

Fuzzy 1

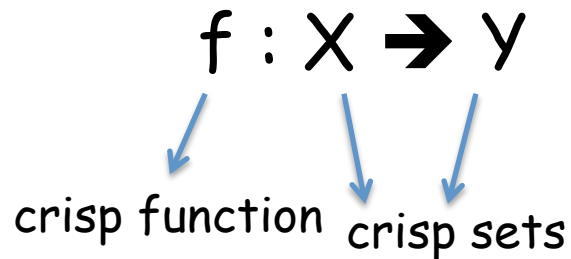
Murat Osmanoglu

Kinds of Fuzzy Functions

fuzzy functions can be categorized as three groups :

- crisp function with fuzzy constraint
- crisp function that propagates the fuzziness of independent variable to dependent variable
- fuzzifying function

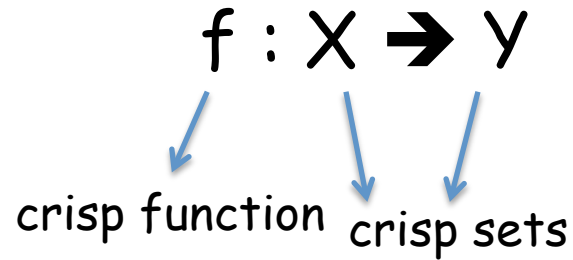
Crisp Function with Fuzzy Constraint



- let $A \subseteq X$ and $B \subseteq Y$ be fuzzy sets.

the function with fuzzy constraint $\mu_A(x) \leq \mu_B(f(x))$ on A and B

Crisp Function with Fuzzy Constraint



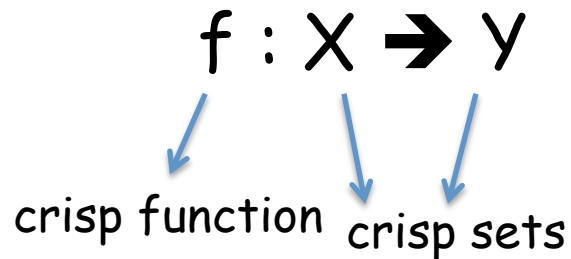
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- consider the function $f : Z^+ \rightarrow Z^+$ with the rule $f(x) = 2x$, and two fuzzy sets $A, B \subseteq Z^+$ defined as

$$A = \{(1,0.3), (2,1.0)\} \text{ and } B = \{(2,0.6), (4,1.0)\}$$

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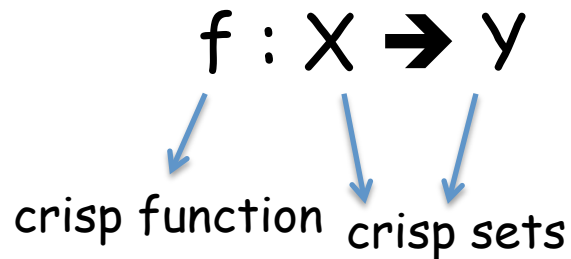
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f can be considered as the crisp function with the fuzzy constraint $\mu_A(x) \leq \mu_B(f(x))$

Crisp Function with Fuzzy Constraint

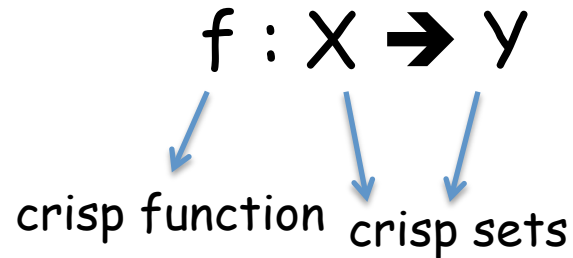


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- let X be the set of salesmen and Y be the set of yearly income

