<u>Chapter 29b:</u>

Force on an Electric Charge

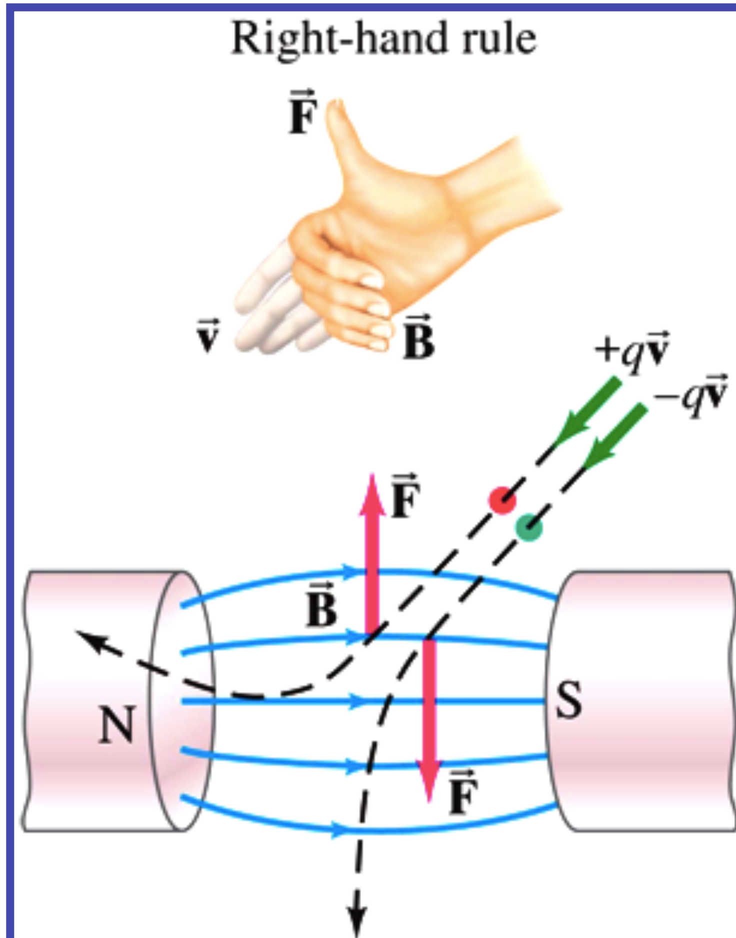
<u>*Moving*</u> in a Magnetic Field

Force on a *Moving* Charge in a Uniform Magnetic Field

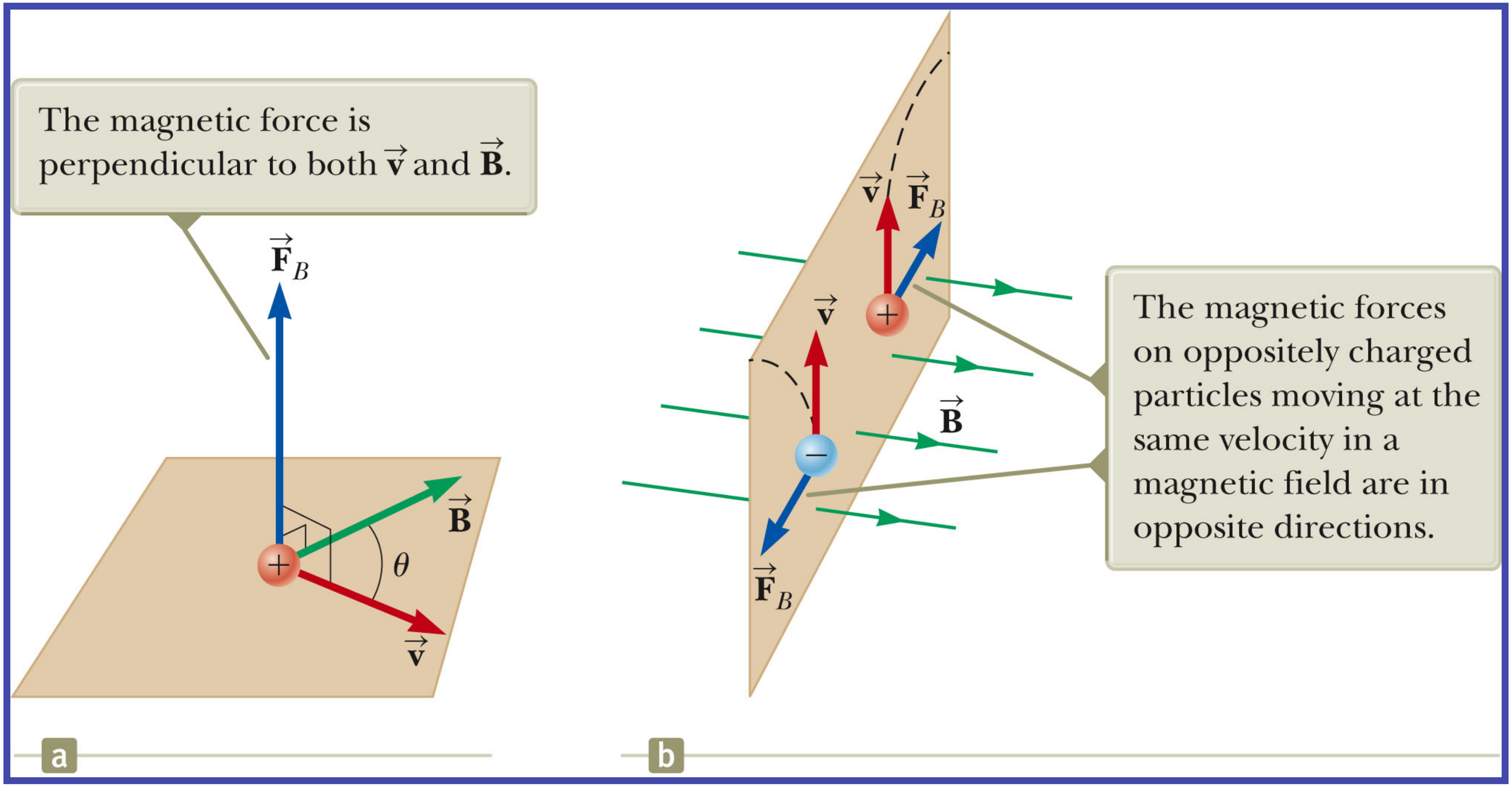
- For this case, the **force F** is obviously related to the force on a current & is given by:

