Chapter 23 Section 5-6-7:
Continous Charge Distributions

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Reference Book: "Physics for Scientists and Engineers" by R. A. Serway & J. W. Hewett

Similar Book: "Physics for Scientists&Engineers" by D.C.Giancoli

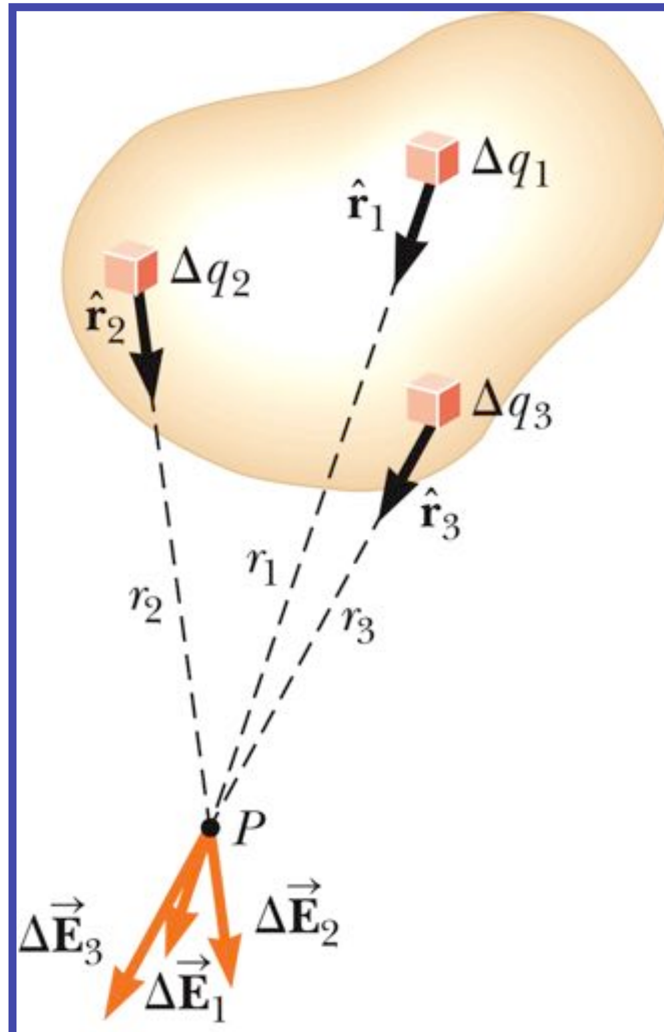
Continuous Charge Distributions

- The distances between charges in a group of charges may be **much smaller than the distance between the group and a point of interest.**
- In this situation, **the system of charges can be modeled as continuous.**
- The system of closely spaced charges is equivalent to a total charge that is continuously distributed along some line, over some surface, or throughout some volume.

Continuous Charge Distributions

Procedure

- Divide the charge distribution into small elements, each containing a small **charge Δ*q*.**
- Calculate the electric field due to one of these elements at point **P**.
- Evaluate the total field by summing the contributions of all of the charge elements.



- For the **individual charge elements:**

$$\Delta\vec{\mathbf{E}} = k_e \frac{\Delta q}{r^2}\hat{\mathbf{r}}$$

- Because **the charge distribution is continuous:**

$$\vec{\mathbf{E}} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^2}\hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}}$$

Charge Densities

Volume Charge Density

- When a charge **Q** is distributed evenly throughout a volume **V**, the ***Volume Charge Density*** is defined as:

$\rho \equiv (Q/V)$ (Units are C/m^3)

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<u>Surface Charge Density</u>

- When a charge **Q** is distributed evenly over a surface area **A**, the ***<u>Surface Charge Density</u>*** is defined as:

$\boldsymbol{\sigma \equiv Q/A}$ (Units are C/m^2)

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Volume Charge Density

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  $\rho \equiv (Q/V)$ (Units are C/m^3)

Surface Charge Density

- When a charge **Q** is distributed evenly over a surface area **A**, the ***Surface Charge Density*** is defined as:

  $\sigma \equiv Q/A$ (Units are C/m^2)

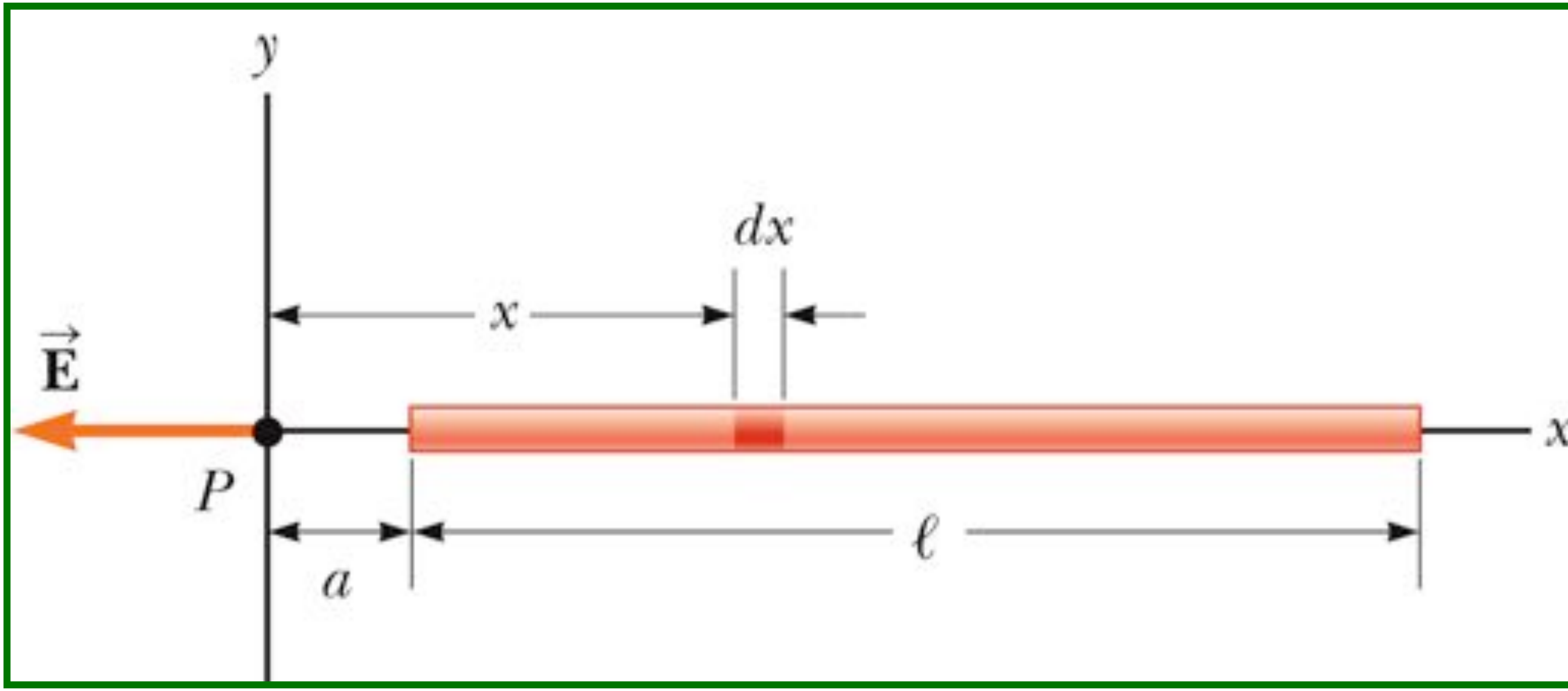
Linear Charge Density

- When a charge **Q** is distributed along a line ℓ, the ***Line Charge Density*** is defined as:

