Fuzzy 9

Murat Osmanoglu

Larsen Fuzzy Inference

Singleton Input

- the fact is : x is 3 and y is 4

  the rule is : If x is A and y is B, then z is C

  the result is : z is C'

where $A = (0, 2, 6)$, $B = (3, 6, 7)$, and $C = (1, 3, 5)$

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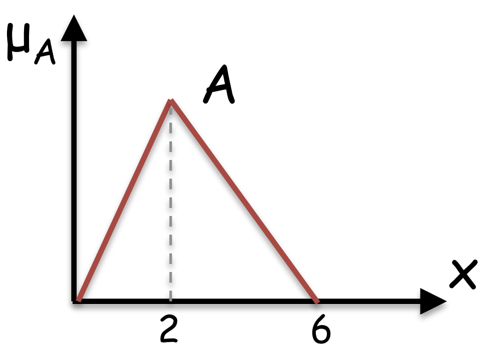

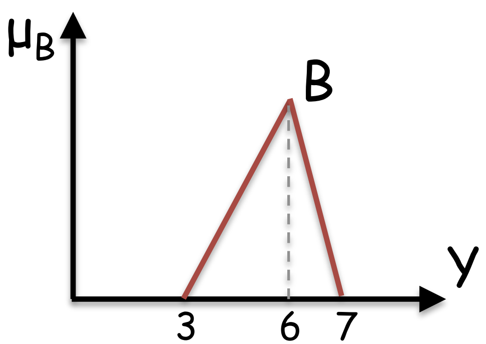

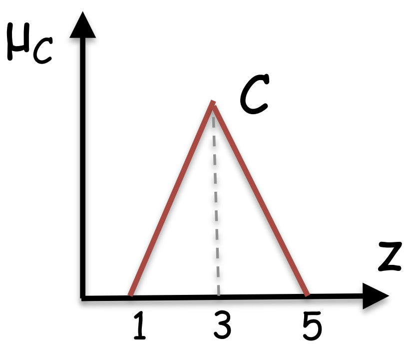

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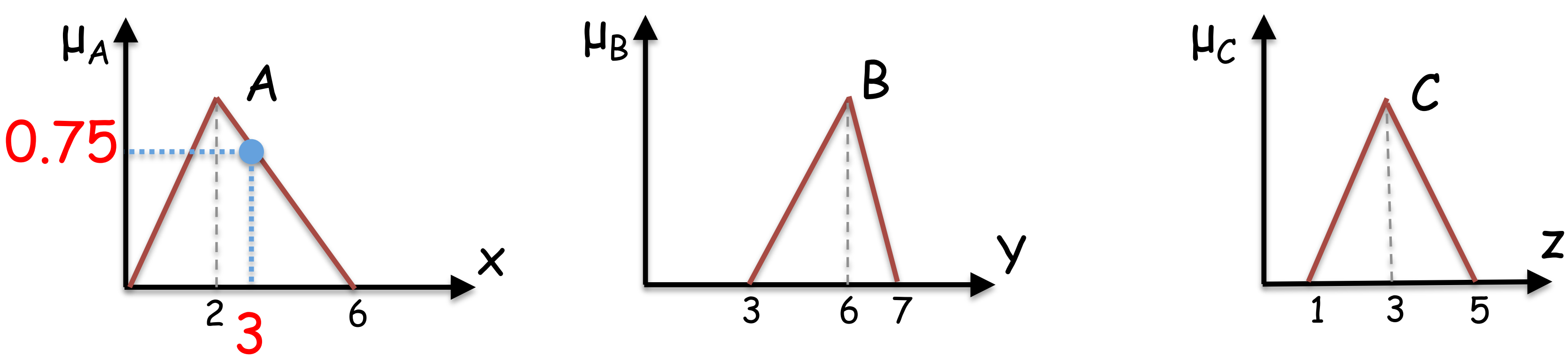

