

# Fuzzy 8

Murat Osmanoglu

# Mamdani Fuzzy Inference

## Fuzzy Input

- the fact is :  $x$  is  $A'$
- the rule is : If  $x$  is  $A$ , then  $z$  is  $C$
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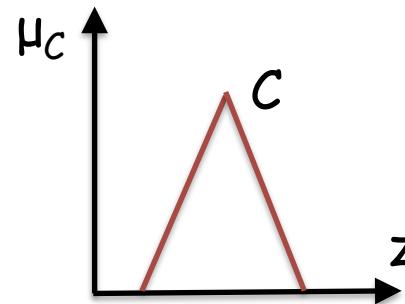
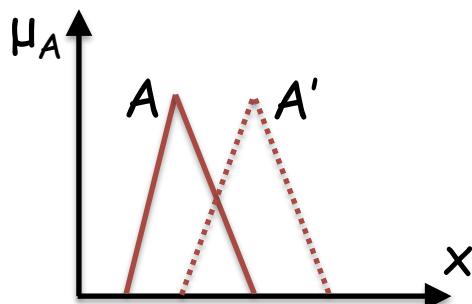
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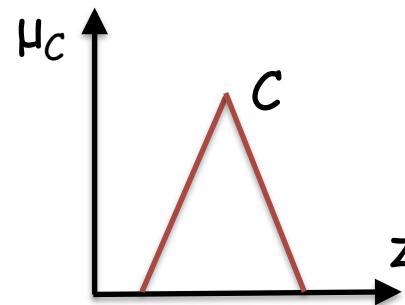
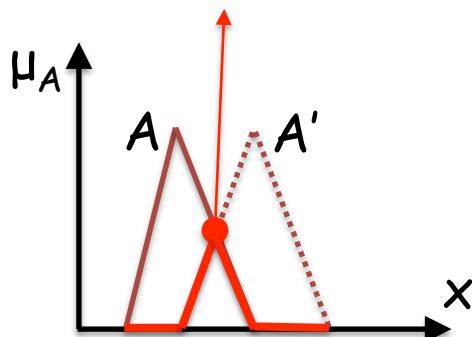
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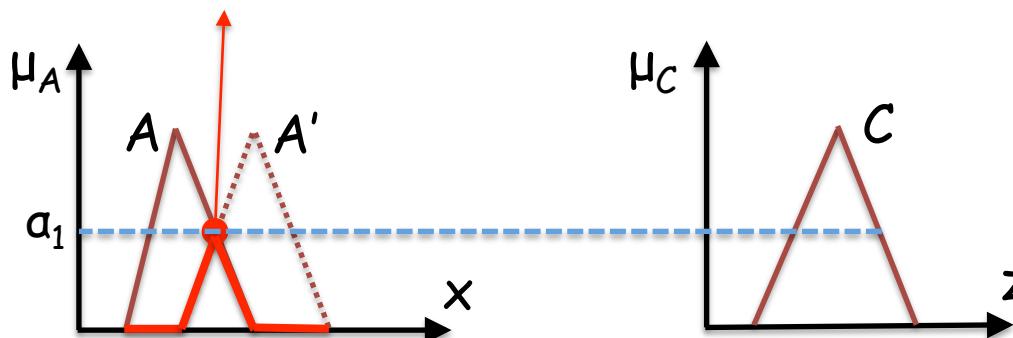
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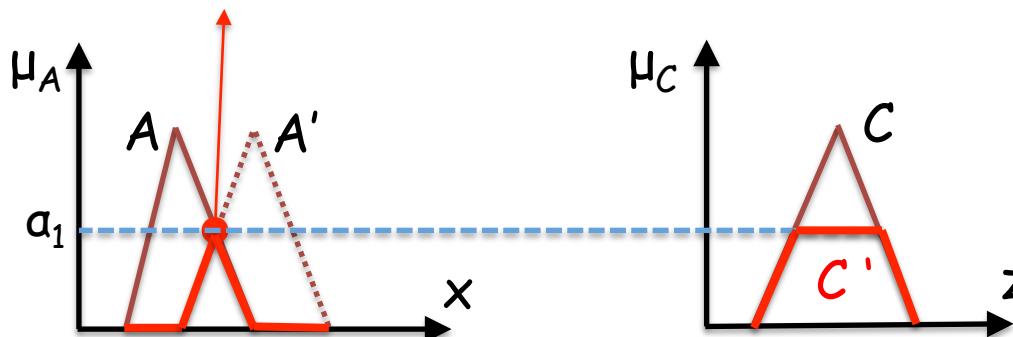
