# Introduction to Algorithms

Murat Osmanoglu

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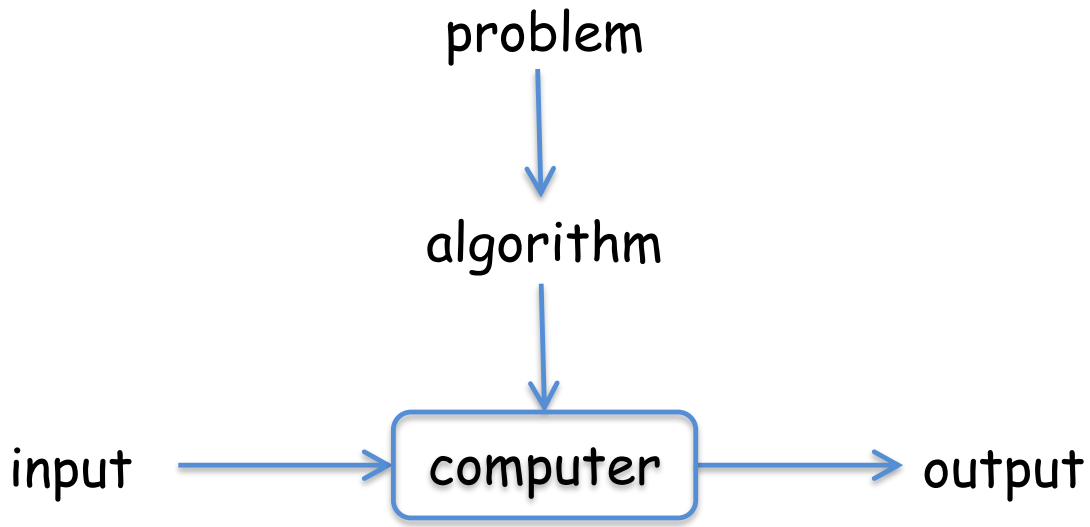

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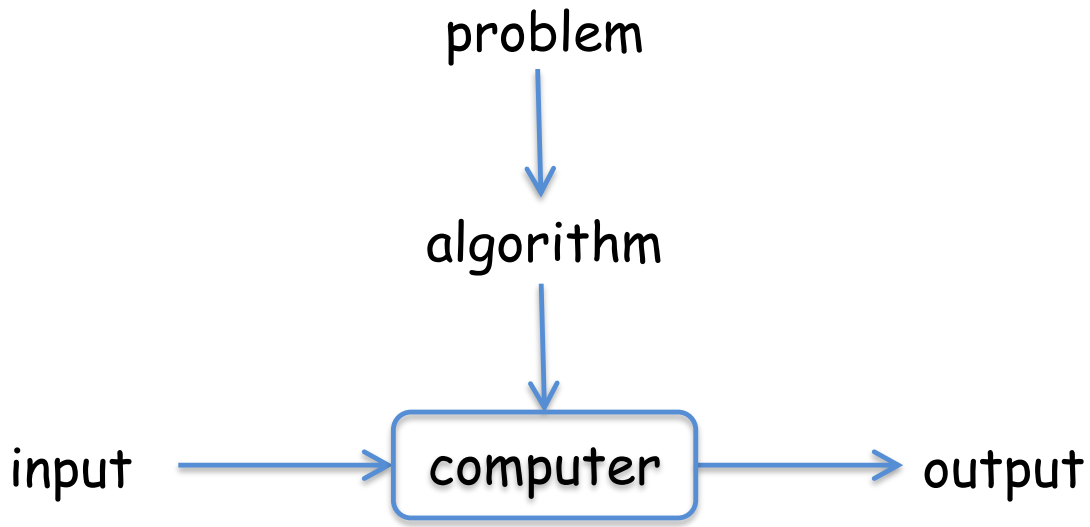


