: ^{196}Au can decay by β^-, β^+, and ε. Find the Q values for the three decay modes.

c) $\quad$ *Electron capture* $^{196}\mathrm{Au}+\mathrm{e}^- \rightarrow {}^{l96}Pt+\nu$

Here we assume that the electron capture event occurs in the K shell.

$$Q_\varepsilon = \left[m\left(^{196}Au \right) - m\left(^{196}Pt \right) \right] c^2 - B_K$$

$$= [195.966544u - 195.964926u](931.502\ MeV/u) - 0.078395\ MeV = 1.429 MeV$$

Question 19: The maximum kinetic energy of the positron spectrum emitted in the decay $^{11}C \rightarrow {}^{11}B$ is 1.983 MeV. Use this information and the known mass of ^{11}B to calculate the mass of ^{11}C.

$m_e = 5.485803(10^{-4})u$, $m(^{11}B) = 11.009305u$

$$Q_{\beta^+} = \left[m\left(^{11}C\right) - m\left(^{11}B\right) - 2m_e \right] c^2$$

$$m\left(^{11}C\right) = \left(\frac{Q_{\beta^+}}{c^2} \right) + m\left(^{11}B\right) + 2m_e = \left[\left(\frac{1.983}{931.502} \right) + 11.009305u + 2\left(5.485803\right)\left(10^{-4}\right)u \right]$$

$$= 11.012531u$$

Question 20: The isotope ^{113}Cd captures a very low-energy neutron, leading to an excited state of ^{114}Cd, which emits a γ ray leading directly to the ^{114}Cd ground state. Find the energy of the γ ray, neglecting the nuclear recoil.

$$^{113}Cd + n \rightarrow {}^{114}Cd^* \rightarrow {}^{114}Cd + \gamma$$

γ-ray energy

$$\begin{aligned} E_\gamma &= \left[m(^{113}Cd) + m_n - m(^{114}Cd) \right] c^2 \\ &= \left[112.904400u + 1.008665u - 113.903357u \right] \left(931.502\ MeV / u \right) \\ &= 9.043\ MeV \end{aligned}$$

Question 21: The isotope ^{254}Cf decays almost exclusively by spontaneous fission, with a half-life of 60.5 days. The energy released is about 225 MeV per fission. Calculate the total fission power produced by 1.0 μg of ^{254}Cf.

$$N = \left(\frac{10^{-6}\,g}{254\;g\,/\,mol}\right)(6.02)\left(10^{23}\;atom\,/\,mol\right) = 2.37\left(10^{15}\right)$$

$$A = \lambda N = 2.37\left(10^{15}\right)\left[\frac{0.693}{(60.5\,day)(86400\;s\,/\,day)}\right] = 3.14\left(10^{8}\right)s^{-1}$$

$$P = (225\;MeV\,/\;fission)(3.14)\left(10^{8}\;fission\,/\,s\right)(1.6)\left(10^{-13}\;J\,/\,MeV\right) = 0.0113\,W$$

Question 22: Given that the activation energy of ^{236}U is 6.2 MeV, what is the minimum-energy α particle that can produce fission following bombardment of a ^{232}Th target?

$$^{232}Th + {}^{4}He \rightarrow {}^{236}U^{*}$$

$$E_{ex} = \left[m(^{236}U^{*}) - m(^{236}U) \right] c^2 + K_\alpha = \left[m(^{232}Th) + m\left(^{4}He \right) - m(^{236}U) \right] c^2 + K_\alpha$$

$$= \left(232.038051u + 4.002603u - 236.045563u \right)\left(931.502\ MeV/u \right) + K_\alpha$$

$$= -4.51\, MeV + K_\alpha > 6.2\ MeV \quad \Rightarrow \quad K_\alpha > 10.8\, MeV$$

Question 23: Compare the excitation and activation energies for thermal neutron-induced fission of (a) ^{239}Pu (b) ^{231}Pa

Activation energies of the ^{239}Pu and ^{231}Pa are 6.0 and 7.6 MeV, respectively.