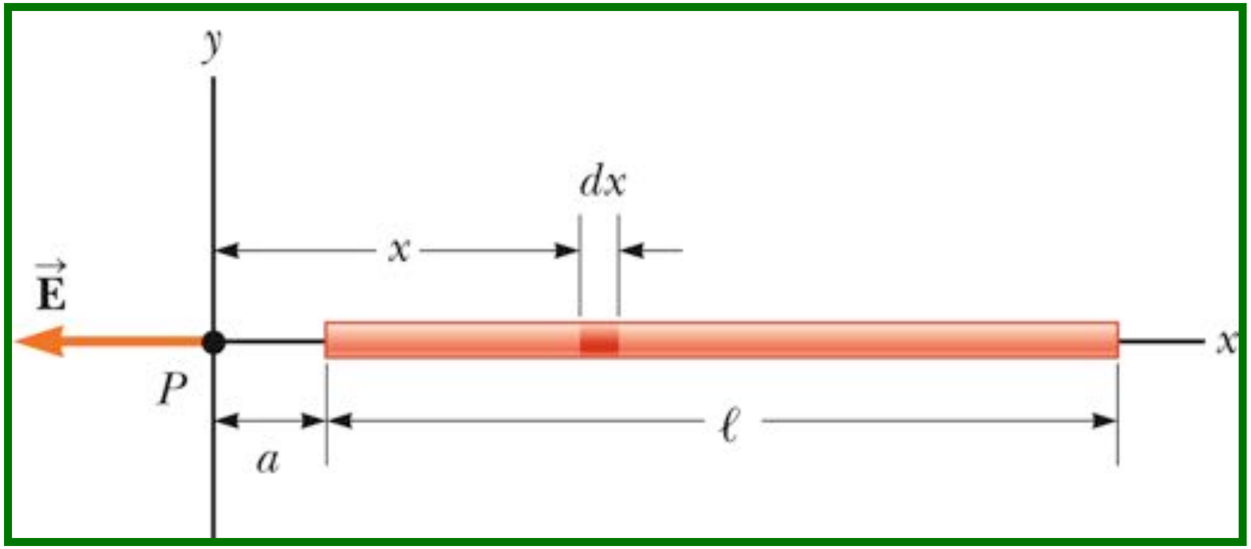
  $\lambda \equiv (Q/\ell)$ (Units are C/m)

Example 23.7:

