Sets

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Definitions

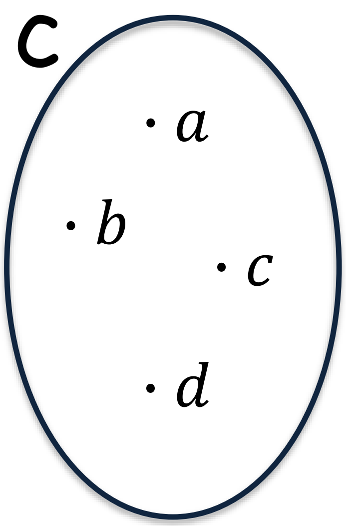
- *A* set is an unordered (well-defined) collection of objects.
- These objects are called elements (or members) of the set.
- $x \in A$, x is an element of the set A
- $x \notin A$, x is not an element of the set A

Definitions

- Z = {2, -22, 12, 0, 43, -1287, . . .}
  Z = {. . ., -3, -2, -1, 0, 1, 2, 3, . . .}

- $A = \{x \in Z^{+} | x < 10\}$ (set builder or common property)
  $B = \{x \in Z | x^2 < 10\}$

- Venn Diagram



Definitions

- Two sets are equal if and only if they have same elements. A and B are equal if and only if $\forall x(x \in A \leftrightarrow x \in B)$.

  $A = \{x \in Z^+ | x < 6\}$ and $B = \{1, 2, 3, 4, 5\}$, A = B

- The universal set, denoted by U, contains all possible elements under the consideration

- The empty set, denoted by ∅, has no element

Definitions

- A set A is a subset of a set B if and only if every element of A is also an element of B.

$A \subseteq B \leftrightarrow \forall x[x \in A \rightarrow x \in B]$

$$
\begin{aligned}
A \not\subseteq B &\leftrightarrow \sim\forall x[x \in A \rightarrow x \in B] \\
&\leftrightarrow \exists x \sim[x \in A \rightarrow x \in B] \\
&\leftrightarrow \exists x \sim[\sim x \in A \lor x \in B] \qquad (p \rightarrow q \equiv \sim p \lor q) \\
&\leftrightarrow \exists x[x \in A \land \sim x \in B] \\
&\leftrightarrow \exists x[x \in A \land x \notin B]
\end{aligned}
$$

Definitions

- A set A is a subset of a set B if and only if every element of A is also an element of B.

  $A \subseteq B \leftrightarrow \forall x[x \in A \rightarrow x \in B]$

- $\emptyset \subseteq A$ and $A \subseteq A$.

- $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

Definitions

of A is

$A = \{x | x = 4k + 1 \text{ for some } k \in Z\}$,
$B = \{x | x = 4k - 3 \text{ for some } k \in Z\}$
Show that whether the sets A and B are equal or not.

(A⊆B) For any $x \in A$, $x = 4k + 1 \text{ for some } k \in Z$

$$x = 4k + 1 + 3 - 3$$
$$x = 4(k + 1) - 3$$
$$x = 4m - 3 \text{ for some } m \in Z, \text{ so } x \in B$$

(B⊆A) For any $x \in B$, $x = 4k - 3 \text{ for some } k \in Z$

$$x = 4k - 3 + 1 - 1$$
$$x = 4(k - 1) + 1$$
$$x = 4m + 1 \text{ for some } m \in Z, \text{ so } x \in A$$

Thus, A=B.

Definitions

- A set A is a subset of a set B if and only if every element of A is also an element of B.

  $A \subseteq B \leftrightarrow \forall x[x \in A \rightarrow x \in B]$

- $\emptyset \subseteq A$ and $A \subseteq A$.

- A = B if and only if $A \subseteq B$ and $B \subseteq A$

- A set A is a proper subset of a set B if and only if $A \subseteq B$ and $A \neq B$

  $B = \{x \in Z^+ | x < 10\}$ and $A = \{1, 2, 3, 4, 5\}$, $A \subseteq B$

Definitions

- The cardinality of a set A is defined as the size of A. It's denoted by |A|. (only for finite set)

  For the set $A = \{x \in Z^{+} | x < 10\}$, |A| = 9

- The power set of a given set is the set of all possible subsets.

  S={1} P(S)={∅, {1}} |P(S)|=2

  S={a, b} P(S)={∅, {a}, {b}, {a, b}} |P(S)|=4

- If |S|=n, then |P(S)|=2^n

Set Operations

- The union of A and B, denoted by A∪B, contains elements that are either in A or B.

$$\mathrm{A} \cup \mathrm{B} = \{x | x \in A \ \vee \ x \in B\}$$

- The intersection of A and B, denoted by A∩B, contains elements that are in both A or B.

$$\mathrm{A} \cap \mathrm{B} = \{x | x \in A \ \wedge \ x \in B\}$$

The sets A and B are called disjoint sets if $\mathrm{A} \cap \mathrm{B} = \emptyset$

- The difference of A and B, denoted by A-B, contains elements that are in A but not in B

$$\mathrm{A} \cap \mathrm{B} = \{x | x \in A \ \wedge \ x \notin B\}$$

- The complement of A, denoted by $\overline{\mathrm{A}}$, contains elements that are in U but not in A.

$$\overline{\mathrm{A}} = \{x \in U | x \notin A\}$$

Set Operations

- $A \cup \emptyset = A$
  $A \cap U = A$
- $A \cup U = U$
  $A \cap \emptyset = \emptyset$
- $A \cup A = A$
  $A \cap A = A$
- $A \cup B = B \cup A$
  $A \cap B = B \cap A$
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- $A \cap \overline{A} = \emptyset$
  $A \cup \overline{A} = U$
- $\overline{(\overline{A})} = A$
- $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$ (De Morgan)
  $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Cartesian Products

- The cartesian product of A and B, denoted by AxB, is the set of all pairs (x,y) where $x \in A$ and $y \in B$

$$A\, x B = \{(x, y) | x \in A \;\; \wedge \;\; y \in B\}$$

- A={a,b}, B={1, 2, 3}

AxB = {(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)}

|AxB| = |A| . |B|

- *The Cartesian products of the sets* $A_1, A_2, . . ., A_n$ *is the set of ordered n-tuples* $(a_1, a_2, . . ., a_n)$ *where* $a_1 \in A_1, a_2 \in A_2, . . ., a_n \in A_n$.

$$A_1 x \dots x A_n = \{(a_1, a_2, . . ., a_n) | a_i \in A_i, i = 1..n\}$$