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}.$$

- Again, the direction of **F** is given by a ***Right-Hand Rule***.

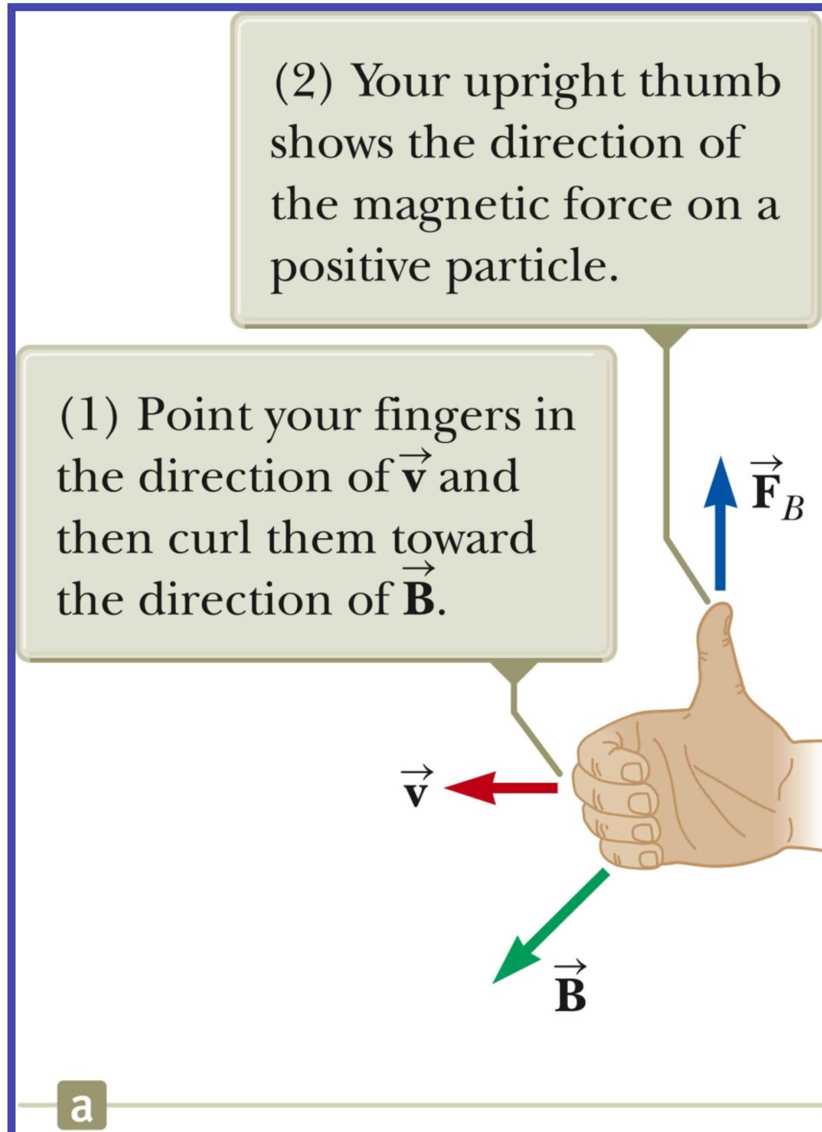


More About Direction



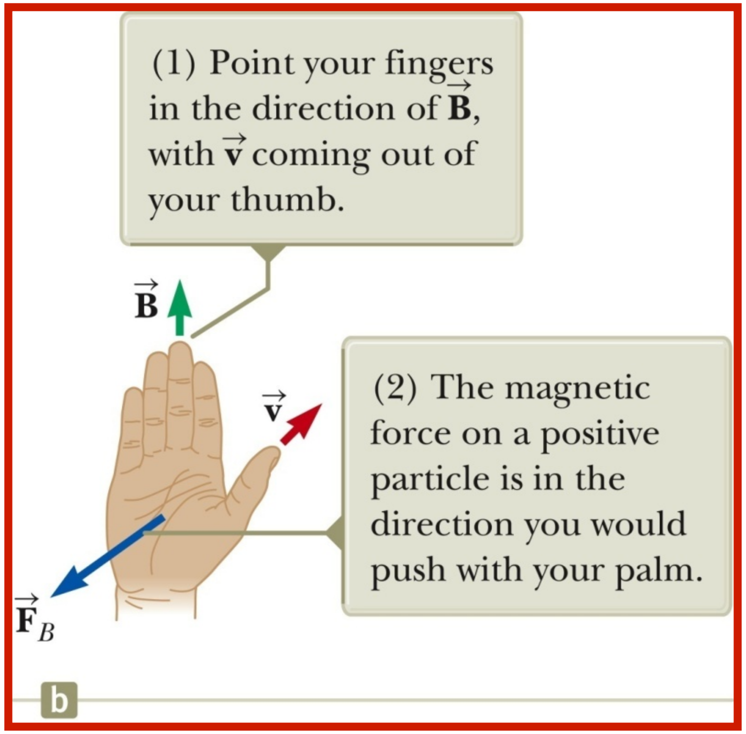
Direction: Right-Hand Rule #1

- This rule is based on the right-hand rule for the cross product.
- Your thumb is in the direction of the force if **q** is positive.
- The force is in the opposite direction of your thumb if **q** is negative.