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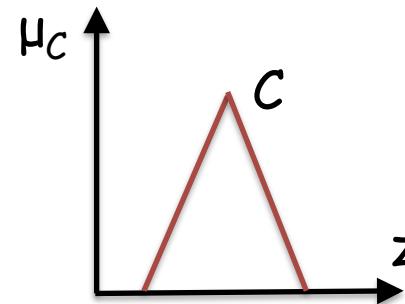
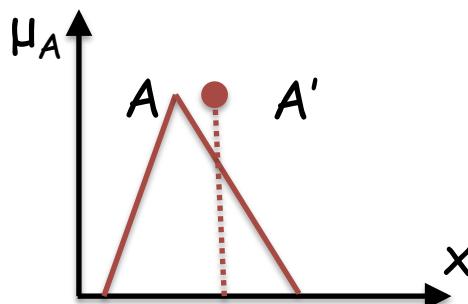
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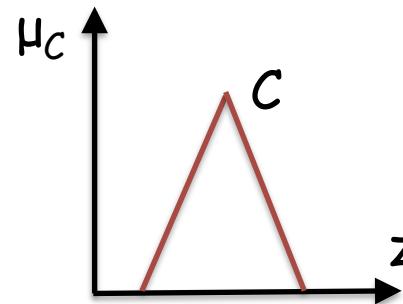
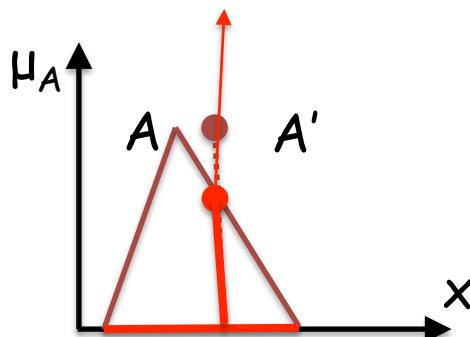
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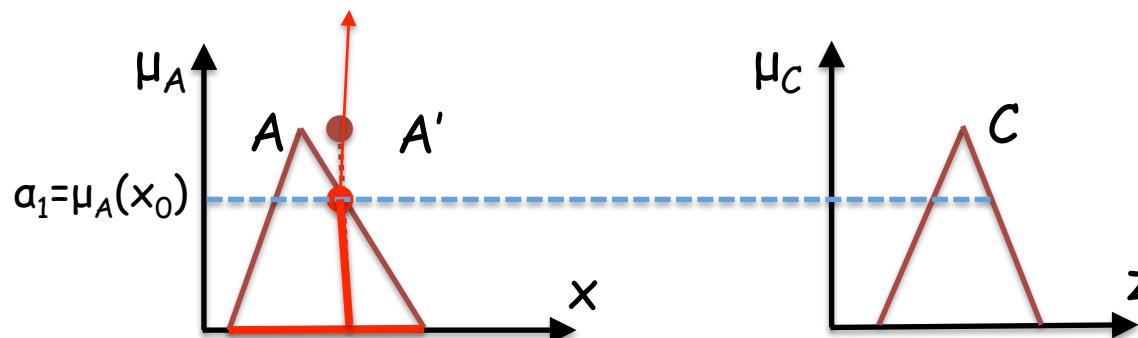
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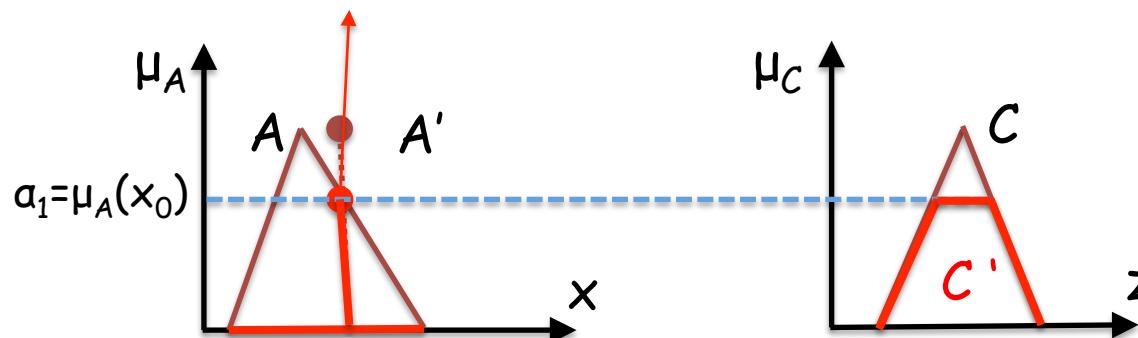
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## Single Input Single Output

