# Circuits Containing Resistors \& Capacitors (RC Circuits) 

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"In an increasingly complex world, sometimes old questions require new answers."

## RC Circuits

When the switch is closed, the capacitor will begin to charge. As it does, the voltage across it increases, and the current through the resistor decreases.


- To find the voltage as a function of time, use the Loop Rule to write the equation for the voltage changes around the loop:

$$
\mathscr{E}=I R+\frac{Q}{C}
$$

- $\mathbf{I}=\mathbf{d Q} / \mathbf{d t}$, so integrate to find the charge as a function of time:

$$
Q=C \mathscr{E}\left(1-e^{-t / R C}\right) .
$$

- The voltage across the capacitor is $\mathbf{V}_{\mathbf{C}}=\mathbf{Q} / \mathbf{C}$. The current $I$ at time $\mathbf{t}$ is found by differentiating the charge:

$$
I=\frac{d Q}{d t}=\frac{\mathscr{E}}{R} e^{-t / R C}
$$

- The quantity $\mathbf{R C}$ that appears in the exponent is called the time constant of the circuit: $\tau=R C$.


## Example <br> RC Circuit, with EMF.

-The capacitance in the circuit shown is
$\mathbf{C}=\mathbf{0 . 3 0} \boldsymbol{\mu \mathrm { F }}$, the total resistance is
$\mathbf{R}=20 \mathrm{k} \Omega$, the battery emf is
$\mathrm{E}=12 \mathrm{~V}$. Calculate:
(a) the time constant,

(b) the maximum charge the capacitor could acquire,
(c) the time it takes for the charge to reach $\mathbf{9 9 \%}$ of this value,
(d) the current $\mathbf{I}$ when the charge $\mathbf{Q}$ is half its maximum value,
(e) the maximum current,
(f) the charge $\mathbf{Q}$ when the current $\mathbf{I}$ is $\mathbf{0 . 2 0}$ of its maximum value.

- If an isolated charged capacitor is connected across a resistor, it discharges:

$$
Q=Q_{0} e^{-t / R C} .
$$

- The voltage \& current as functions of time can be found from the charge:

$$
V_{\mathrm{C}}=V_{0} e^{-t / R C}
$$


and $I=-\frac{d Q}{d t}=\frac{Q_{0}}{R C} e^{-t / R C}=I_{0} e^{-t / R C}$.

## Example: Discharging $R C$ circuit.

-In the RC circuit shown, the battery has fully charged the capacitor, so $\mathbf{Q}_{0}=\mathbf{C} E$. Then at $\mathbf{t}=\mathbf{0}$ the switch is thrown from position a to b . The battery emf is 20.0 V , and the capacitance $\mathbf{C}=\mathbf{1 . 0 2} \mu \mathbf{F}$. The current $\mathbf{I}$ is observed to decrease to 0.50 of its initial value in $40 \mu$ s.
(a) What is the value of $\mathbf{Q}$, the charge on the capacitor, at $\mathbf{t}=\mathbf{0}$ ?
(b) What is the value of $R$ ? (c) What is $Q$ at $t=60 \mu s$ ?


## Conceptual Example: Bulb in RC circuit.

- In the circuit shown, the capacitor is originally uncharged.
- Describe the behavior of the lightbulb from the instant switch S is closed until a long time later.



## Example

 Resistor in a turn signal. Estimate the order of magnitude of the resistor in a turn-signal circuit.

## Electric Hazards

- Most people can "feel" a current of $\mathbf{1} \mathbf{m A}$; a few mA of current begins to be painful. Currents above $\mathbf{1 0} \mathbf{~ m A}$ may cause uncontrollable muscle contractions, making rescue difficult.
- Currents around $\mathbf{1 0 0} \mathbf{~ m A}$ passing through the torso can cause death by ventricular fibrillation.
- Higher currents may not cause fibrillation, but can cause severe burns.
- Household voltage can be lethal if you are wet and in good contact with the ground. Be careful!

A person receiving a shock has become part of a complete circuit.


Faulty wiring and improper grounding can be hazardous. Make sure electrical work is done by a professional.


- The safest plugs are those with three prongs; they have a separate ground line.
- Here is an example of household wiring - colors can vary, though! Be sure you know which is the hot wire before you do anything.