Direction: Right-Hand Rule #2
Alternative to Rule #1

- The force on a positive charge extends outward from the palm.
- The advantage of this rule is that the force on the charge is in the direction you would push on something with your hand.
- The force on a negative charge is in the opposite direction.



More About the Magnitude of F

- The magnitude of the magnetic force on a charged particle is

$$F_B = |q| \, v \, B \sin\theta$$

- θ is the smaller angle between **v** & **B**
- F_B is zero when the field & velocity are parallel or antiparallel **($\theta = 0$ or 180^o)**
- F_B is a maximum when the field & velocity are perpendicular **$\theta = 90^o$**

Differences Between Electric & Magnetic Fields

Direction of the Force on a Point Charge

- The electric force acts along the direction of the electric field.
- The magnetic force acts perpendicular to the magnetic field.

Motion

- The electric force acts on a charged particle regardless of whether the particle is moving.
- The magnetic force acts on a charged particle only when the particle is in motion.

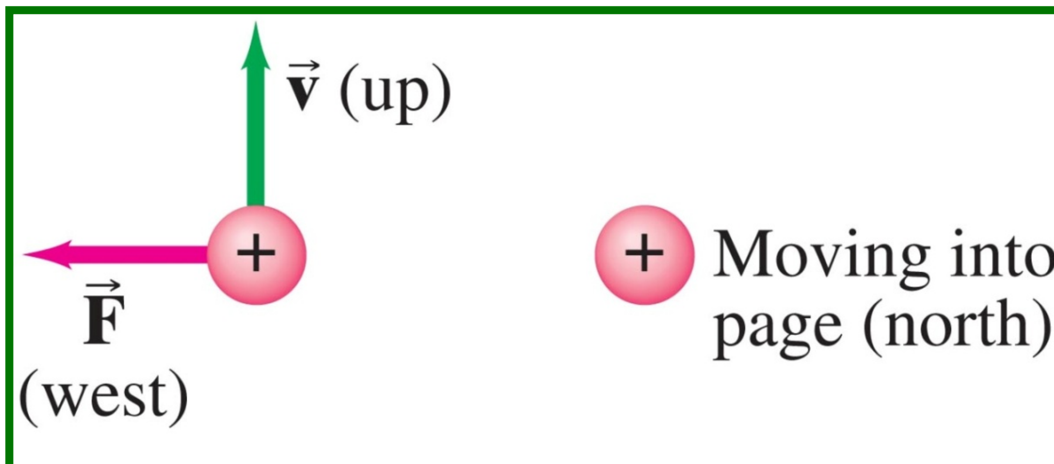
More Differences Between Electric & Magnetic Fields

Work

- The electric force does work in displacing a charged particle.
- The magnetic force associated with a steady magnetic field does no work when a particle is displaced.
- This is because the force is perpendicular to the displacement of its point of application.
- The kinetic energy of a charged particle moving through a magnetic field can't be altered by the magnetic field alone.
- When a charged particle moves with a given velocity through a magnetic field, the field can alter the direction of the velocity, but not the speed or the kinetic energy.

Example: Magnetic Force on a Proton.

A magnetic field exerts a force **F = 8.0 x 10^{-14} N toward** the **west** on a proton moving **vertically up** at a **speed v = 5.0 x 10^{6} m/s**. When moving horizontally in a northerly direction, the force on the proton is zero. Calculate the magnitude & direction of the magnetic field **B** in this region. (Proton charge: ***q* = +*e* = 1.6 x 10^{-19} C**.)



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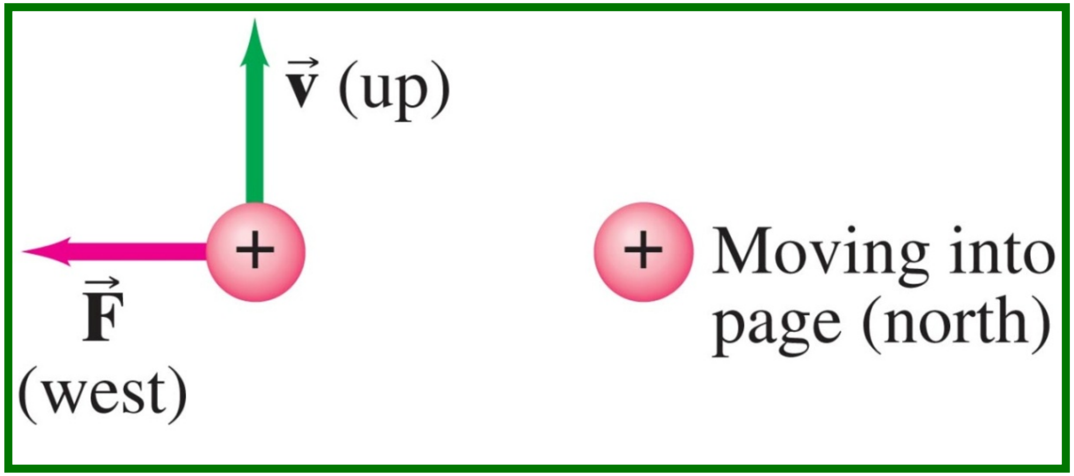
Solution: Use

$$\vec{F} = q\vec{v} \times \vec{B}.$$



Solve & get:

B = 0.1 T

Example

Magnetic Force on Ions During a Nerve Pulse.