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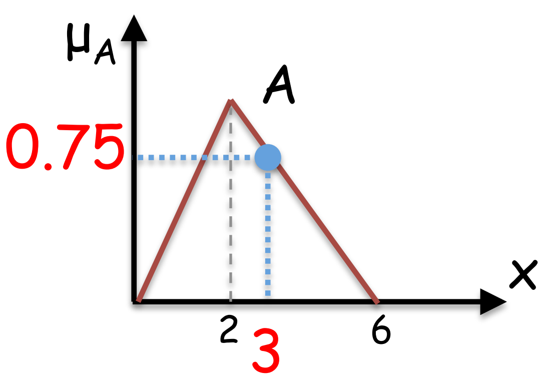

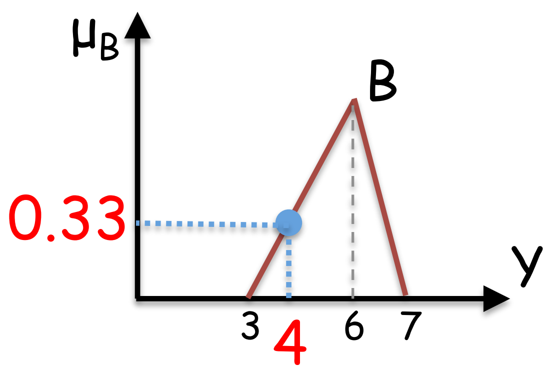

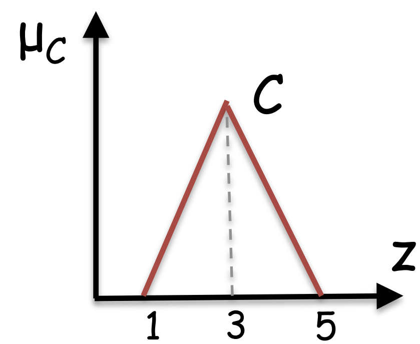

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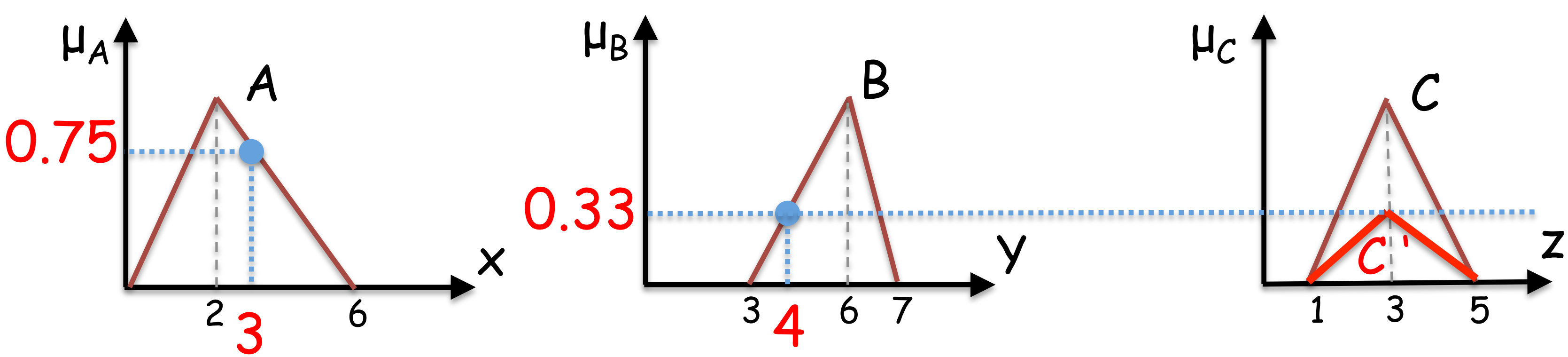