- input :  $x$  is  $A'$

$R_1$  : if  $x$  is  $A_1$ , then  $z$  is  $C_1$  :  $A_1 \rightarrow C_1$

$R_2$  : if  $x$  is  $A_2$ , then  $z$  is  $C_2$  :  $A_2 \rightarrow C_2$

output :  $z$  is  $C'$

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$R_2$  : if  $x$  is  $A_2$ , then  $z$  is  $C_2$  :  $A_2 \rightarrow C_2$

output :  $z$  is  $C'$

- $C' = A' \circ (R_1 \cup R_2) = A' \circ [(A_1 \rightarrow C_1) \cup (A_2 \rightarrow C_2)]$

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$R_2$  : if  $x$  is  $A_2$ , then  $z$  is  $C_2$  :  $A_2 \rightarrow C_2$

output :  $z$  is  $C'$

$$C' = A' \circ (R_1 \cup R_2) = A' \circ [(A_1 \rightarrow C_1) \cup (A_2 \rightarrow C_2)]$$

$$C' = [A' \circ (A_1 \rightarrow C_1)] \cup [A' \circ (A_2 \rightarrow C_2)] = C'_1 \cup C'_2$$

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$$\mu_{C'}(z) = \max \{ \mu_{C'_1}(z), \mu_{C'_2}(z) \}$$

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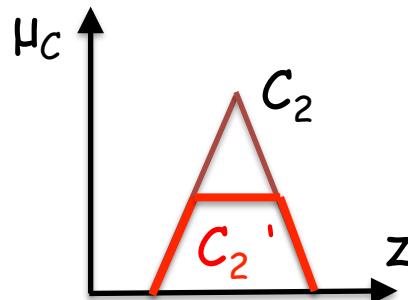
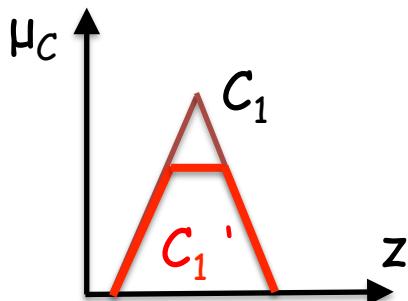
$R_2$  : if  $x$  is  $A_2$ , then  $z$  is  $C_2$  :  $A_2 \rightarrow C_2$

output :  $z$  is  $C'$

- $C' = A' \circ (R_1 \cup R_2) = A' \circ [(A_1 \rightarrow C_1) \cup (A_2 \rightarrow C_2)]$