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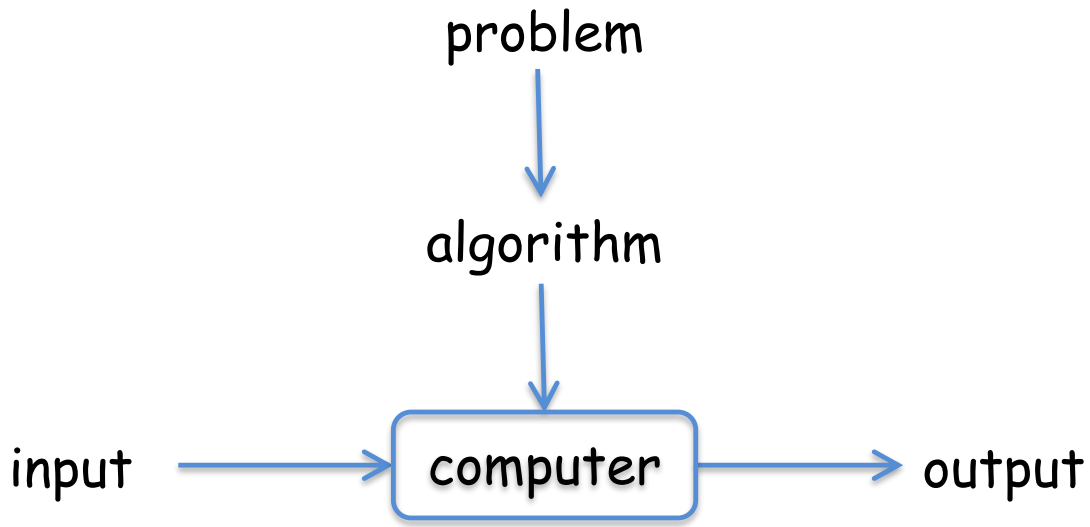


- each step should be expressed in a clear way
- the nature of the input should be specified carefully

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- approximation algorithm can be a sub-procedure of a more sophisticated algorithm

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- Brute-Force, Decrease-and-Conquer, Divide-and-Conquer, Transform-and-Conquer, Dynamic Programming, Greedy Techniques

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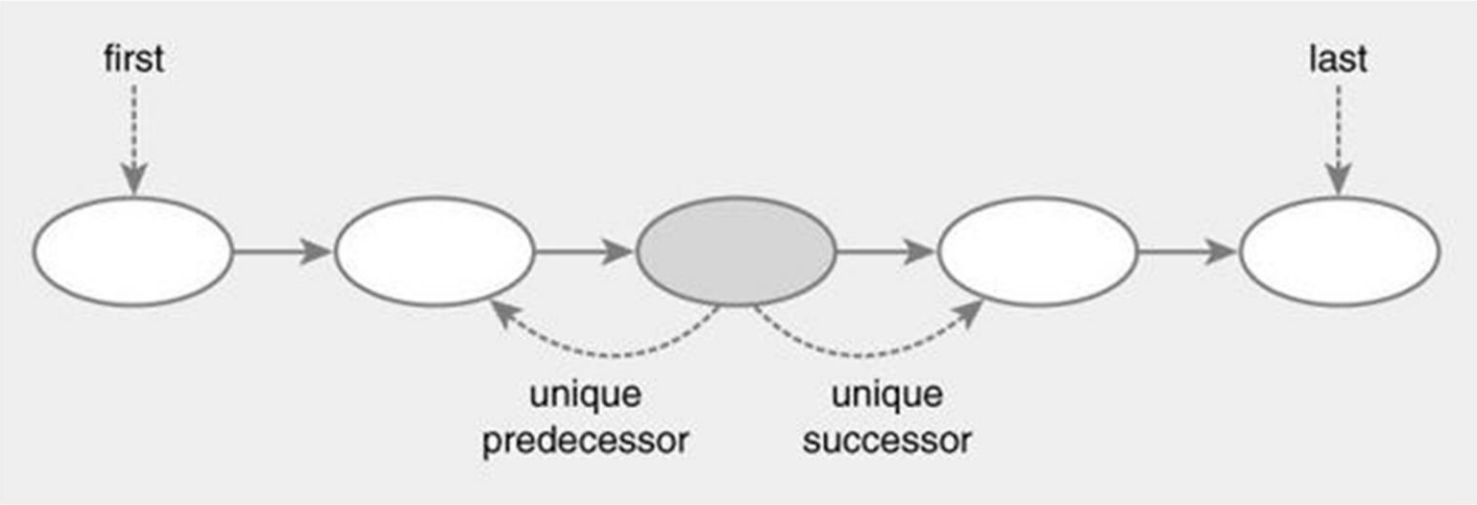
- Linear Data Structures
  - one-to-one relationship between elements in the collection
  - Arrays, Linked Lists, Stacks, Queues