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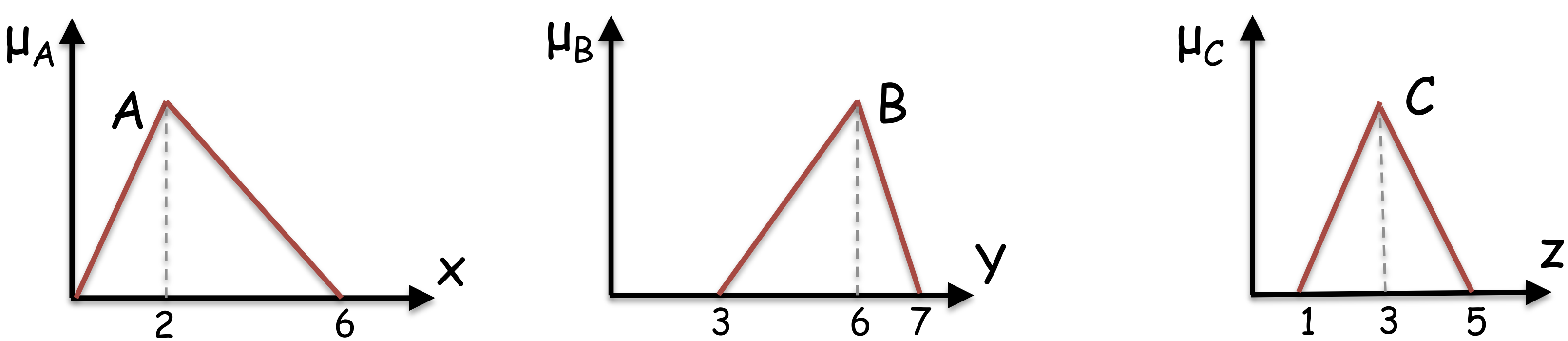

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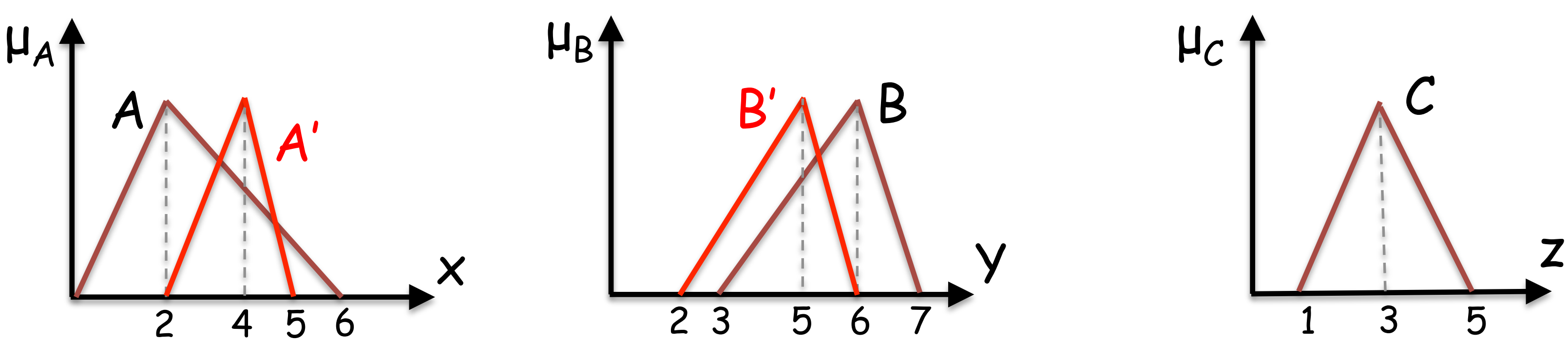

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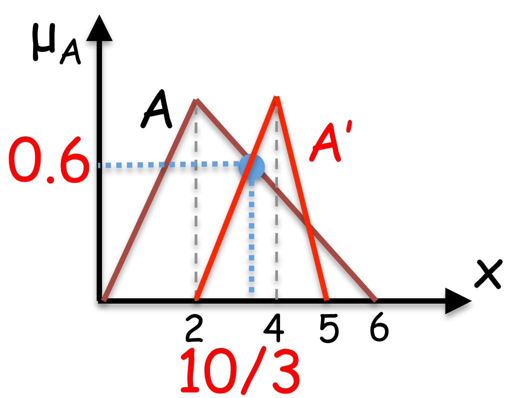