Estimate the magnetic force **F** due to the Earth's magnetic field (**$B = 10^{-4}$ T**) on ions crossing a cell membrane during an action potential. Assume that the speed of the ions is **$v = 10^{-2}$ m/s** & that their charge is **$q = 10^{-19}$ C.**

Example

Magnetic Force on Ions During a Nerve Pulse.

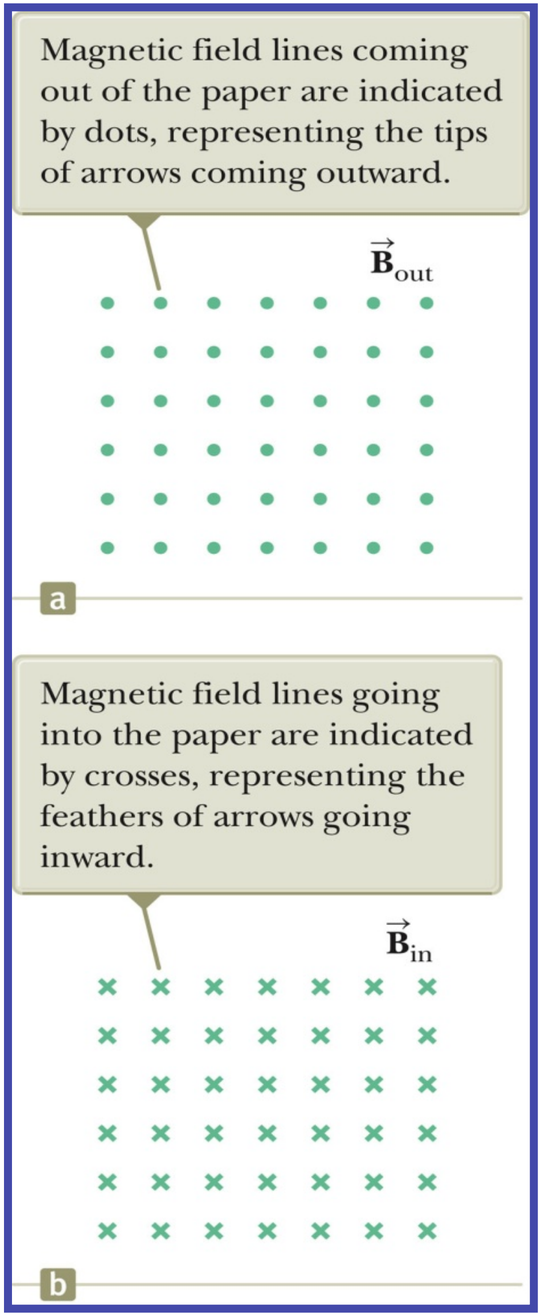
Estimate the magnetic force **F** due to the Earth's magnetic field (**B = 10^{-4} T**) on ions crossing a cell membrane during an action potential. Assume that the speed of the ions is **v = 10^{-2} m/s** & that their charge is **q = 10^{-19} C.**

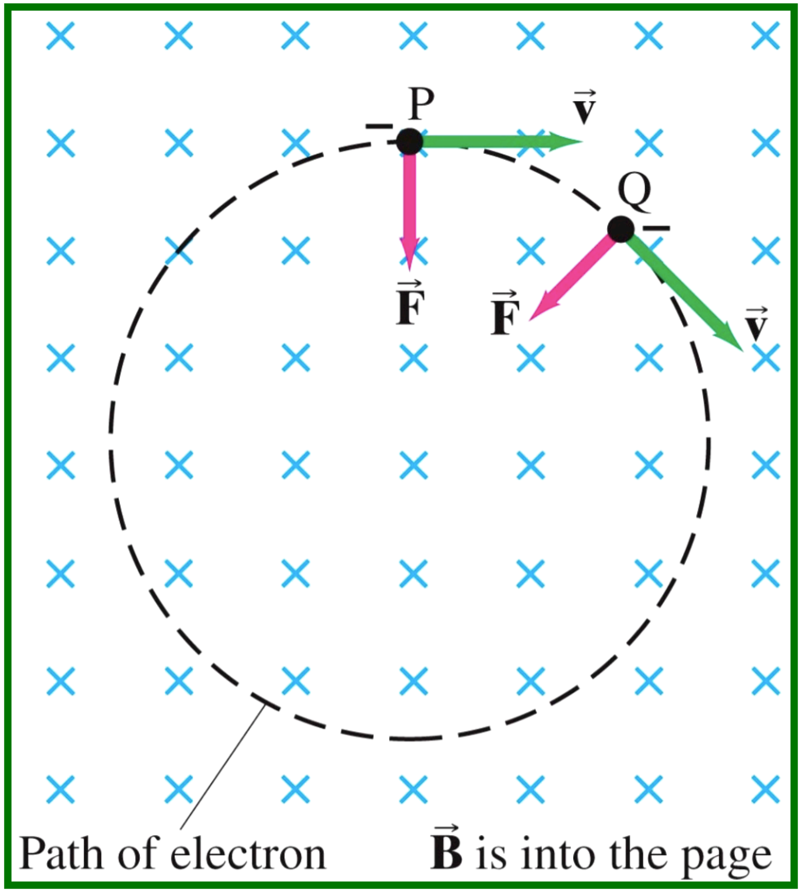
Solution: Use $$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}.$$

Solve & get: F $\simeq$ 10^{-25} N

Notation Notes

- When vectors are perpendicular to the page, dots & crosses are used.
- The dots represent the arrows coming out of the page.
- The crosses represent the arrows going into the page.
- The same notation applies to other vectors.