Electric Field Due to a Charged Rod



Example 23.7:
Electric Field Due to a Charged Rod



$dq = \lambda dx$, so $dE = k_e[dq/(x^2)] = k_e[(\lambda dx)/(x^2)]$

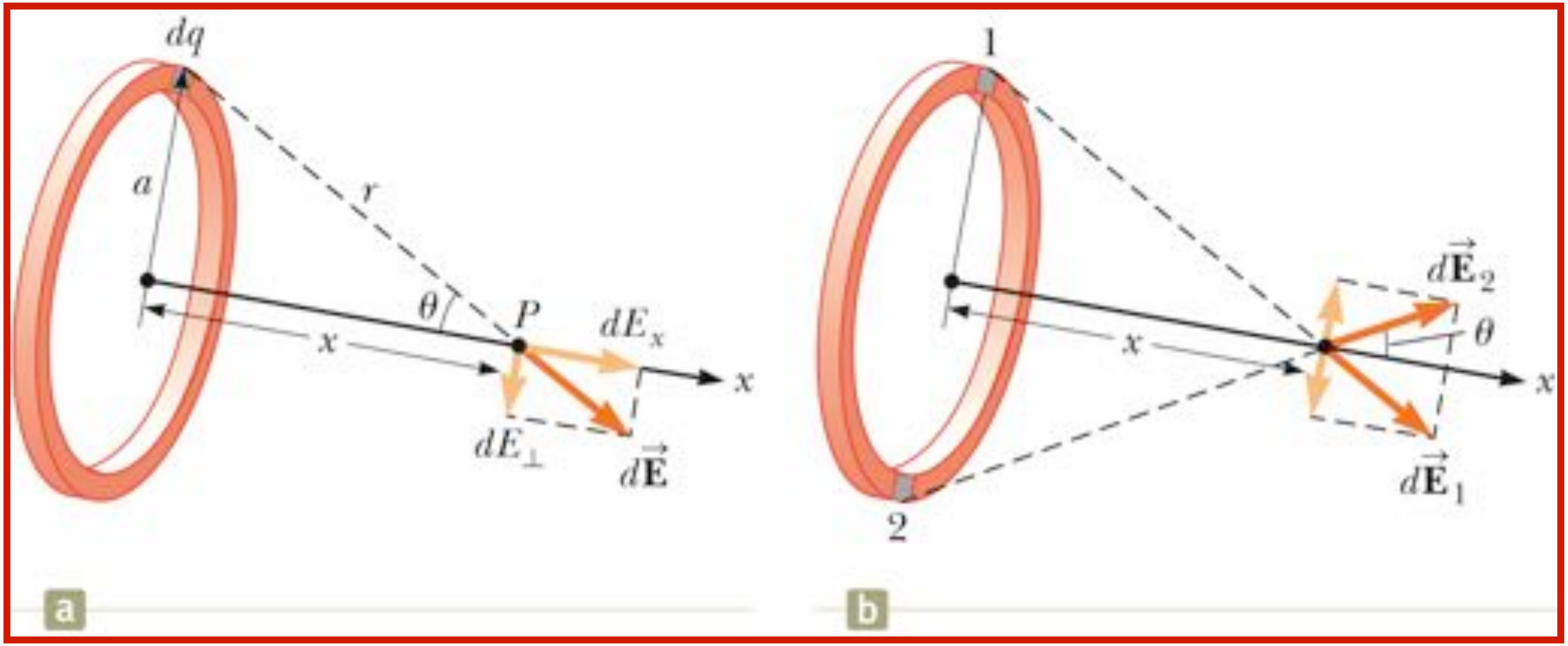
And $E = k_e \lambda \int [(dx)/(x^2)]$

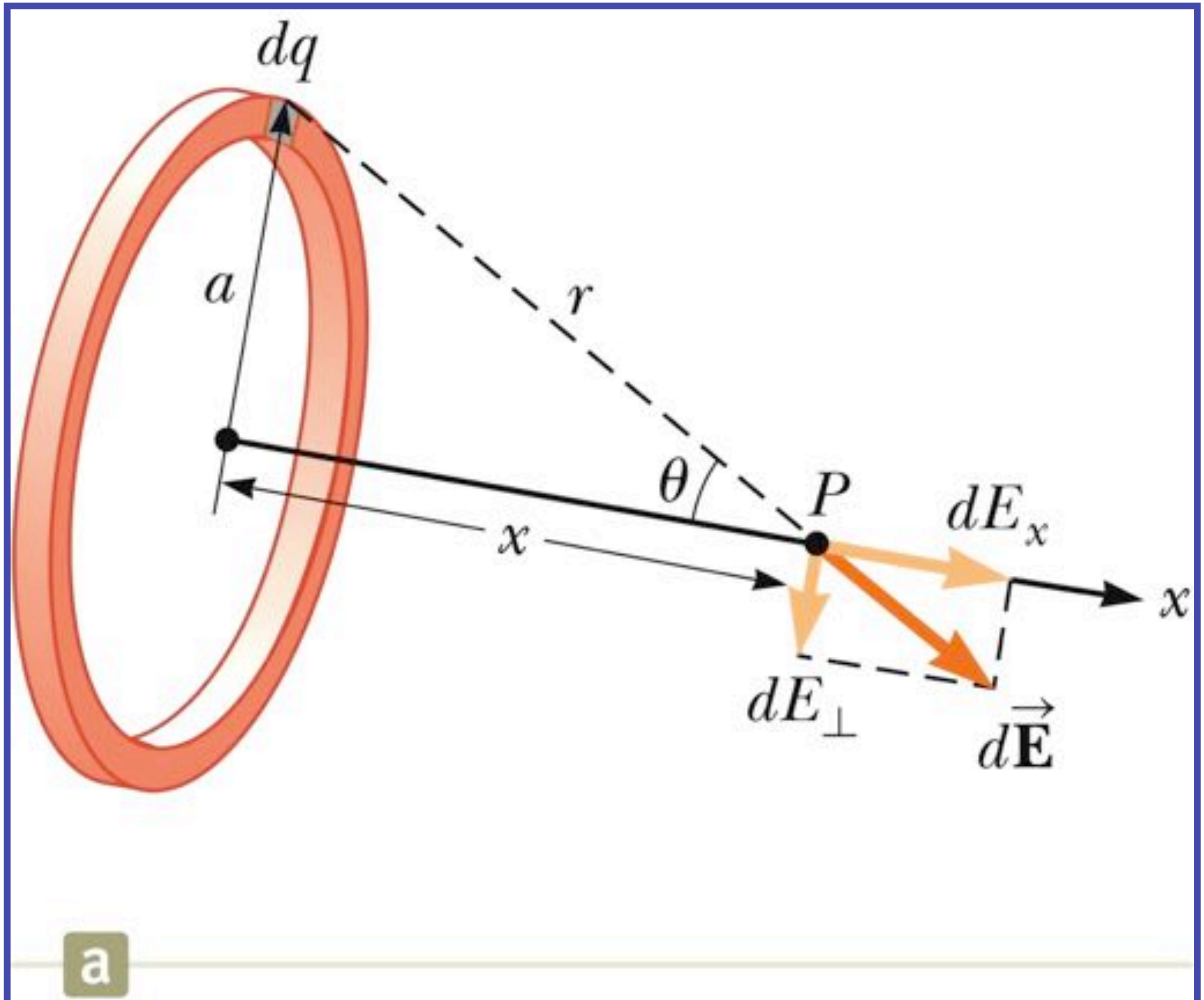
(limits $x = a$ to $x = l$)

$$E = k_e \lambda \, [(1/a) - 1/(a + l)]$$

Example 23.8:

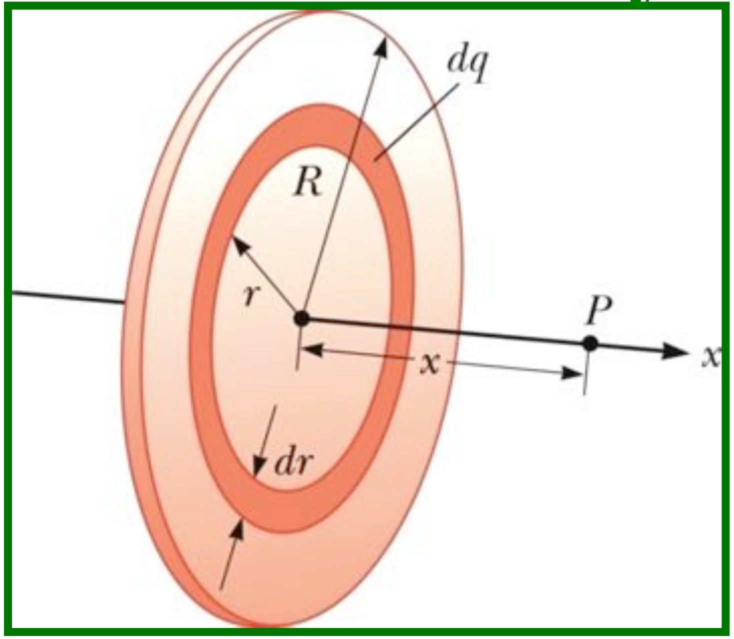
Electric Field of a Uniform Ring of Charge




dq
a
r
θ
P
x
dE_x
x
dE_⊥
dE
a

Example 23.9:

Electric Field of a Uniformly Charged Disk



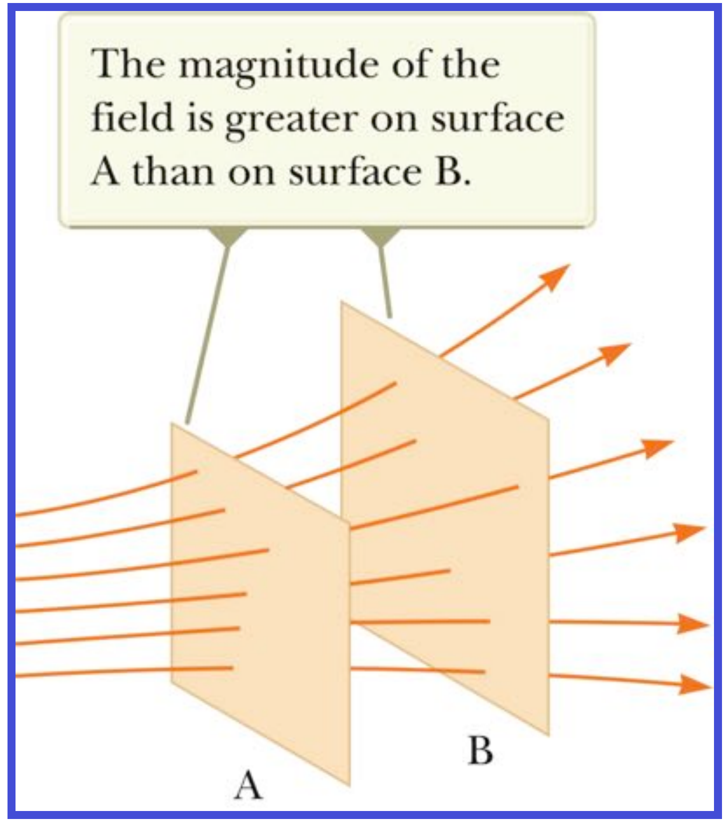
- The disk has radius **R** & uniform charge density **σ**.
- Choose **dq** as a ring of radius **r**.
- The ring has a surface area **2πr dr.**
- Integrate to find the total field.

Electric Field Lines

- **Field lines** give ***a means of representing the electric field pictorially***.
- The electric field vector is **tangent to the electric field line** at each point.
  - The line has a direction that is the same as that of the electric field vector.
- The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region.