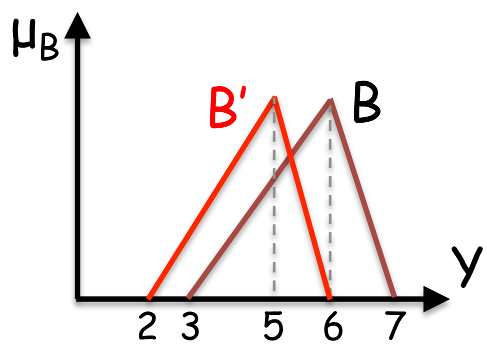

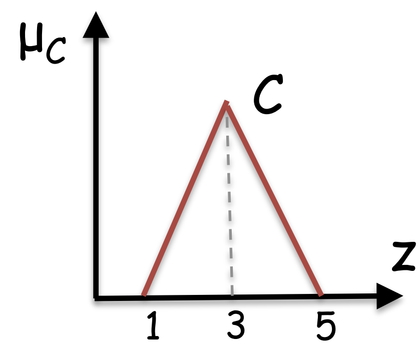

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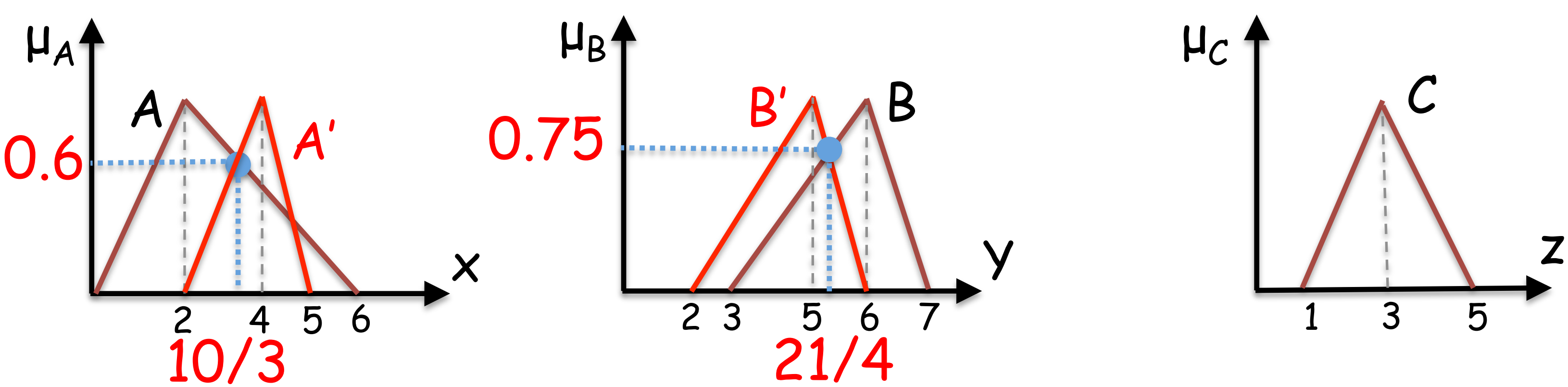

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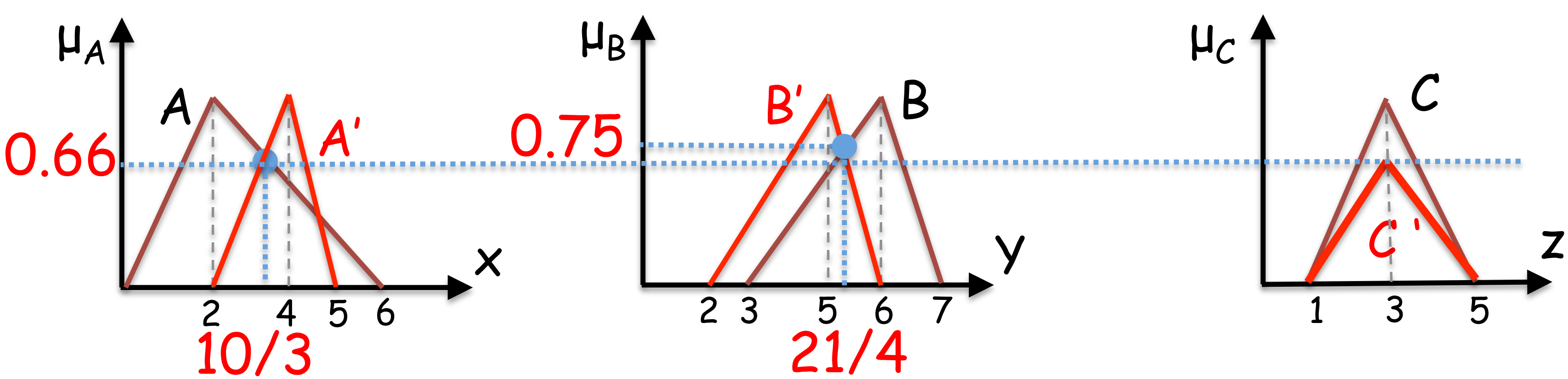

