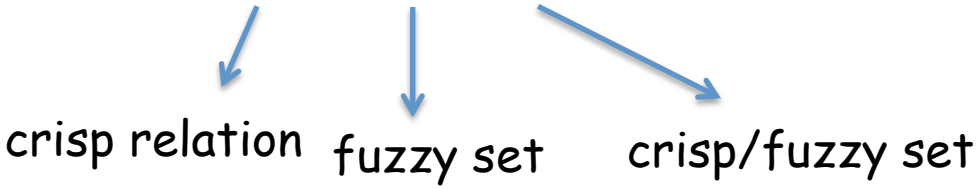
Fuzzy 3

Murat Osmanoglu

Extension of Fuzzy Set

$$R \subseteq A \times B$$



Extension by Crisp Relation

- $B' \subseteq B$ induced by the crisp relation R and the fuzzy set A:

$$B' = \{(y, \mu_{B'}(y)) \mid \mu_{B'}(y) = \max_{x \text{ s.t. } (x,y) \text{ in } R} \mu_A(x)\}$$

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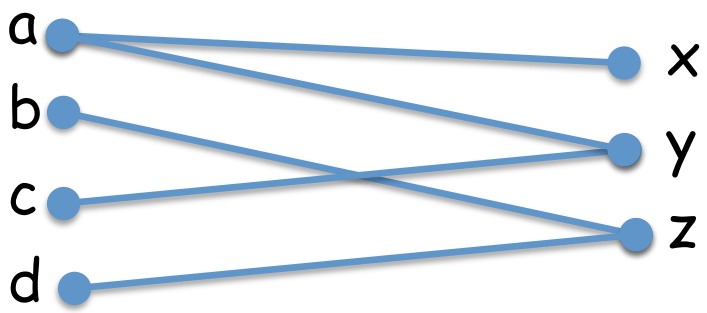

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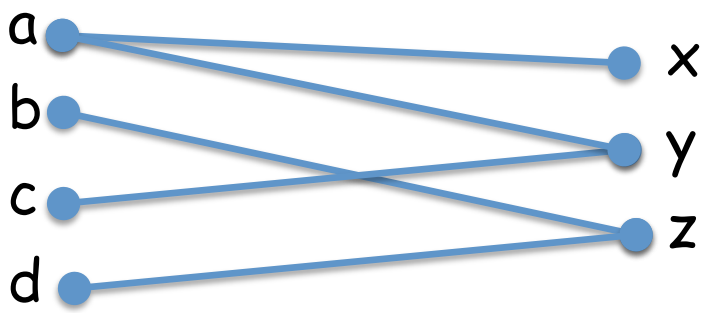

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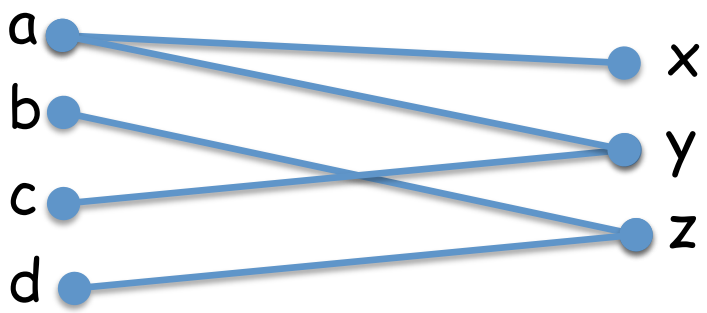

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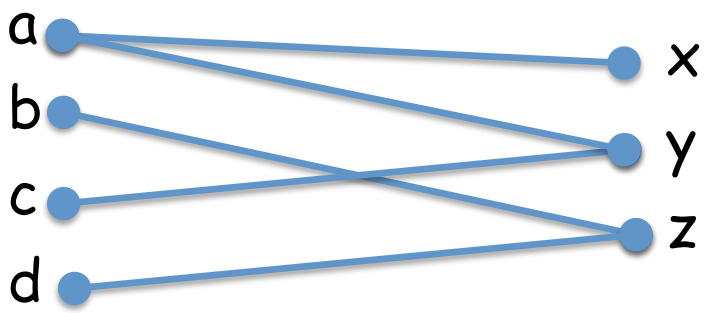