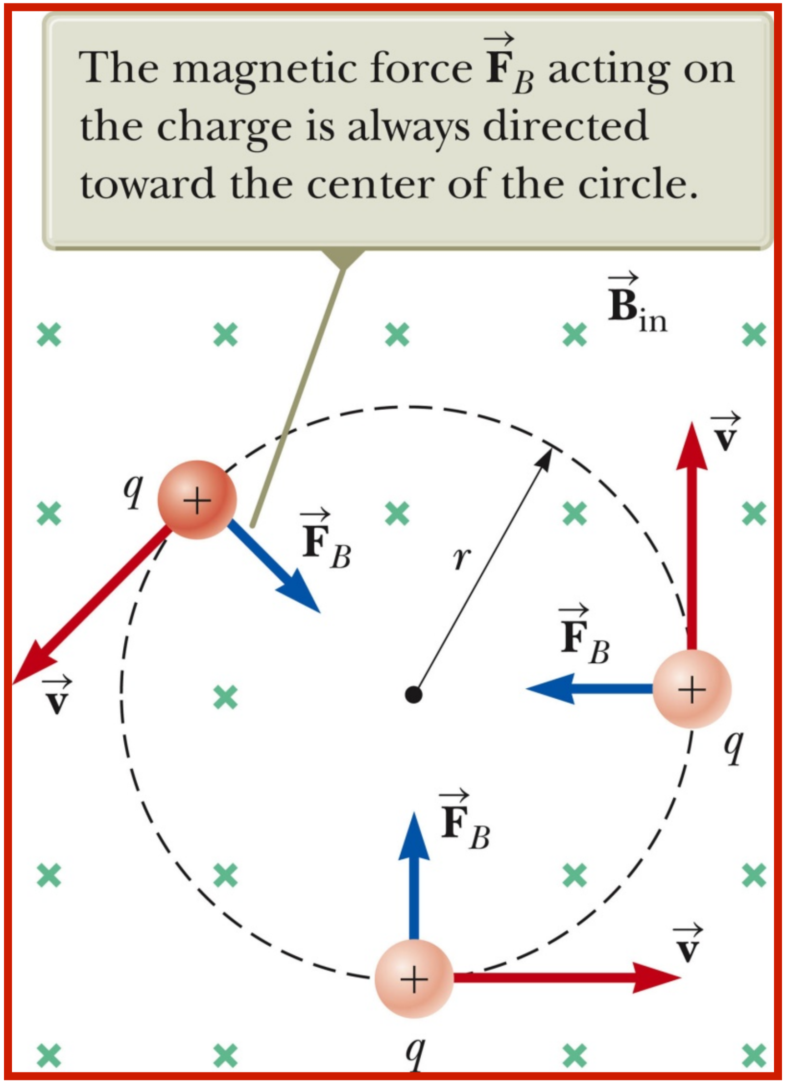




If a particle of charge **Q** moves with velocity **v** perpendicular to a uniform magnetic field **B**, its easy to show that its path will be a circle.

Charged Particle in a Magnetic Field

- Consider a particle moving in an external magnetic field with its velocity perpendicular to the field.
- The force is always directed toward the center of the circular path.
- The magnetic force causes a centripetal acceleration, changing the direction of the velocity of the particle.



Force on a Charged Particle

- Use Newton's 2nd Law for a particle in uniform circular motion. Equate the magnetic & centripetal forces:

$$F_B = qvB = \frac{mv^2}{r}$$

- Solve for the circle radius **r**:

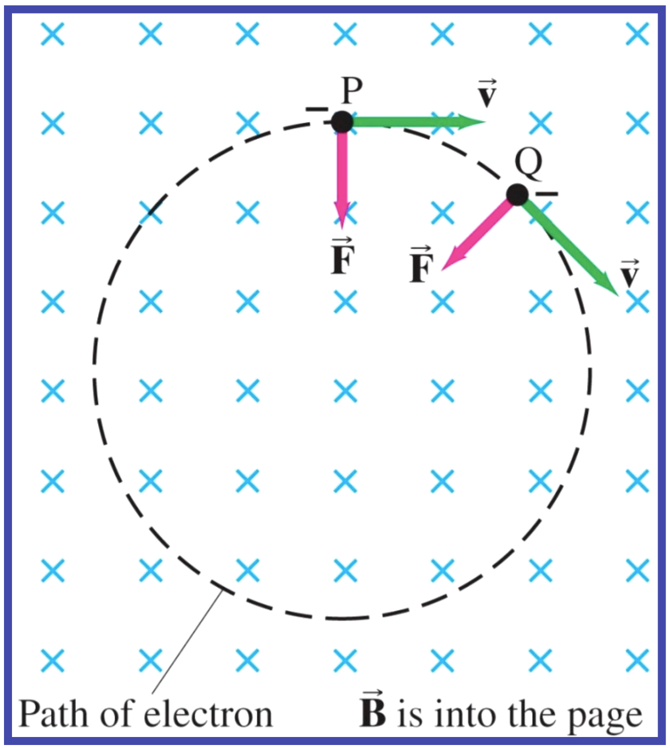
$$r = \frac{mv}{qB}$$

- **r** is proportional to the linear momentum of the particle & inversely proportional to the magnetic field.