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- the consequence of the fuzzy rule is represented by a fuzzy set with a monotonic membership function

- the output for each rule will be a crisp value induced by the rule's matching degree

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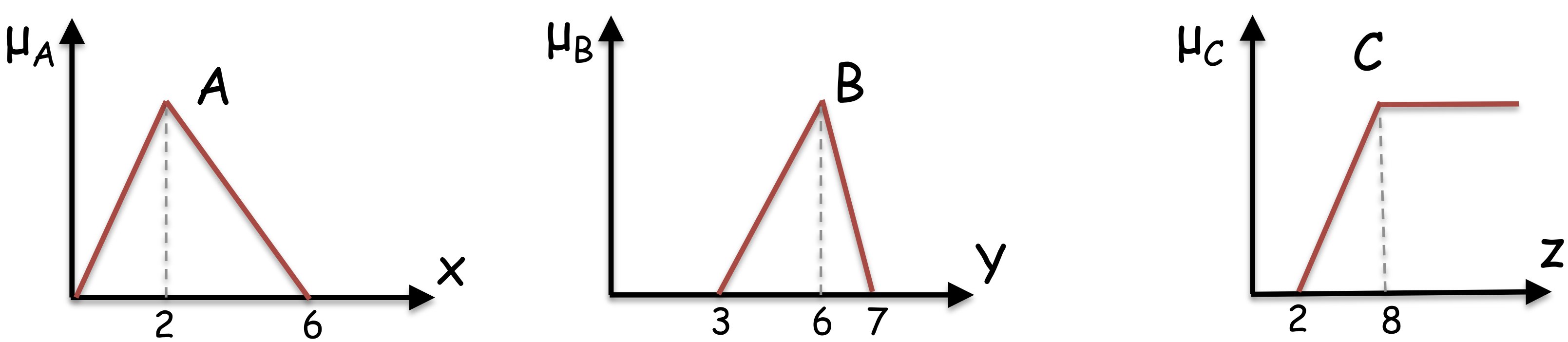

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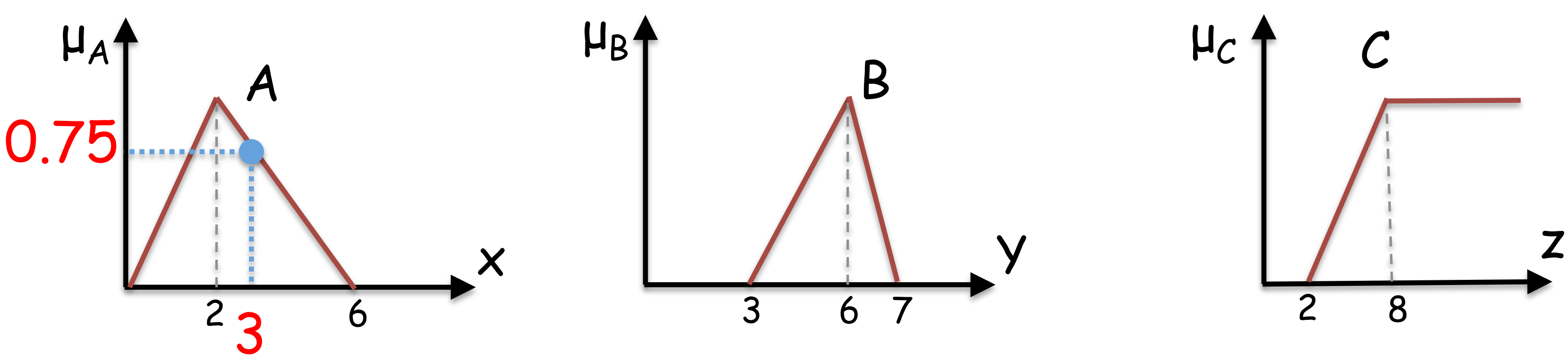