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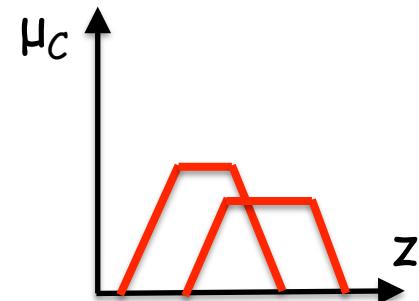
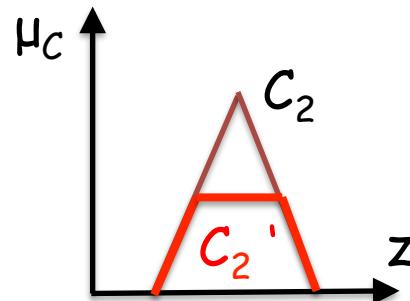
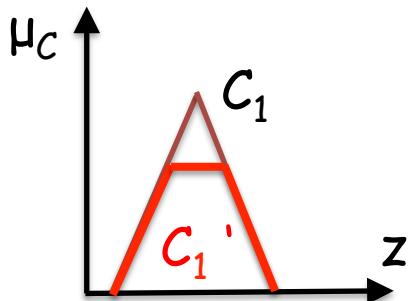
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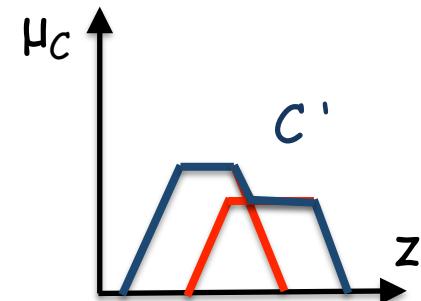
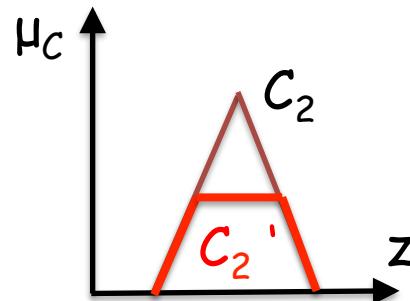
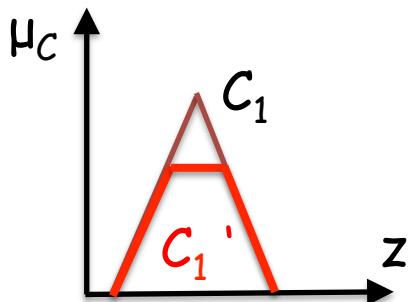
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# Mamdani Fuzzy Inference

## Two Input Single Output

- input :  $x$  is  $A'$  and  $y$  is  $B'$   
 $R$  : if  $x$  is  $A$  and  $y$  is  $B$ , then  $z$  is  $C$  :  $(A \text{ and } B) \rightarrow C$   
output :  $z$  is  $C'$

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## Two Input Single Output

- input :  $x \text{ is } A'$  and  $y \text{ is } B'$   
 $R : \text{if } x \text{ is } A \text{ and } y \text{ is } B, \text{ then } z \text{ is } C : (A \text{ and } B) \rightarrow C$   
output :  $z \text{ is } C'$
- $C' = A' \circ R = A' \circ [(A \text{ and } B) \rightarrow C] = A' \circ [(A \rightarrow C) \cap (B \rightarrow C)]$