Example

Electron's Path in a Uniform Magnetic Field.

An electron travels at speed **v = 2.0 x 10^7 m/s** in a plane perpendicular to a uniform **B = 0.010 T** magnetic field. Describe its path quantitatively.



Solution: Use $\vec{F} = q\vec{v} \times \vec{B}$.

Combine this force with ***Newton's 2nd Law*** & use the knowledge of the centripetal force on a particle moving in a **circle of radius r**:

$$F = (mv^2)/(r) = qvB$$

This gives

$$r = (mv/qB) = 1.1 \text{ cm}$$

More About Motion of Charged Particle

- The **angular speed** of the particle is

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

- The angular speed, ***w***, is also referred to as the ***cyclotron frequency.***
- The period of the motion is

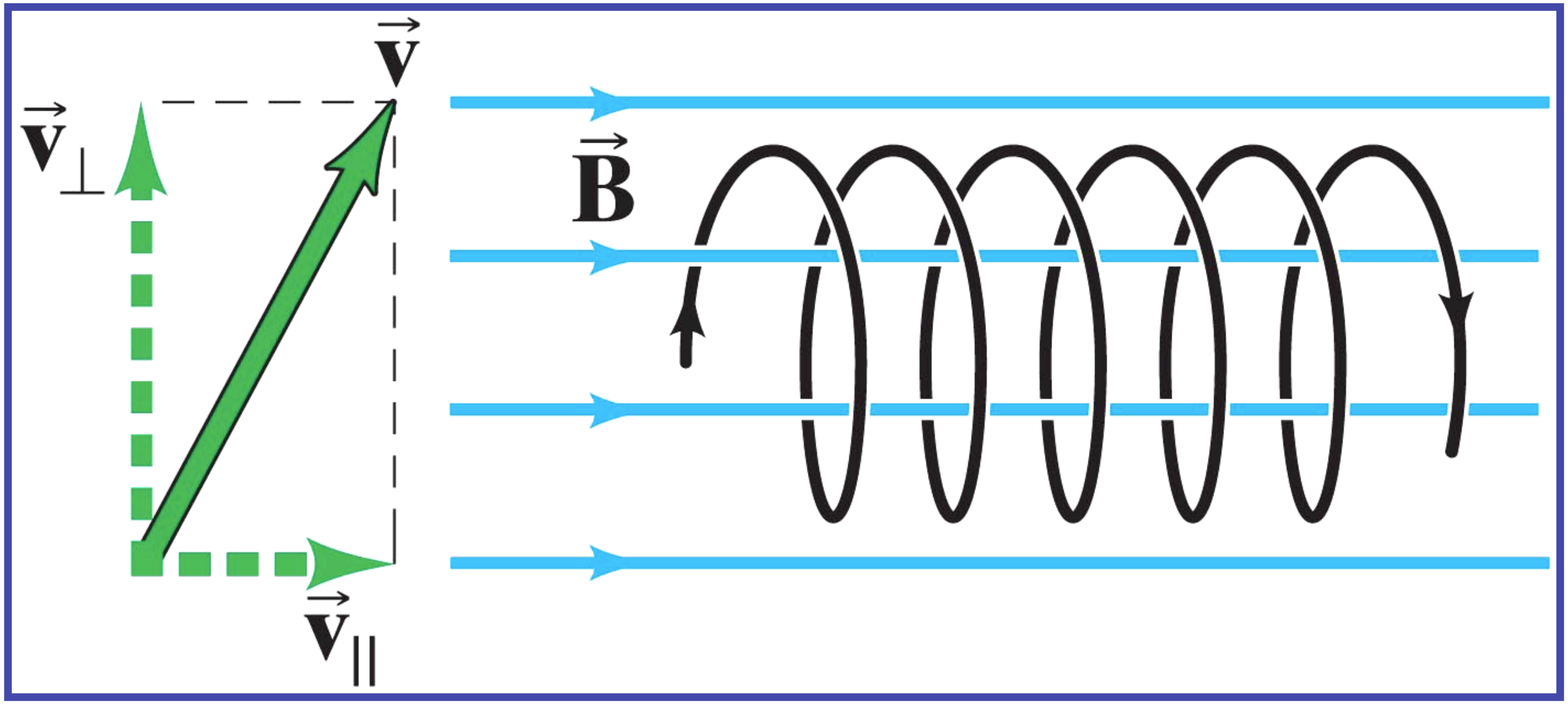
$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

TABLE 27–1 Summary of Right-hand Rules (= RHR)

Physical Situation	Example	How to Orient Right Hand	Result
1. Magnetic field produced by current (RHR-1)	 **Fig. 27–8c**	Wrap fingers around wire with thumb pointing in direction of current I	Fingers point in direction of $\vec{\mathbf{B}}$
2. Force on electric current I due to magnetic field (RHR-2)	 **Fig. 27–11c**	Fingers point straight along current I, then bend along magnetic field $\vec{\mathbf{B}}$	Thumb points in direction of the force $\vec{\mathbf{F}}$
3. Force on electric charge $+q$ due to magnetic field (RHR-3)	 **Fig. 27–15**	Fingers point along particle's velocity $\vec{\mathbf{v}}$, then along $\vec{\mathbf{B}}$	Thumb points in direction of the force $\vec{\mathbf{F}}$