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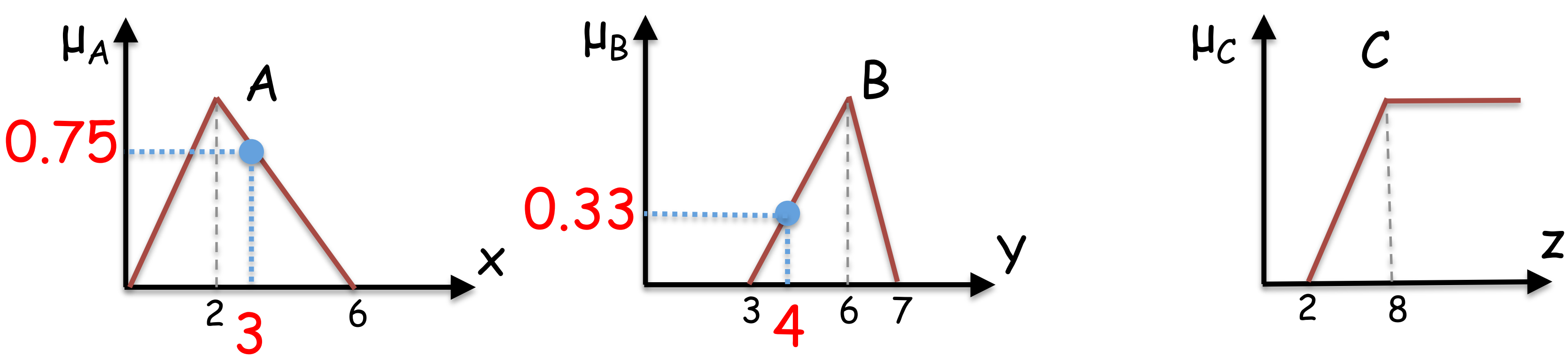

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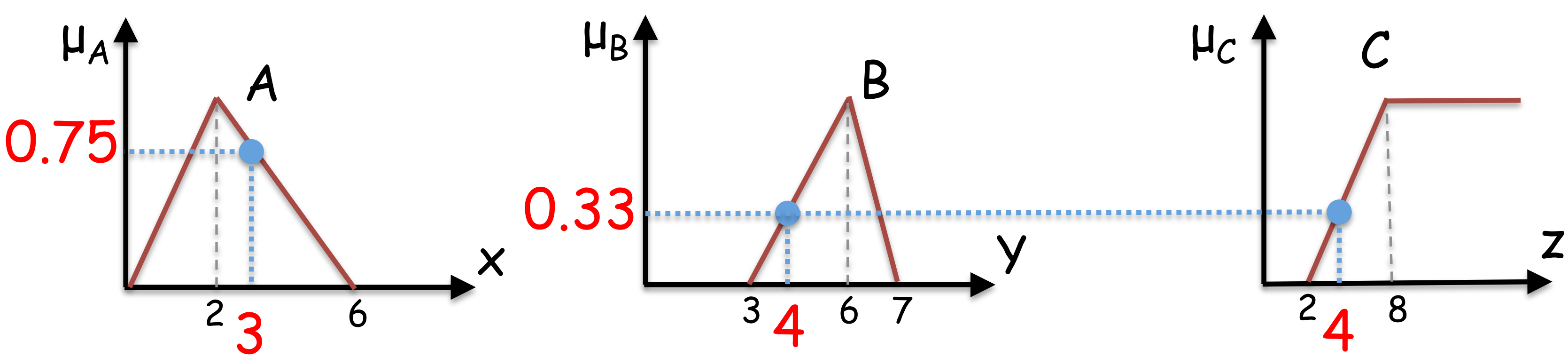