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- $C' = A' \circ R = A' \circ [(A \text{ and } B) \rightarrow C] = A' \circ [(A \rightarrow C) \cap (B \rightarrow C)]$   
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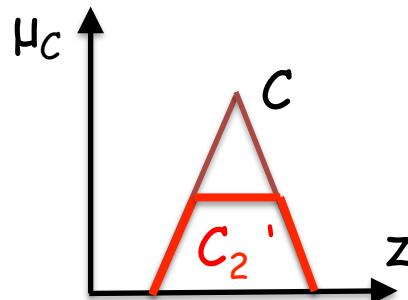
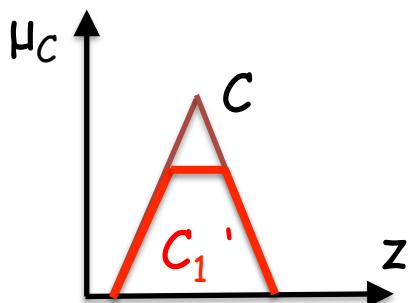
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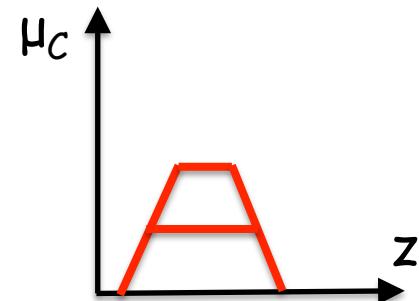
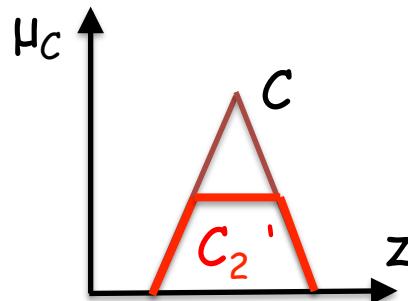
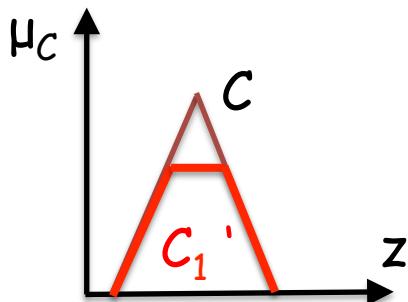
- input :  $x$  is  $A'$  and  $y$  is  $B'$   
 $R$  : if  $x$  is  $A$  and  $y$  is  $B$ , then  $z$  is  $C$  :  $(A \text{ and } B) \rightarrow C$   
output :  $z$  is  $C'$
- $C' = A' \circ R = A' \circ [(A \text{ and } B) \rightarrow C] = A' \circ [(A \rightarrow C) \cap (B \rightarrow C)]$   
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# Mamdani Fuzzy Inference