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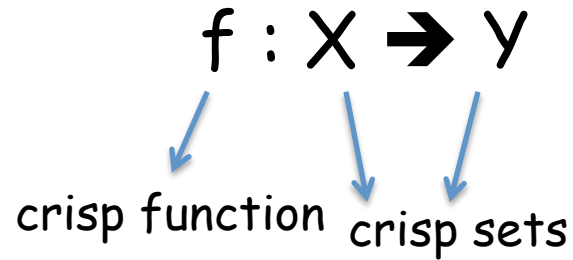
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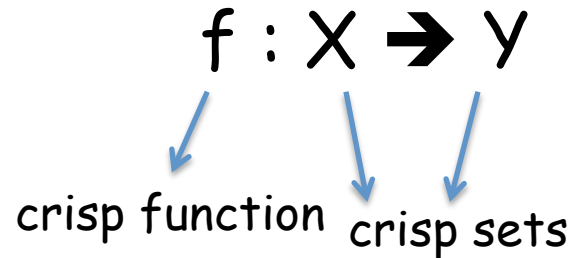
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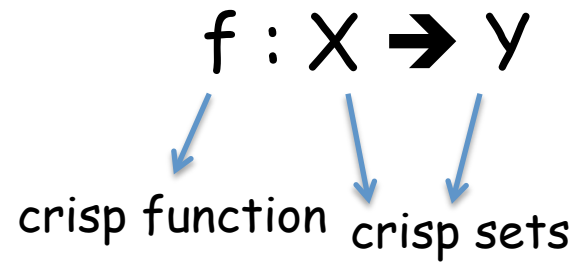
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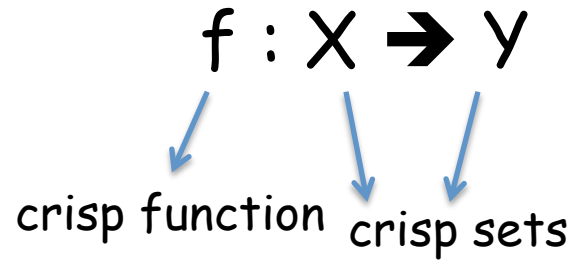
the constraint here 'a competent salesman gets higher income'

Fuzzy Extension Function



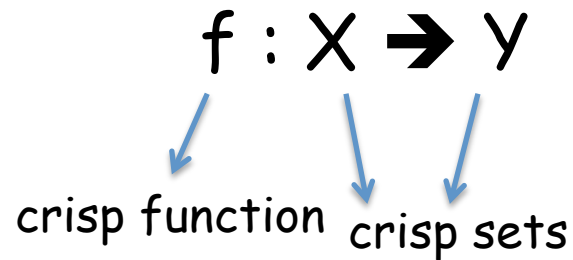
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Fuzzy Extension Function



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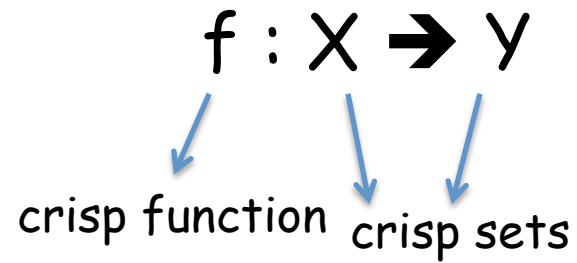
Fuzzy Extension Function



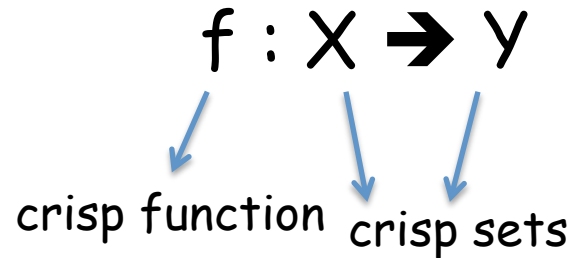
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$$\mu_{f(A)}(y) = \max_{x \text{ s.t. } f(x)=y} \mu_A(x)$$

Fuzzy Extension Function

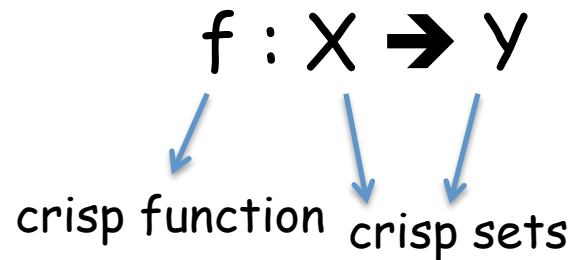


Fuzzy Extension Function



- Let $A = \{(-2, 0.2), (-1, 0.7), (0, 1.0), (1, 0.6), (2, 0.3)\}$ be a fuzzy set and $f : Z \rightarrow Z$ be fuzzy extension function.