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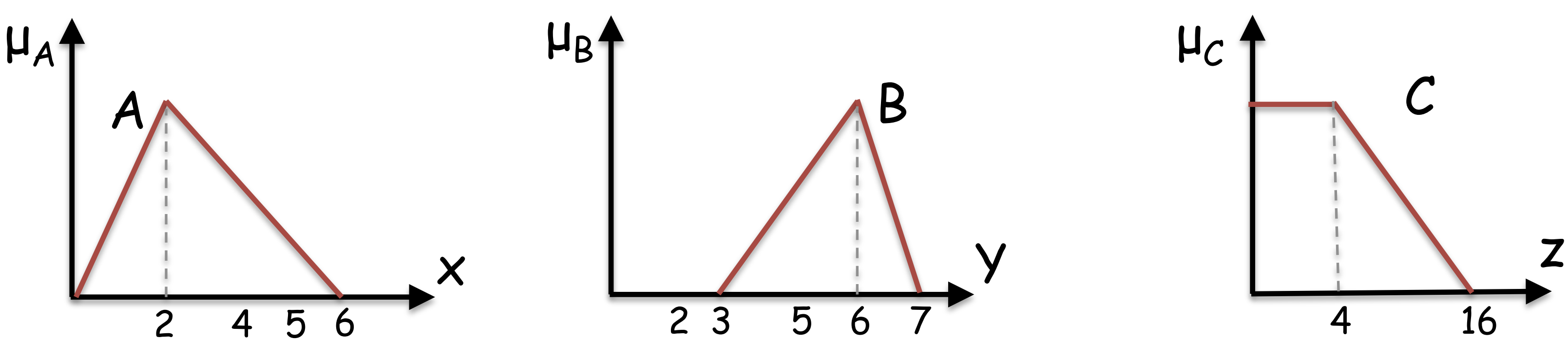

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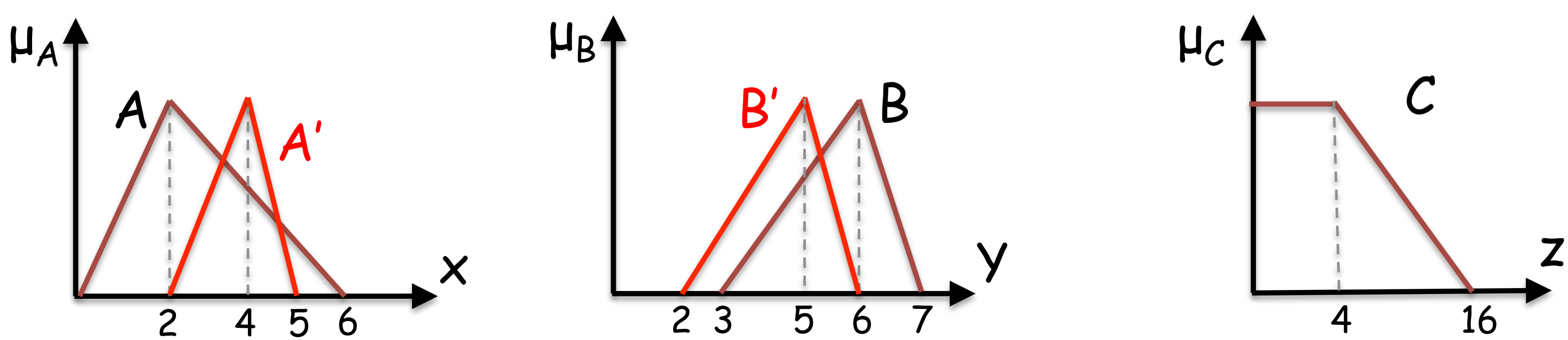

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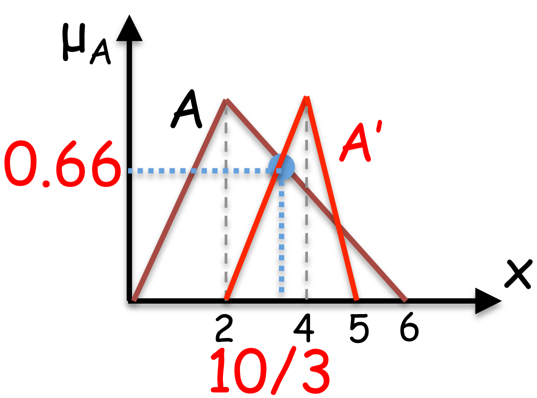