Electric Field Lines, General

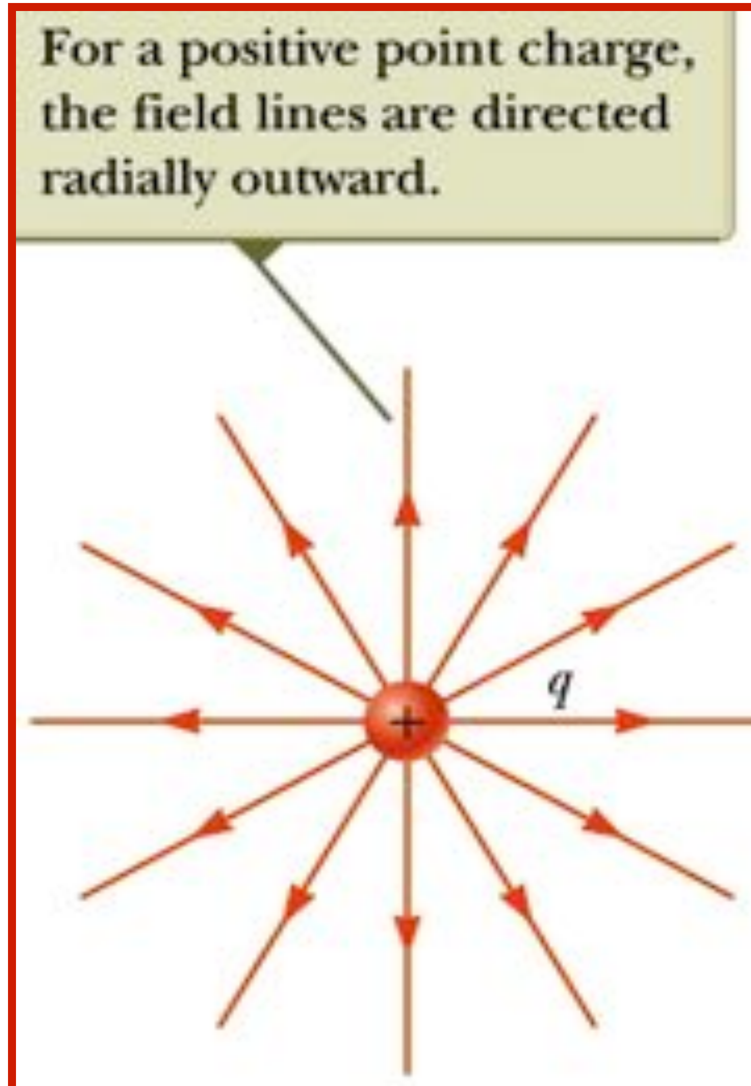
- In the figure, the density of lines through surface A is greater than through surface B.
- The **magnitude of the electric field is greater on surface A than B.**
- The lines at different locations point in different directions.
  - This indicates that the field is nonuniform.



Electric Field Lines:

Positive Point Charge

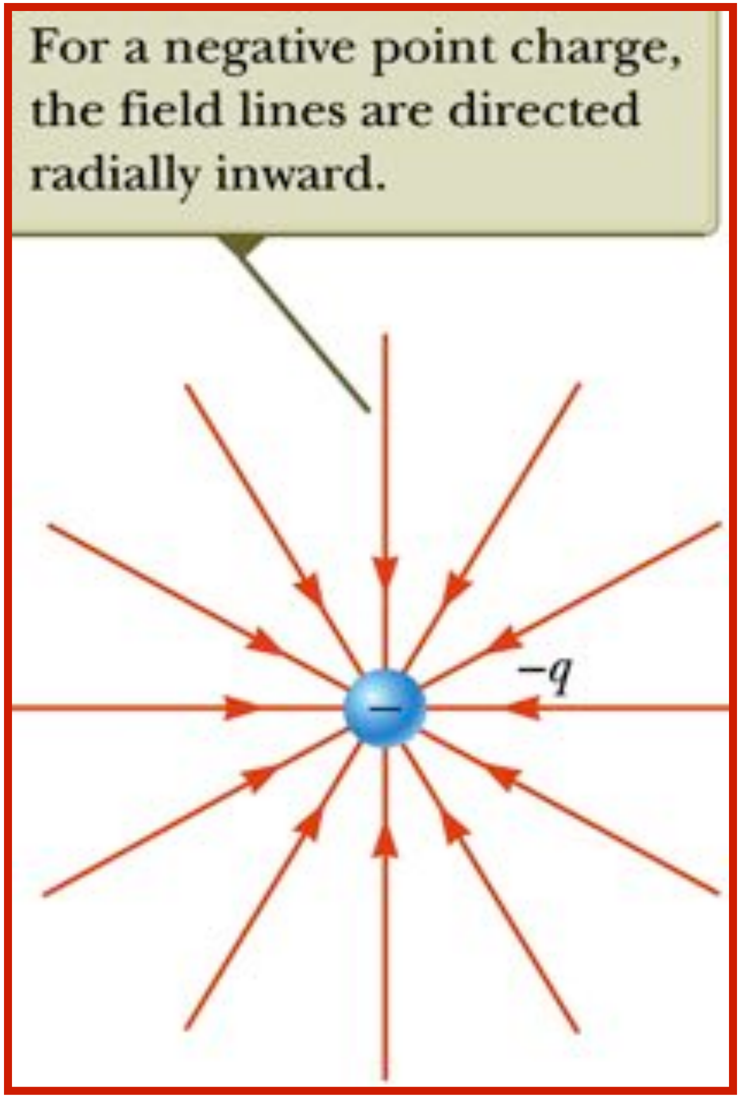
- The ***field lines radiate outward*** in all directions.
- In **three dimensions**, the ***distribution is spherical***.
- The field lines are **Directed *away*** from a **Positive Source Charge.**
- So, a positive test charge would be repelled away from the positive source charge.



Electric Field Lines:

Negative Point Charge

- The ***field lines radiate inward*** in all directions.
- In **three dimensions**, the ***distribution is spherical***.
- The field lines are **Directed *Towards*** a **Negative Source Charge.**
- So, a positive test charge would be attracted to the negative source charge.



Electric Field Lines – Rules for Drawing

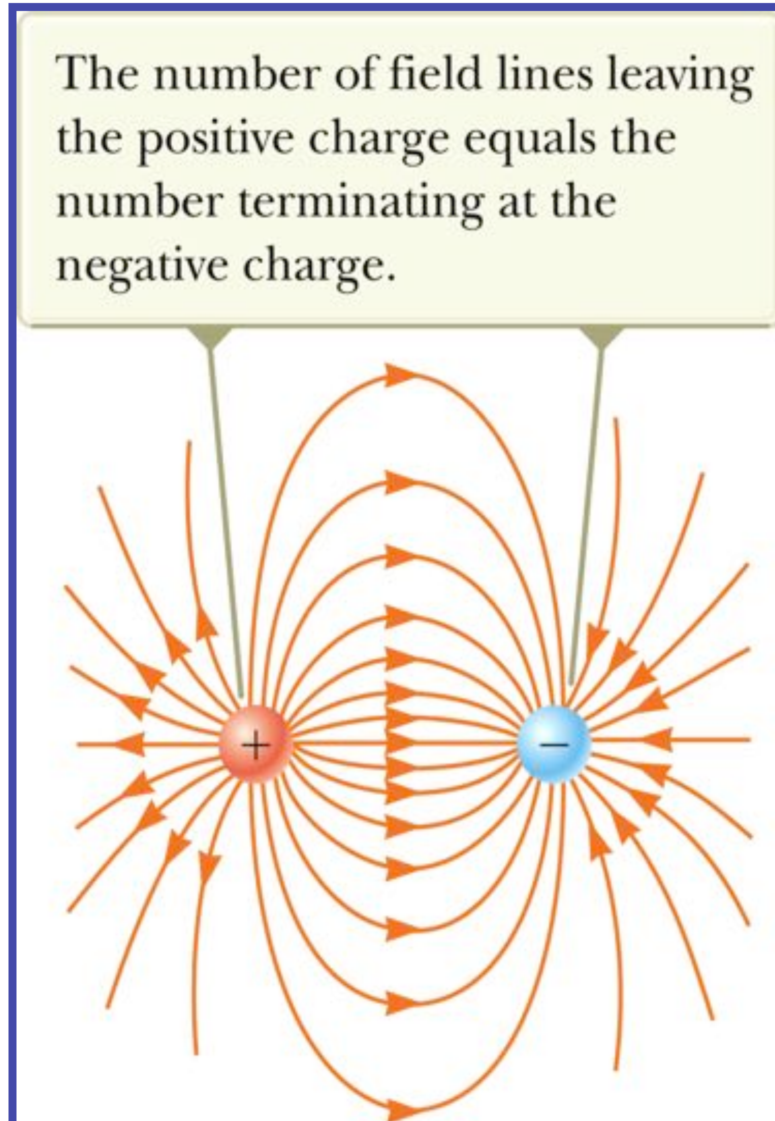
- The lines must ***begin on a positive charge*** & ***terminate on a negative charge***.
- In the case of an excess of one type of charge, some lines will begin or end infinitely far away.
- The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.