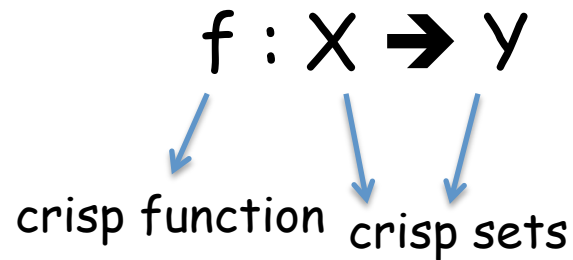
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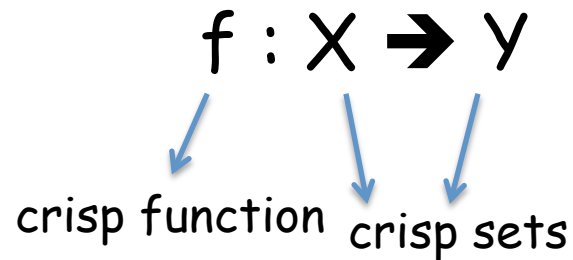


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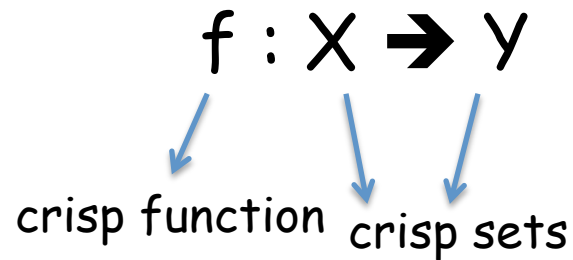
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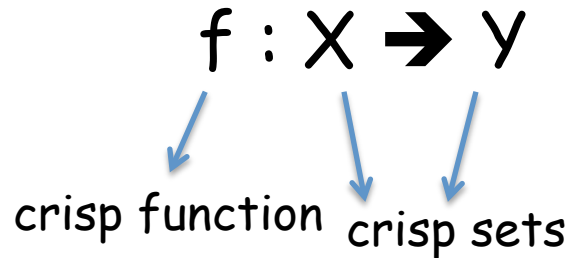
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$$\begin{cases} \mu_A(1) = 0.6, f(1) = 1 \\ \mu_A(-1) = 0.7, f(-1) = 1 \end{cases}$$

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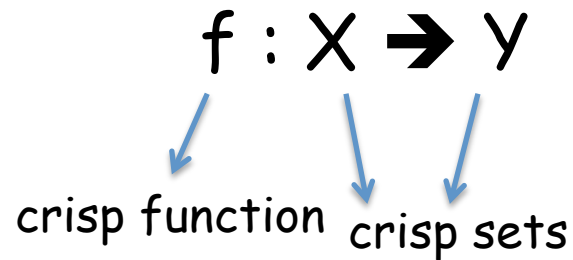
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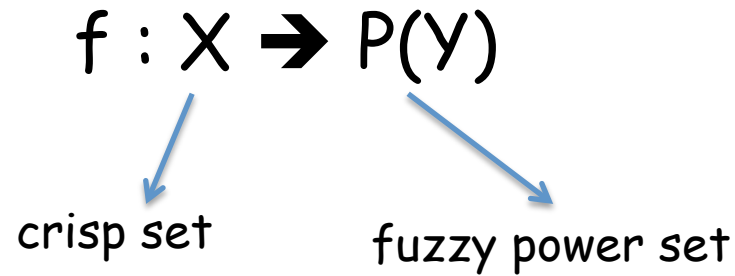
$B' \subseteq Z$ induced by f ($f(x) = x^2$) will be

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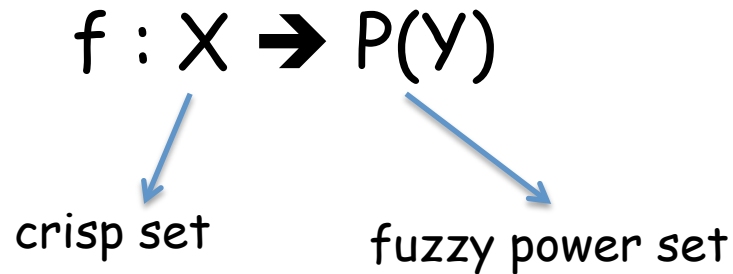
$$\max \begin{cases} \mu_A(1) = 0.6, f(1) = 1 \\ \mu_A(-1) = 0.7, f(-1) = 1 \end{cases}$$