## Two Input Single Output

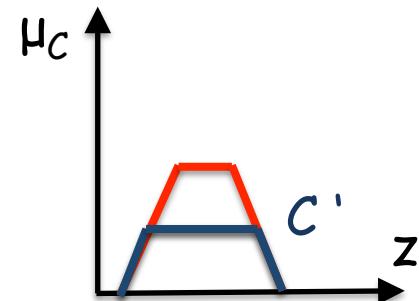
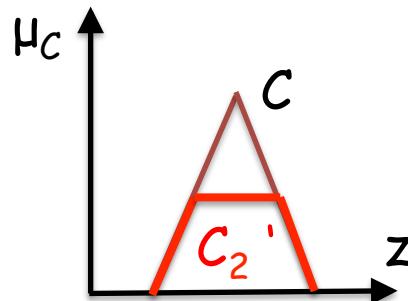
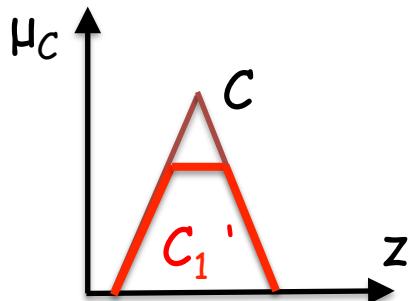
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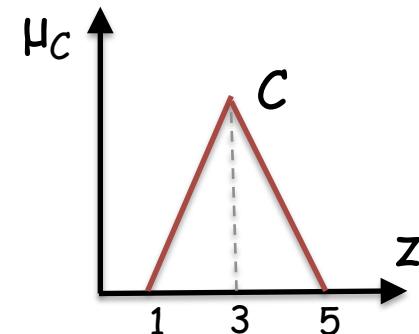
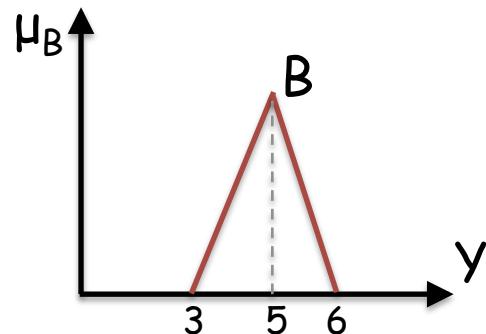
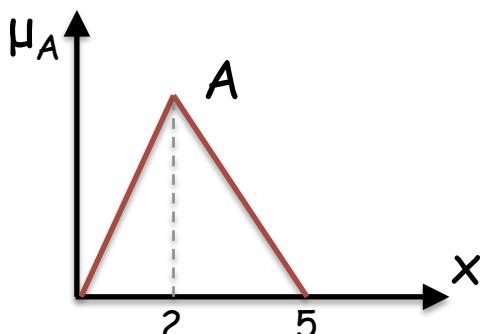
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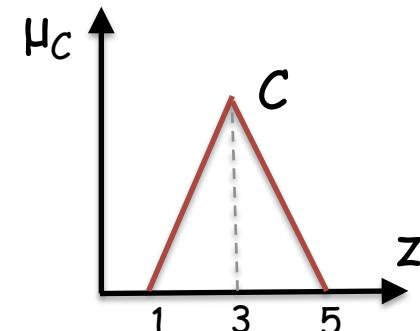
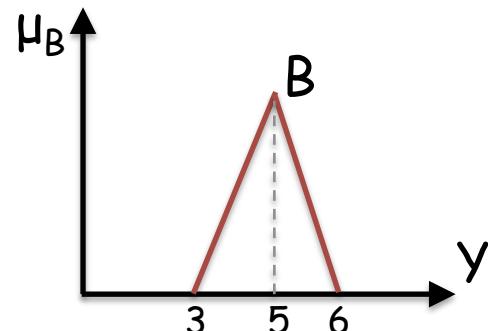
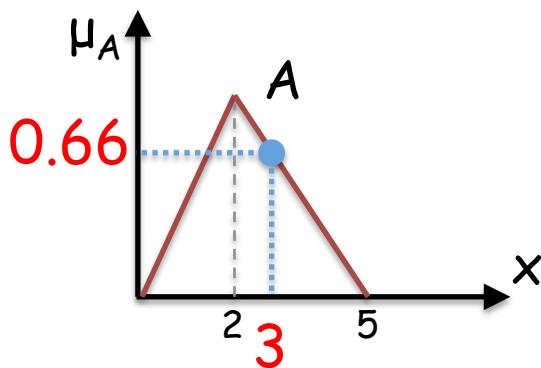