No two field lines can cross.

- Remember the field lines are **not** material objects, they are a ***pictorial representation*** used to qualitatively describe the electric field.

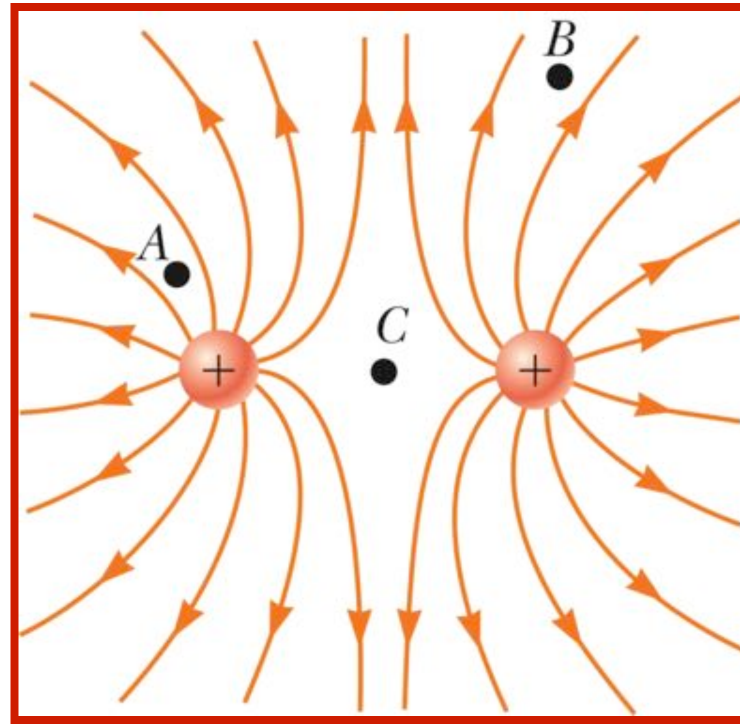
Electric Field Lines – Electric Dipole

- The charges are equal & opposite.
- The number of field lines leaving the positive charge equals the number of lines terminating on the negative charge.



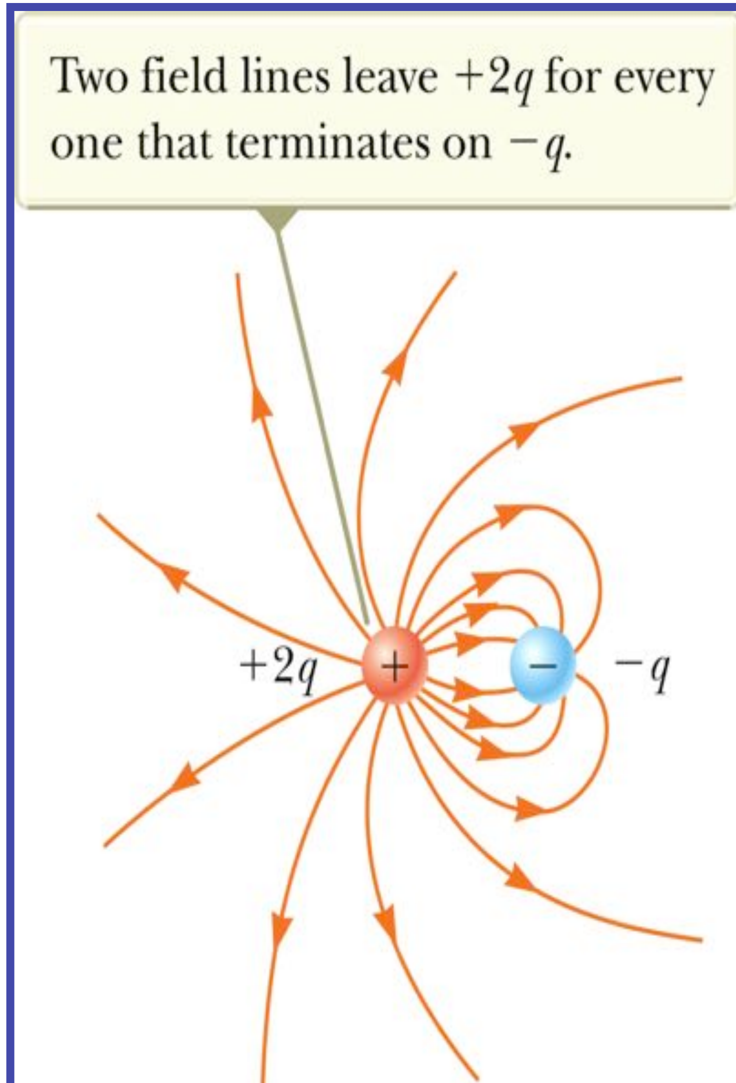
Electric Field Lines – Like Charges

- The charges are equal & positive.
- The same number of lines leave each charge since they are equal in magnitude.
- At a great distance away, the field is approximately equal to that of a single charge of **2*q***.
- Since there are no negative charges available, the field lines end infinitely far away.



Electric Field Lines, Unequal Charges

- The positive charge is twice the magnitude of the negative charge.
- Two lines leave the positive charge for each line that terminates on the negative charge.
- At a great distance away, the field would be approximately the same as that due to a single charge of **+*q***.



Motion of Charged Particles

- When a charged particle is placed in an electric field, it experiences an electric force.
- If this is the only force on the particle, it must be the net force.
- The net force will cause the particle to accelerate according to

<u>Newton's 2nd Law.</u>

Motion of a Charged Particle in an Electric Field

- From **Coulomb's Law**, the force on an object of charge q in an electric field **E** is:

$$\mathbf{F} = q\mathbf{E}$$

- So, if the mass & charge of a particle are known, its motion in an electric field can be calculated by

Combining Coulomb's Law with Newton's 2nd Law:

$$\mathbf{F} = m\mathbf{a} = q\mathbf{E}$$

$$\vec{F}_e = q\vec{E} = m\vec{a}$$

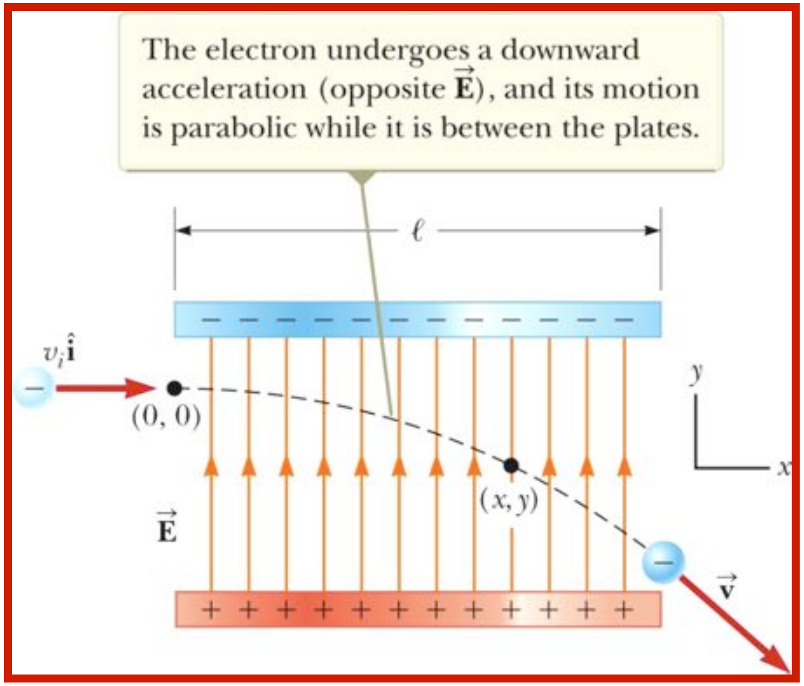
- If the field **E** is uniform, then the acceleration is constant. So, to describe the motion of a particle in the field, the kinematic equations from Physics I can be used, with the acceleration