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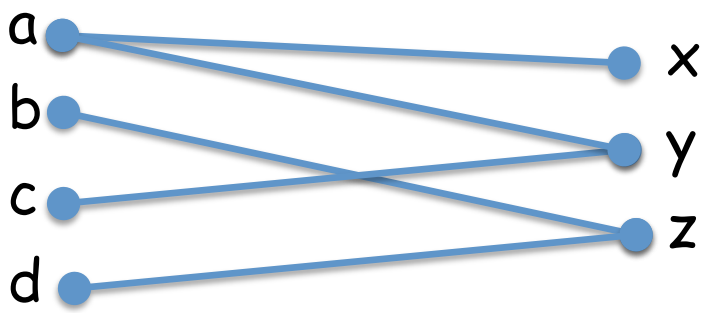

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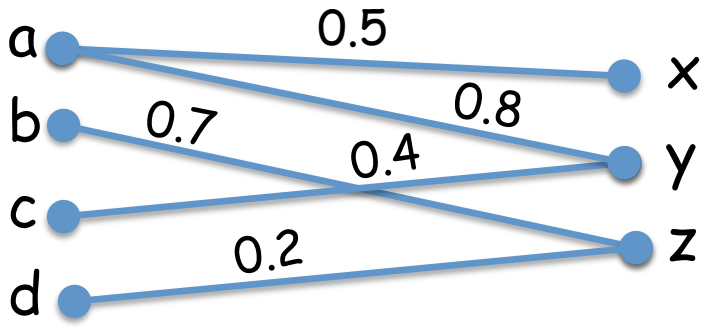

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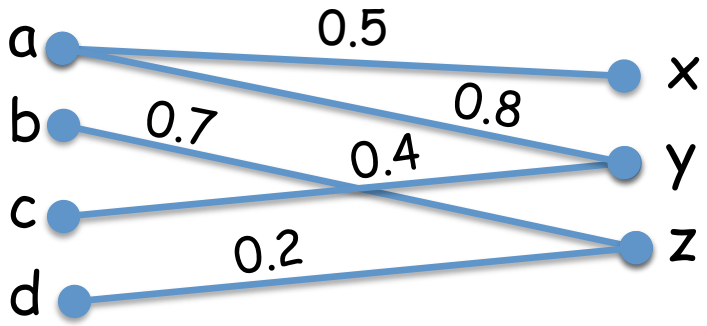

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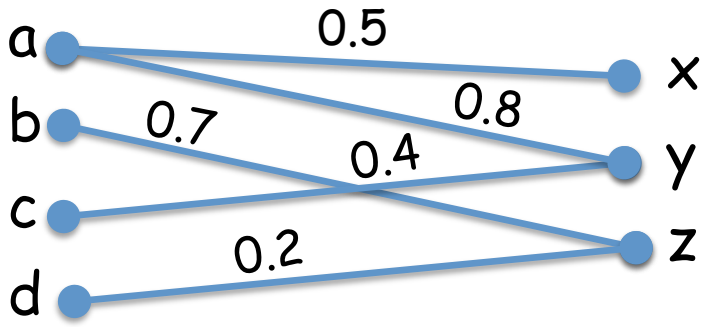

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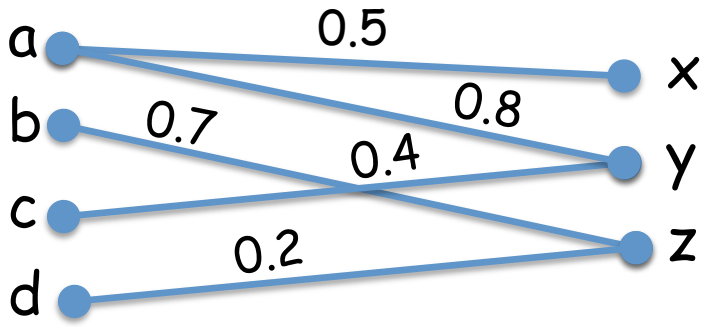

B' = {(x, 0.5), (y, 0.6), (z, 0.7)}

Fuzzy Distance

- calculate the fuzzy distance between the fuzzy sets
  $A = \{(1, 0.5), (2, 1.0), (3, 0.7)\}$ and $B = \{(3, 0.6), (4, 1.0), (5, 0.3)\}$

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