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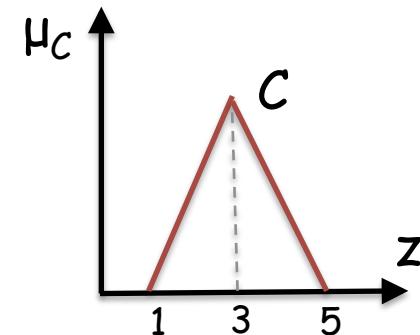
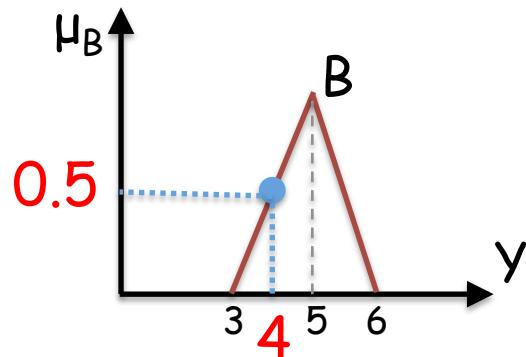
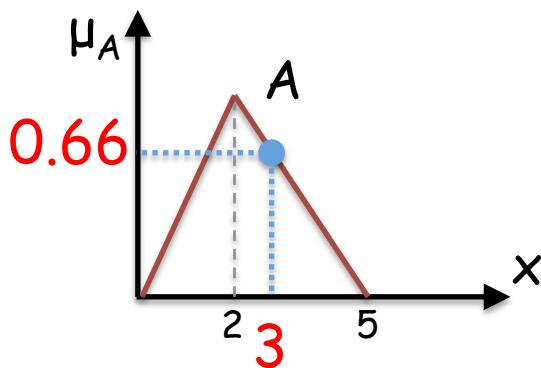
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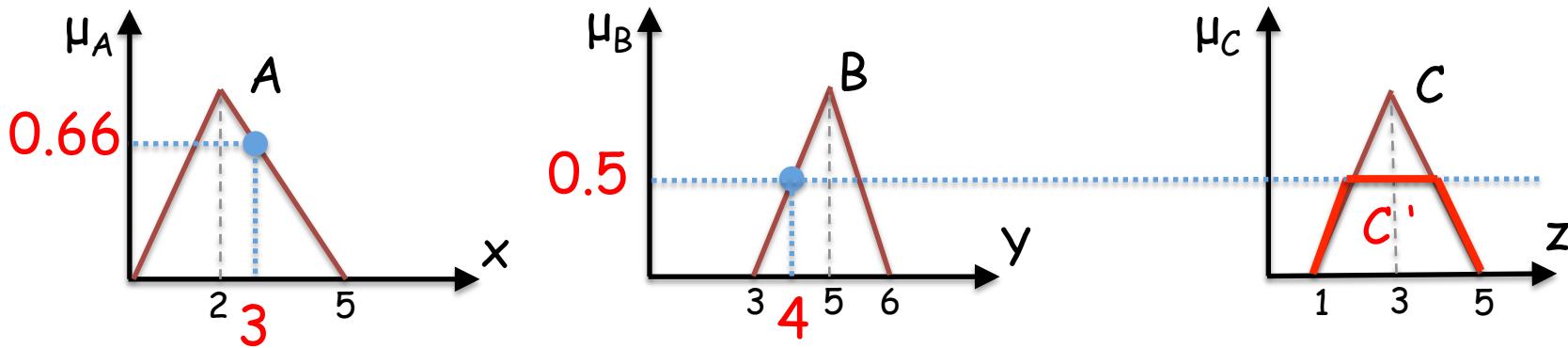
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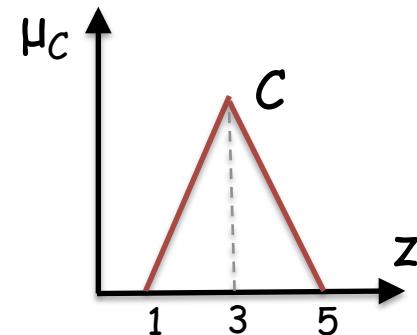
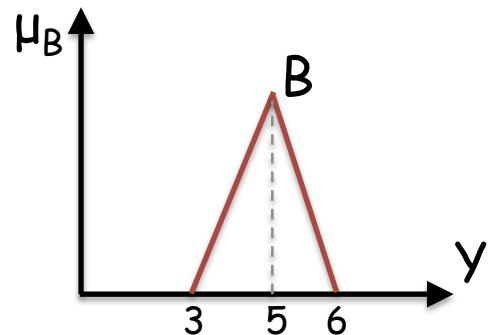
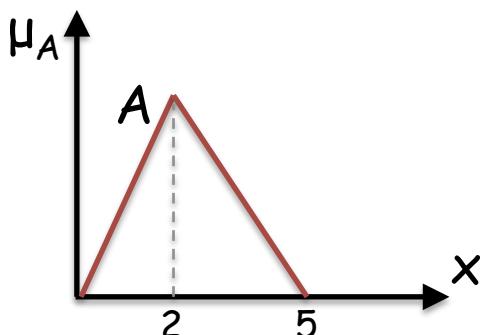
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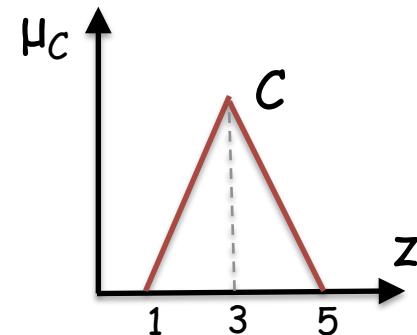