$$\mathbf{a = q(E/m)}$$

- If the particle has a positive charge, its acceleration is in the direction of the field.
- If the particle has a negative charge, its acceleration is in the direction opposite the electric field.

Example 23.11:
Electron in a Uniform E Field

- An electron is projected horizontally into a uniform electric field **E** (Figure). It undergoes a downward acceleration.
  - It is a negative charge, so the acceleration is opposite to the direction of the field.



- Its motion follows a parabolic path while it is between the plates.
- Solving this problem is identical mathematically to the problem of projectile motion in Physics I!!!

Section 21-7: Electric Field Calculations for Continuous Charge Distributions

To do **electric force** (**F**) & **field** (**E**) calculations for a continuous distribution of charge, treat the distribution as a succession of infinitesimal (point) charges.

The total field* E *is then the integral of the infinitesimal fields due to each bit of

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2}.$$

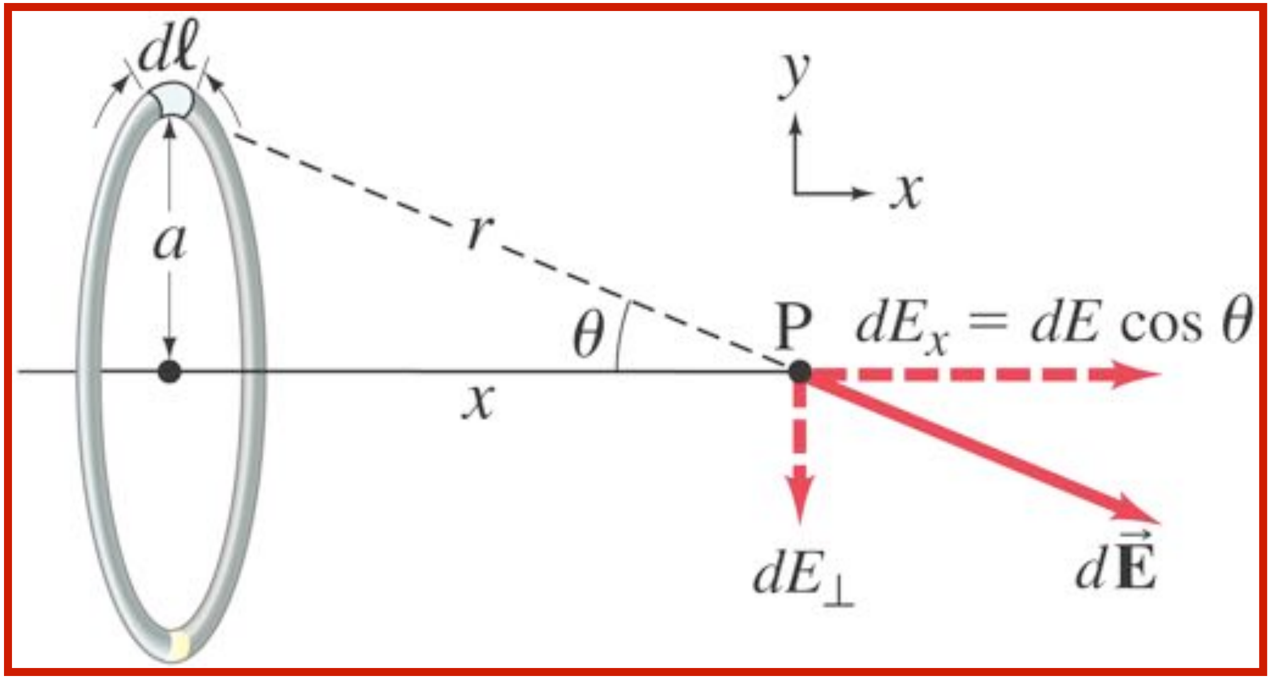
$$\vec{\mathbf{E}} = \int d\vec{\mathbf{E}}.$$

The electric field E is a vector, so *a separate integral for each component* is needed.

Example 21-9: A ring of charge.

A thin, ring-shaped object of radius ***a*** holds a total charge **+*Q*** distributed uniformly around it. Calculate the electric field **E** at a point **P** on its axis, a distance ***x*** from the center. Let ***λ*** be the charge per unit length (C/m).

Note: For a uniform ring of charge **Q**, = ***λ*** **= Q/(2πa)**



Conceptual Example 21-10:
Charge at the Center of a Ring

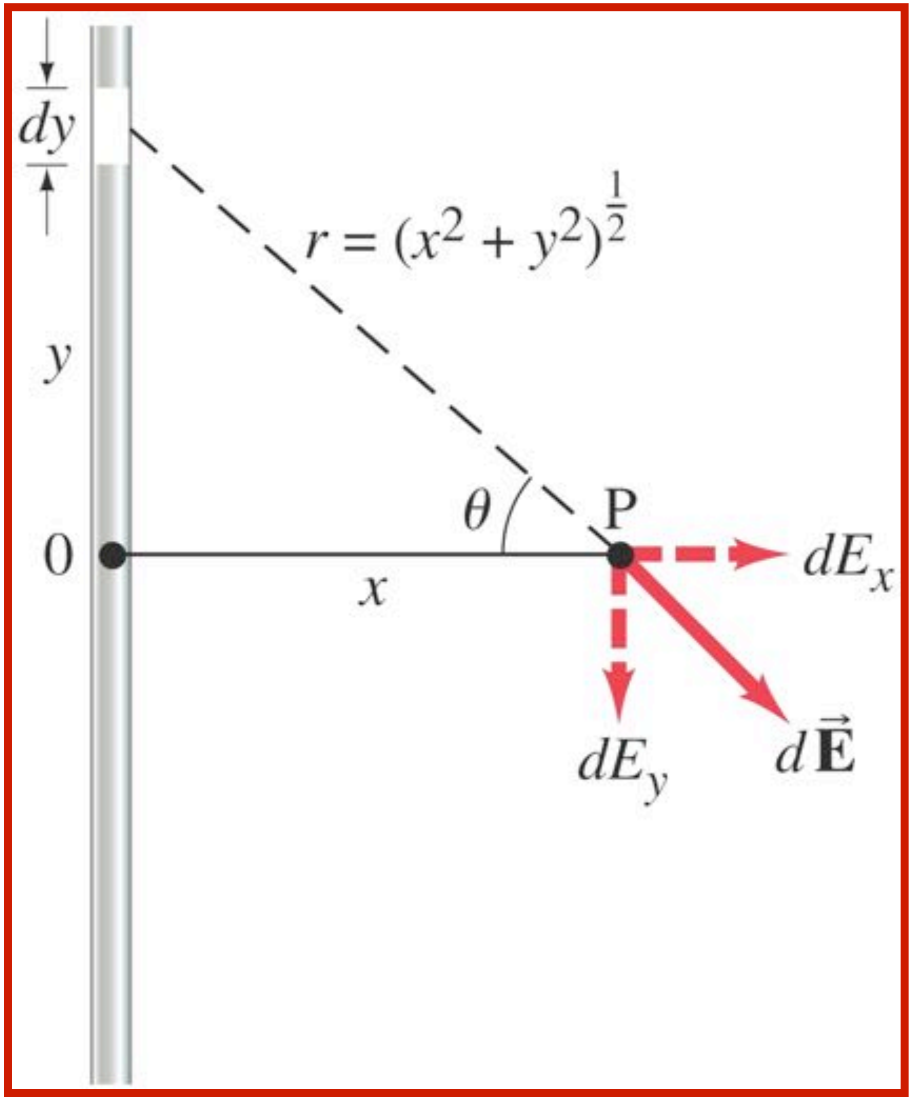
- A small positive charge q is put at the center of a nonconducting ring which has a uniformly distributed negative charge.
- Is the positive charge in equilibrium (total force on it = 0) if it is displaced slightly from the center along the axis of the ring, and if so is it stable?
- What if the small charge is negative?
- Neglect gravity, as it is much smaller than the electrostatic forces.