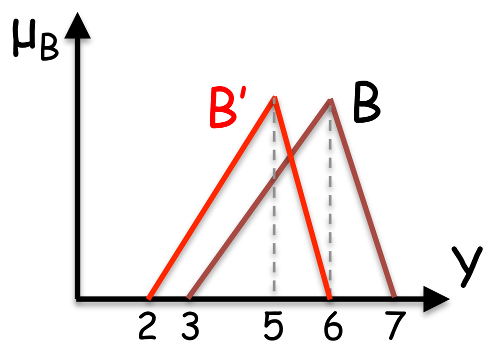

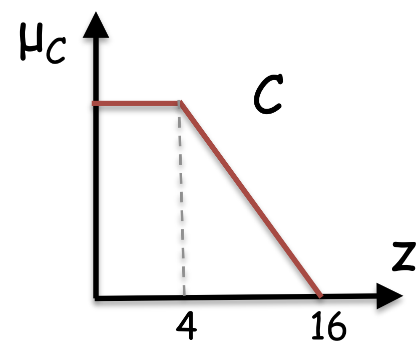

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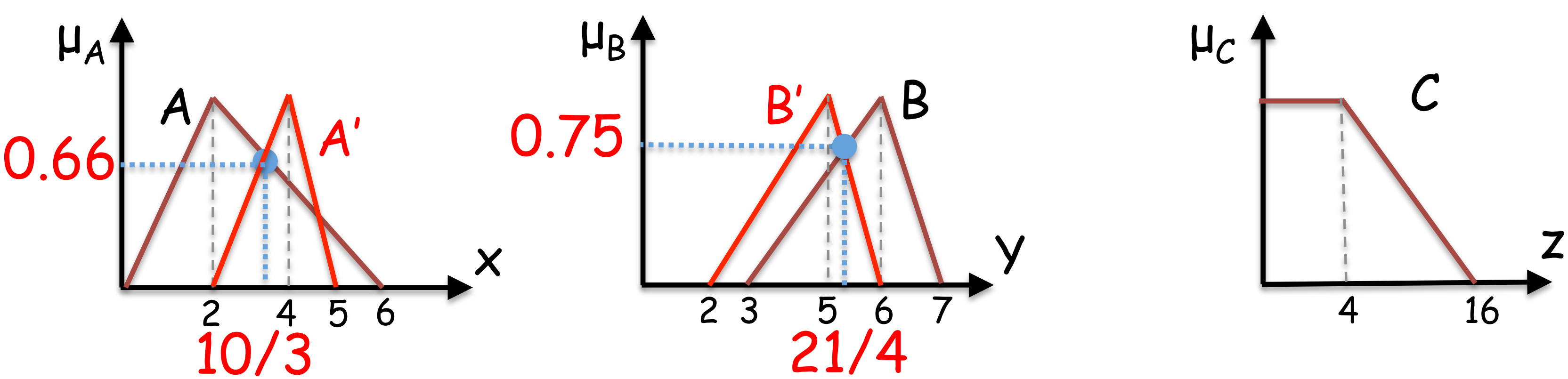
where A = (0, 2, 6) and B = (3, 6, 7)

- $\alpha_1 = \max_x \{\min(\mu_A(x), \mu_{A'}(x))\}$ and $\alpha_2 = \max_y \{\min(\mu_B(y), \mu_{B'}(y))\}$

  $z = \mu_C^{-1}(\alpha)$ where $\alpha = \alpha_1 \wedge \alpha_2$



Tsukamoto Fuzzy Inference

Fuzzy Input

- the fact is : x is A' and y is B' --A'=(2, 4, 5) and B'=(2, 3, 5)--

  the rule is : If x is A and y is B, then z is C

  the result is : $z = z_0$

where A = (0, 2, 6) and B = (3, 6, 7)

- $\alpha_1 = \max_x \{\min(\mu_A(x), \mu_{A'}(x))\}$ and $\alpha_2 = \max_y \{\min(\mu_B(y), \mu_{B'}(y))\}$

  $z = \mu_C^{-1}(\alpha)$ where $\alpha = \alpha_1 \wedge \alpha_2$