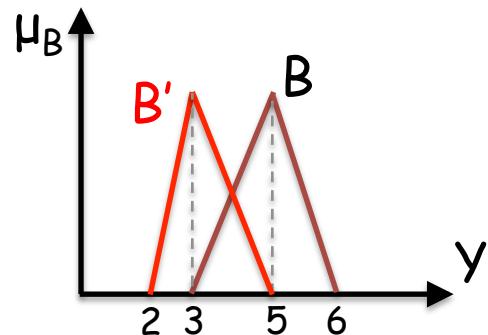
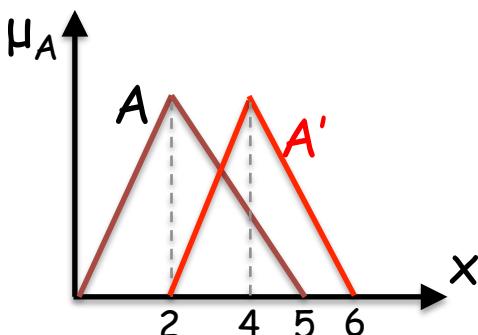
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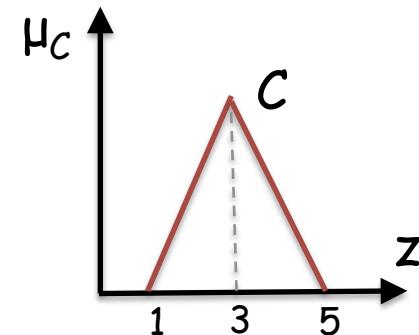
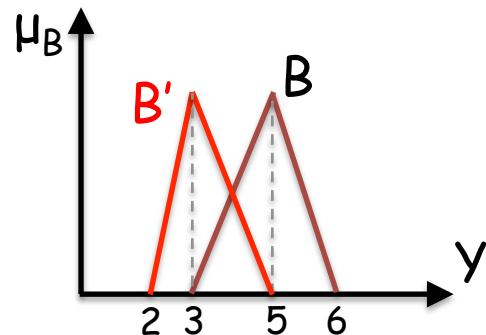
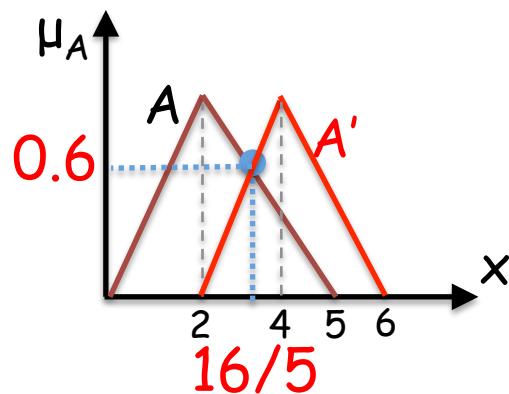
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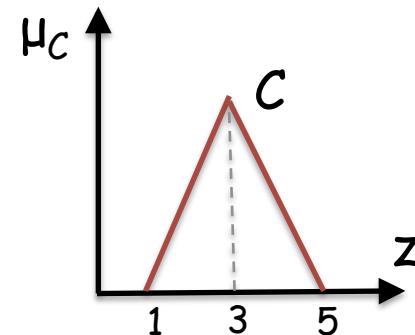
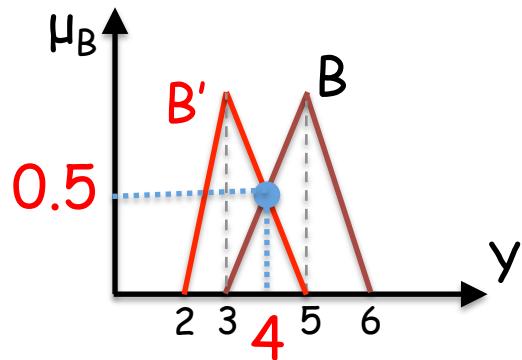
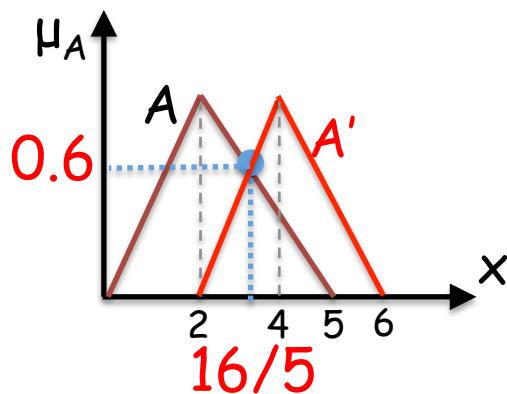
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