Example 21-11:

Long Line of Charge.

Calculate the magnitude of the electric field **E** at any point **P** a distance ***x*** from a very long line (a wire, say) of uniformly distributed charge. Assume that ***x*** is much smaller than the length of the wire.

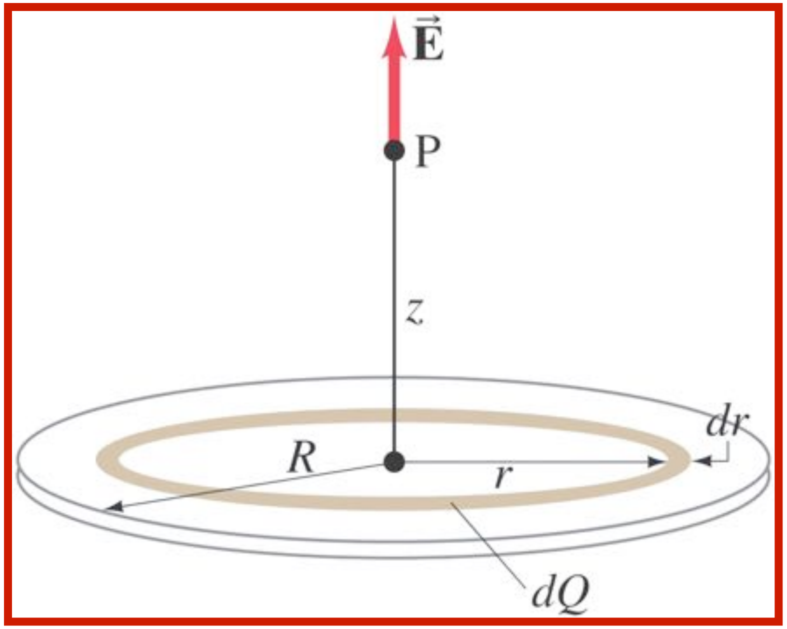
Let λ be the charge per unit length (C/m).



Example 21-12

Uniformly charged disk.

Charge is distributed uniformly over a thin circular disk of radius ***R***. The charge per unit area (C/m^2) **is σ.** Calculate the electric field **E** at a point **P** on the axis of the disk, a distance ***z*** above its center.



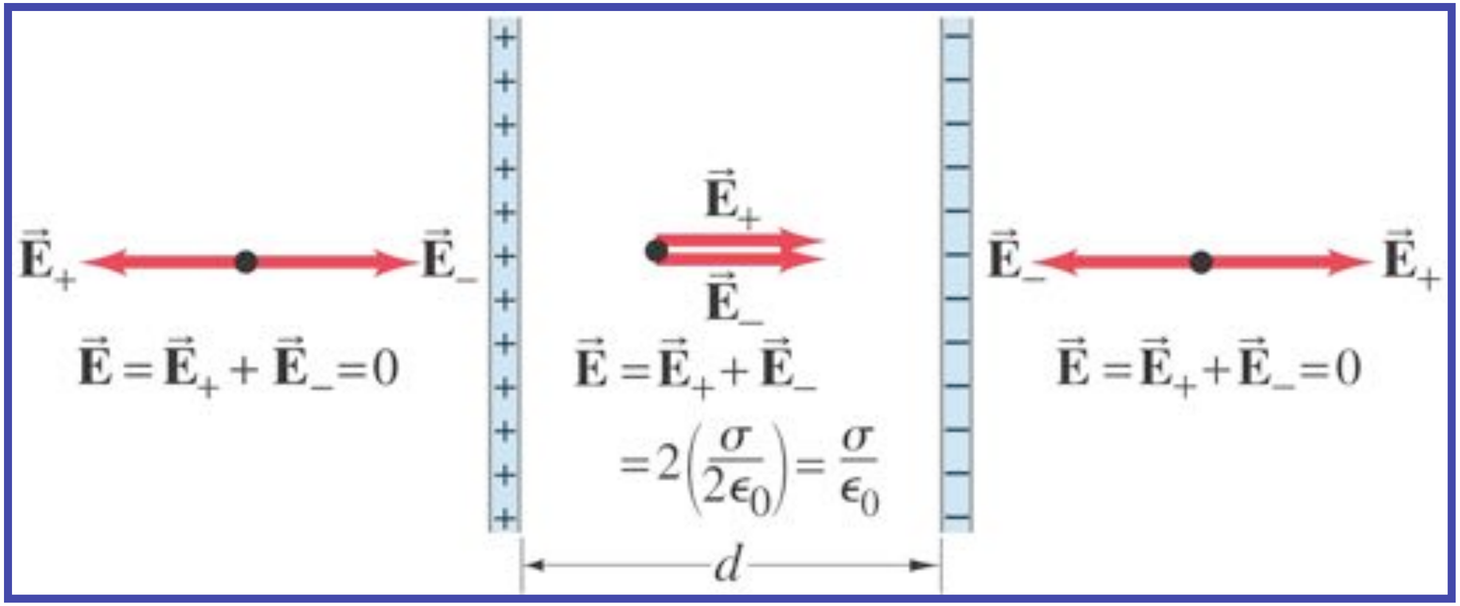
In the previous example, if we calculate the electric field **E** in the approximation that **z** is very close to the disk (that is, **z << R**), the electric field is:

$$E = \frac{\sigma}{2\epsilon_0}.$$

This is the field due to an infinite plane of charge.

Example 21-13: Two parallel plates.