Fuzzifying Function of Crisp Value



Single Fuzzifying Function

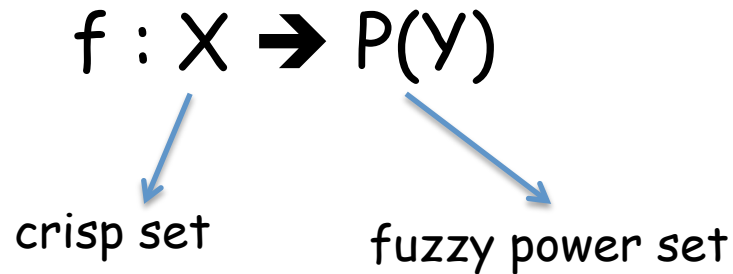
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Single Fuzzifying Function

Let A and B two crisp sets $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 5, 6, 7\}$

Fuzzifying Function of Crisp Value

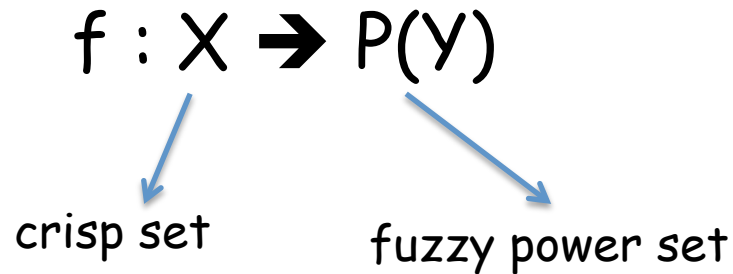


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$f(1) = B_1$, $f(2) = B_2$, $f(3) = B_3$ where B_1, B_2, B_3 in $P(B)$

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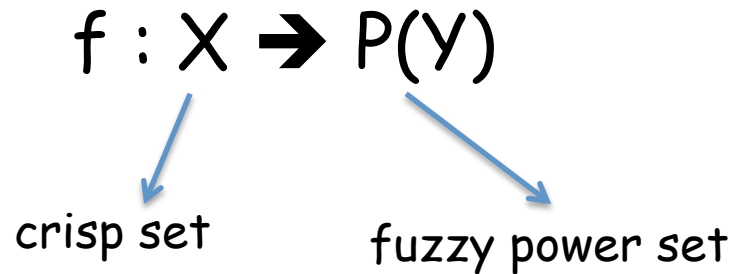
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$B_1 = \{(1, 0.5), (2, 1.0), (3, 0.5)\}$

$B_2 = \{(3, 0.5), (4, 1.0), (5, 0.5)\}$

$B_3 = \{(5, 0.5), (6, 1.0), (7, 0.5)\}$

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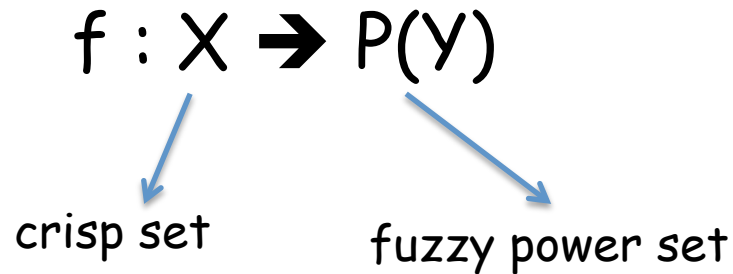
$$B_1 = \{(1, 0.5), (2, 1.0), (3, 0.5)\}$$

$$f : 1 \rightarrow \{1, 2, 3\} \quad \alpha = 0.5$$

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$$f : 1 \rightarrow \{2\} \quad \alpha = 1.0$$

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Fuzzifying Function of Crisp Value

Fuzzy bunch of functions

- fuzzy set of crisp functions:

$$F = \{(f_1, \mu_F(f_1)), (f_2, \mu_F(f_2)), \dots, (f_n, \mu_F(f_n))\}$$

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