Conceptual Example: A helical path

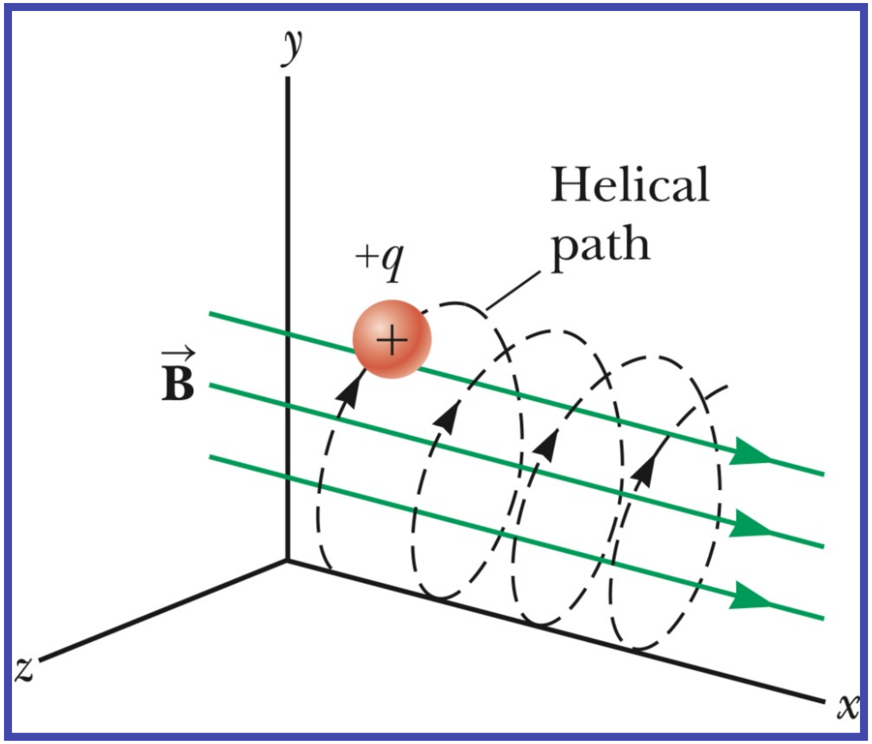
What is the path of a charged particle in a uniform magnetic field if its velocity is not perpendicular to the magnetic field?



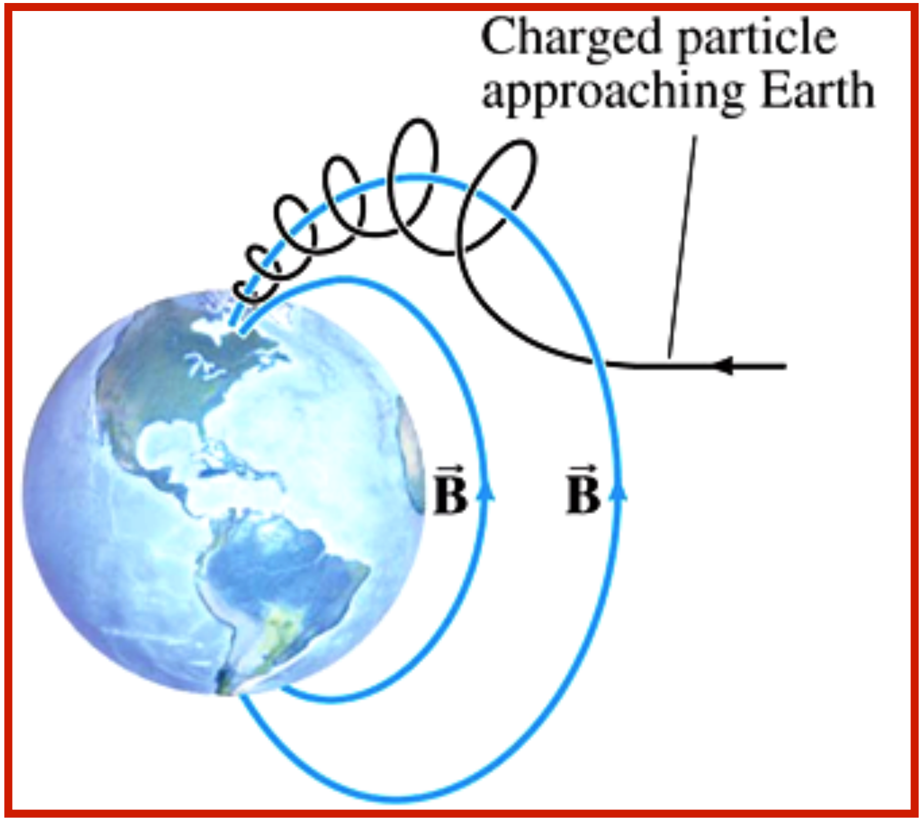
Motion of a Particle, General

If a charged particle moves in a magnetic field at some arbitrary angle with respect to the field, its path is a helix. The same equations apply, with **v** replaced by