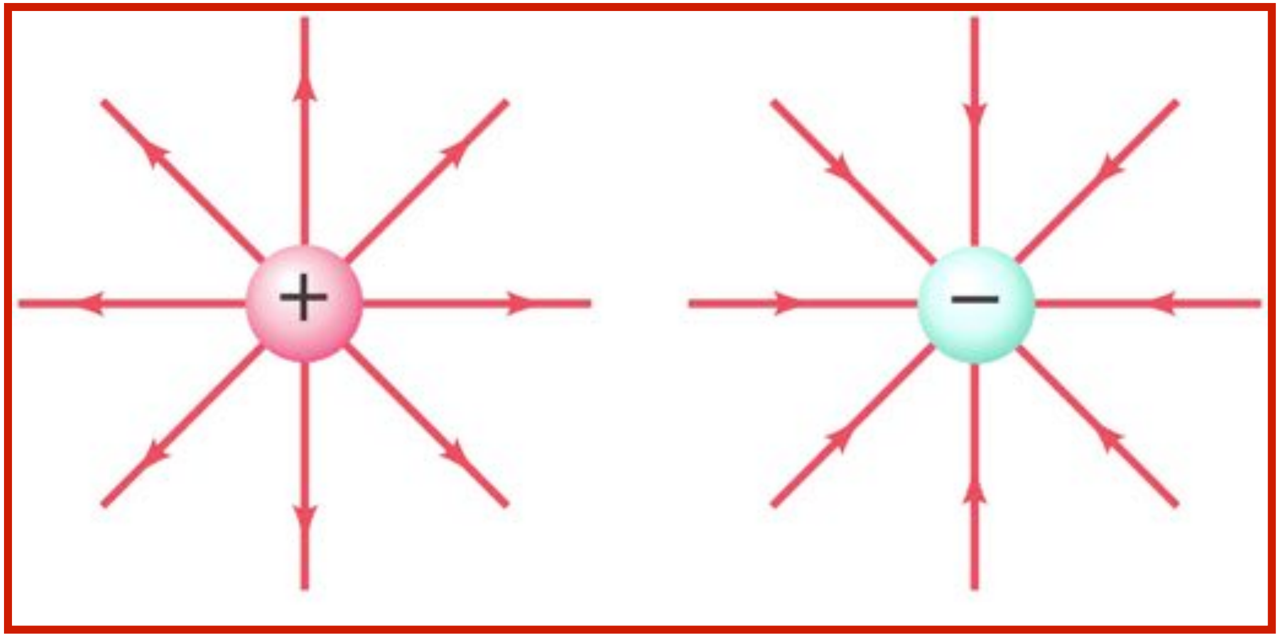
Calculate the electric field **E** between two large parallel plates (sheets), which are very thin & are separated by a distance ***d*** which is small compared to their height & width. One plate carries a uniform surface charge density **σ** . The other carries a uniform surface charge density **–σ** as shown (the plates extend upward & downward beyond the part shown).



Section 21-8: Field Lines

The electric field can be represented by ***FIELD LINES.***

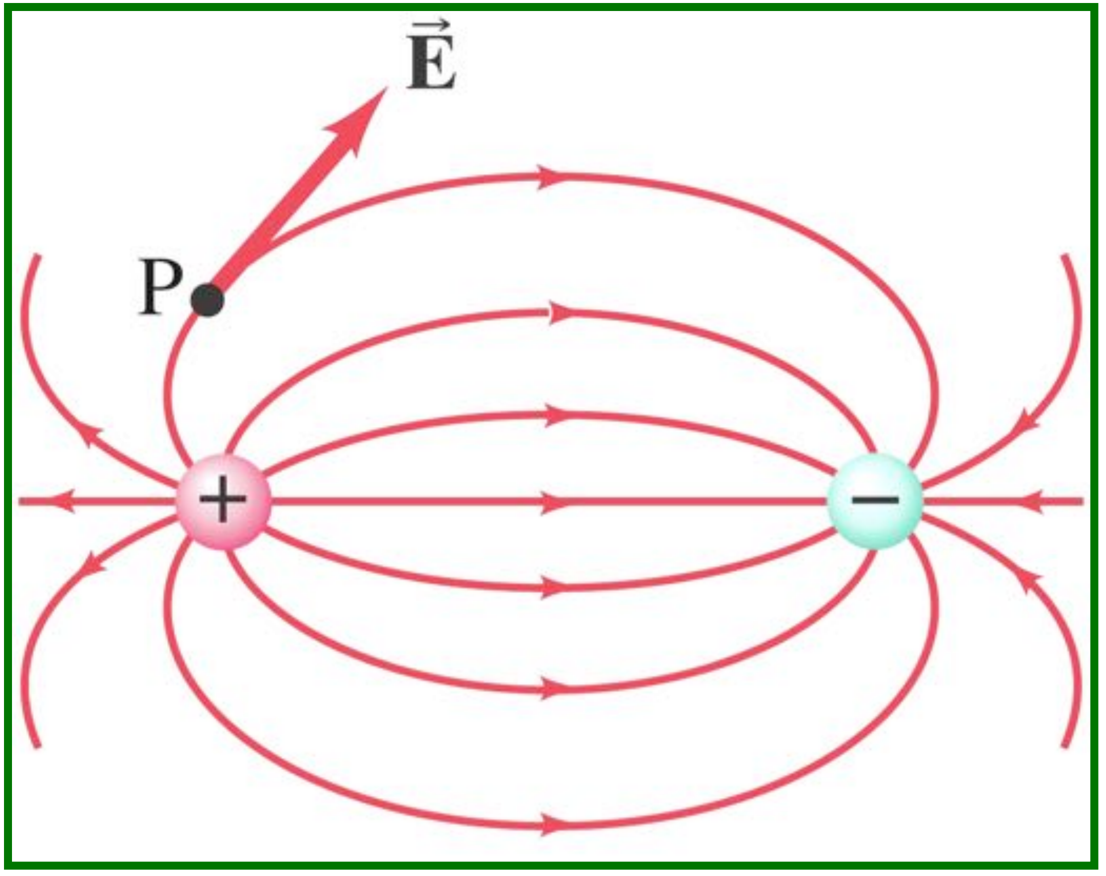
These start on a positive charge & end on a negative charge.

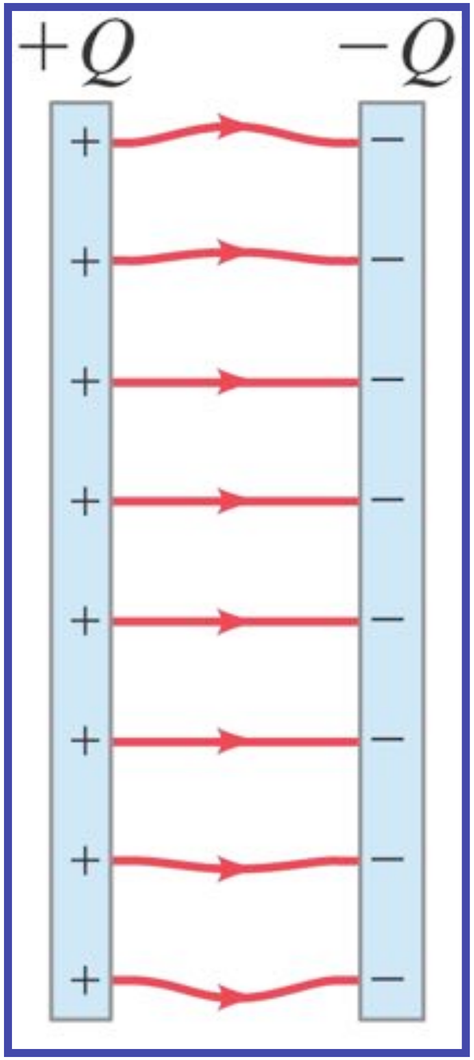


The number of field lines starting (ending) on a positive (negative) charge is proportional to the magnitude of the charge. The electric field is stronger where the field lines are closer together.

Electric Dipole

Two equal charges, opposite in sign.





Between two closely spaced, oppositely charged parallel plates,

the electric field

E ***is a constant.***

Summary of Field Lines

1. Field lines indicate the direction of the field; the field is tangent to the line.
2. The magnitude of the field is proportional to the density of the lines.
3. Field lines start on positive charges & end on negative charges; the number is proportional to the magnitude of the charge.