$$a)\quad {}^{239}Pu + n \rightarrow {}^{240}Pu^{*}$$

$$E_{ex} = \left[m({}^{240}Pu^{*}) - m({}^{240}Pu) \right] c^{2} = \left[m({}^{239}Pu) + m_{n} - m({}^{240}Pu) \right] c^{2}$$

$$= \left(239.052158u + 1.008665u - 240.053808u \right)\left(931.502\ MeV/u \right)$$

$$= 6.5\ MeV$$

$$b)\quad {}^{231}Pa + n \rightarrow {}^{232}Pa^{*}$$

$$E_{ex} = \left[m({}^{232}Pa^{*}) - m({}^{232}Pa) \right] c^{2} = \left[m({}^{231}Pa) + m_{n} - m({}^{232}Pu) \right] c^{2}$$

$$= \left(231.035880u + 1.008665u - 232.038565u \right)\left(931.502\ MeV/u \right)$$

$$= 5.6\ MeV$$

According to these results, while Pu is fissioned with thermal neutrons, Pa must be bombarded with MeV-energized neutrons to undergo fission.

Question 24: Compute the Q values for the reactions

(a) ^{6}Li + p $\longrightarrow$ ^{3}He +^{4}He; (b) ^{59}Co + p $\longrightarrow$ n + ^{59}Ni;

(c) ^{40}Ca + α $\longrightarrow$ n + ^{43}Ti.

$$a)\quad {}^{6}Li + p \to {}^{3}He + {}^{4}He$$

$$Q = \left[m\left({}^{6}Li\right) + m\left({}^{1}H\right) - m\left({}^{3}He\right) - m\left({}^{4}He\right) \right] c^{2}$$

$$= \left(6.015121u + 1.007825u - 3.016029u - 4.002603u\right)\left(931.502\ MeV/u\right)$$

$$= 4.018\ MeV$$

$$b)\quad {}^{59}Co + p \to n + {}^{59}Ni$$

$$Q = \left[m\left({}^{59}Co\right) + m\left({}^{1}H\right) - m_{n} - m\left({}^{59}Ni\right) \right] c^{2}$$

$$= \left(58.933198u + 1.007825u - 1.008665u - 58.934349u\right)\left(931.502\ MeV/u\right)$$

$$= -1.855 MeV$$

The threshold kinetic energy for this endoergic reaction

$$E_{th} = \left|-Q\right| \left(1 + \frac{m_{x}}{M_{X}}\right) = 1.855\left(1 + \frac{1.007825u}{59.933198u}\right) = 1.886\ MeV$$

Question 24: Compute the Q values for the reactions

(a) $^{6}\text{Li} + p \rightarrow {}^{3}\text{He} + {}^{4}\text{He}$;   (b) $^{59}\text{Co} + p \rightarrow n + {}^{59}\text{Ni}$;

(c) $^{40}\text{Ca} + \alpha \rightarrow n + {}^{43}\text{Ti}$.

$$c)\quad {}^{40}Ca + \alpha \rightarrow n + {}^{43}Ti$$

$$Q = \left[m\left({}^{40}Ca\right) + m_{\alpha} - m_{n} - m\left({}^{43}Ti\right)\right]c^{2}$$

$$= \left(39.962591u + 4.002603u - 1.008665u - 42.968523u\right)\left(931.502\ MeV/u\right)$$

$$= -11.172 MeV$$

The threshold kinetic energy for this endoergic reaction

$$E_{th} = \left|-Q\right|\left(1 + \frac{m_x}{M_X}\right) = 11.172\left(1 + \frac{4.002603u}{39.96259u}\right) = 12.291\,MeV$$

Question 25: How many C^{14} nuclei are required to achieve 1 Ci of activity. $T_{1/2}$ =5730 year

Activity

$$A = \lambda N = \frac{0.693}{5730\,(365)(86400s)} \frac{m\left(6.02x10^{23}\ atom/mol\right)}{14\ g/mol} = 1x3.7\left(10^{10}\ \frac{atom}{s}\right)$$

$$m = 0.224\ g$$

Question 26: It is desired to determine the age of a wood timber used to construct an ancient shelter. A sample of the wood is analyzed for its ^{14}C content and gives 2.1 decays per minute. Another sample of the same size form a recently cut tree of the same type gives 5.3 decays per minute. What is the age of the sample.

$$A(t_1) = A_0\, e^{-\lambda t_1} \quad \Rightarrow \quad t_1 = \left(\frac{1}{\lambda}\right)\ln\left(\frac{A_0}{A(t_1)}\right) = \left(\frac{t_{1/2}}{0.693}\right)\ln\left(\frac{A_0}{A(t_1)}\right)$$

$$= \left(\frac{5730y}{0.693}\right)\ln\left(\frac{5.3}{2.1}\right) = 7.65\left(10^3\right)y$$