$$v_{\perp} = \sqrt{v_y^2 + v_z^2}$$



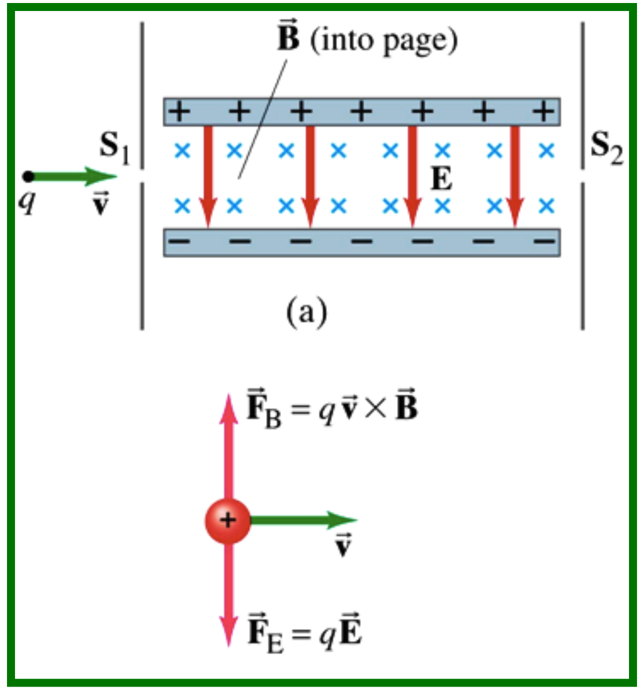
The **aurora borealis** (Northern lights) is caused by charged particles from the solar wind spiraling along the Earth's magnetic field, and colliding with air molecules.



Conceptual Example

Velocity Selector or Filter: Crossed E & B fields

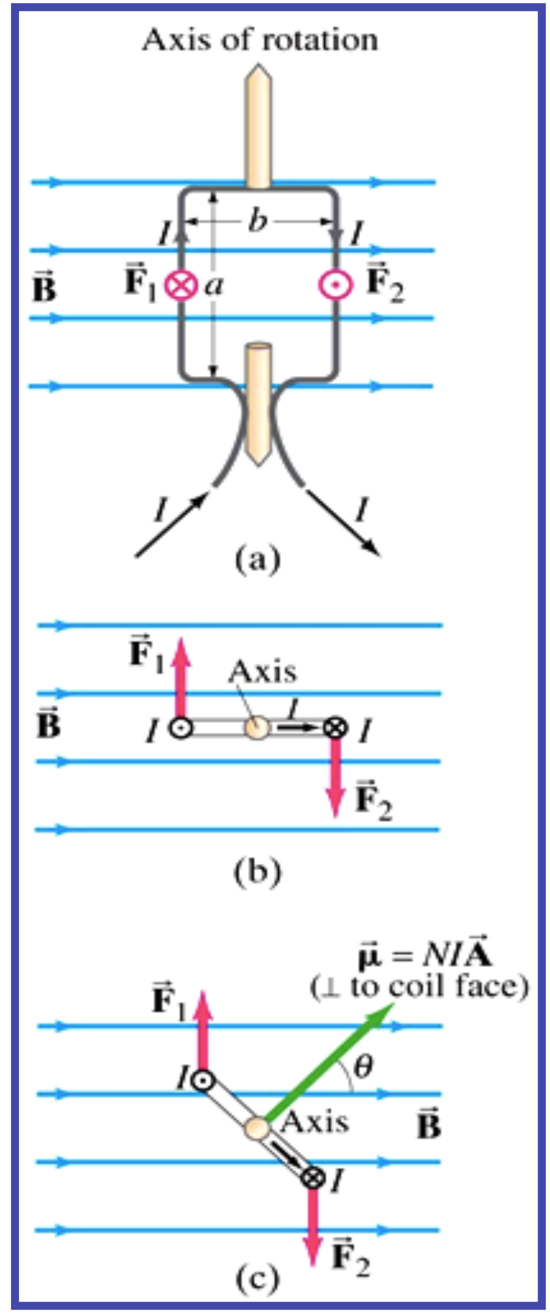
Some electronic devices & experiments need a beam of charged particles all moving at nearly the same velocity. This is achieved using both a **uniform electric field E** & a **uniform magnetic field B** arranged so ***they are at right angles to each other.*** In the figure, particles of charge **q** pass through slit **S_1** & enter the region where **B** points into the page & **E** points down from the positive plate toward the negative plate. If the particles enter with different velocities, show how this device "selects" a particular velocity **v**, & determine what this velocity is



Torque on a Current Loop; Magnetic Dipole Moment

The forces on the opposite sides of a current loop will be ***Equal & Opposite*** (if the field is uniform & the loop is symmetric), but there may be **a torque** τ. For **N** current loops, the magnitude of the torque τ is given by

$$\tau = NIAB\sin\theta.$$



- Consider **N** circular loops of wire of loop area **A** & carrying a current **I**, the quantity **NIA** is called the

Magnetic Dipole Moment, **μ**:

$$\vec{\boldsymbol{\mu}} = NI\vec{\mathbf{A}}.$$

- Also, the potential energy **U** of a loop of dipole moment **μ** depends on its orientation in the field:

$$U = -\mu B \cos\theta = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}.$$

Example: Torque on a Coil

- A circular coil of wire has diameter **d = 20 cm** & contains **N = 10** loops. The current in each loop is **I = 3 A**, & the coil is placed in a external magnetic field **B = 2 T** . **Calculate** the maximum & minimum torque exerted on the coil by the field.

Solution: Use $\tau = NIAB\sin\theta.$

Solve & get: $\tau_{max} = 1.99$ **N m**

and $\tau_{min} = 0$

Example:

Magnetic Moment of a Hydrogen Atom.

Calculate the magnetic dipole moment **μ** of the electron orbiting the proton of a hydrogen atom at a given instant, assuming (in the Bohr model) it is in its ground state with a **circular orbit** of **radius**

$$\mathbf{r = 0.529 \times 10^{-10}\ m}.$$

NOTE: This is a *very* rough, *very* approximate picture of atomic structure, but nonetheless gives an accurate result. A correct treatment of this problem requires the use of

Quantum Mechanics