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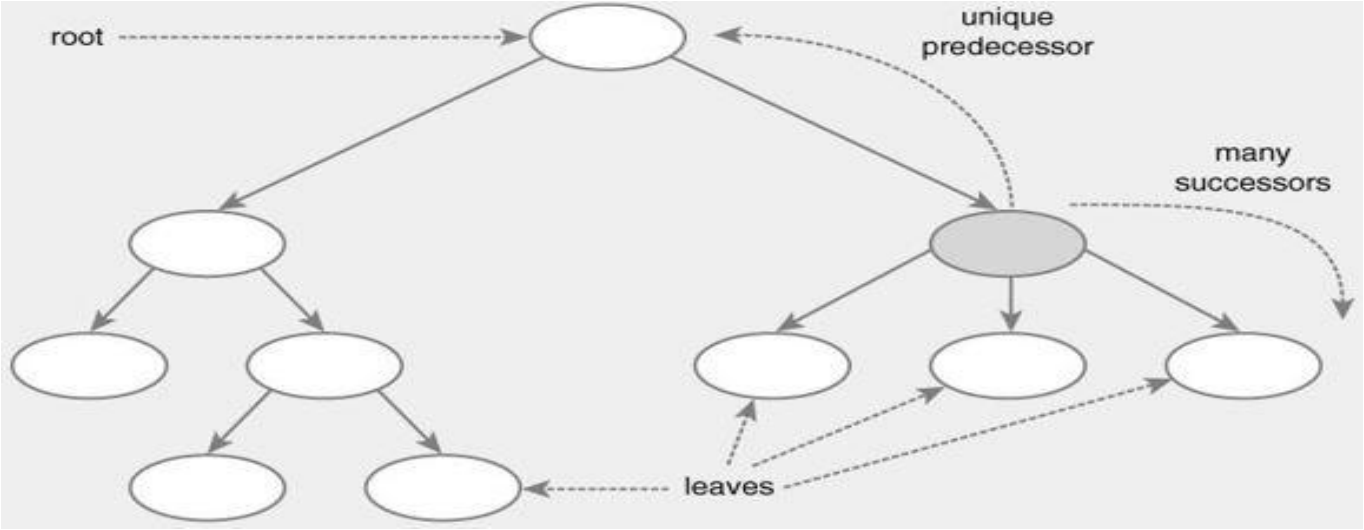
- Hierarchical Data Structures
  - one-to-many relationship between elements in the collection
  - Binary Trees, AVL Trees, Splay Trees, B Trees



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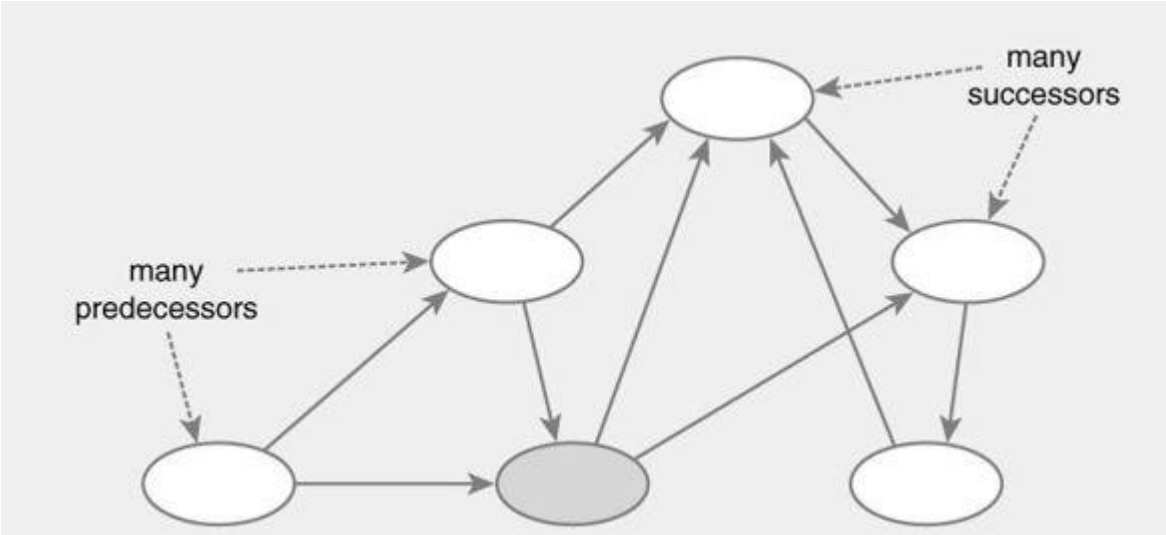

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  **Step 1** : if n = 0, return m; otherwise go to Step 2

  **Step 2** : divide m by n and set the variable r to the remainder

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- **Euclid(m,n)**

**input** : two non-negative, not-both-zero integers m and n
**output**: the greatest common divisor of m and n

**while** $n \neq 0$

$r \leftarrow m \ (mod \ n)$

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- high-level description of algorithms that combines a natural language and familiar structures from a programming language
- use '←' for the assigments and '//' for the comments

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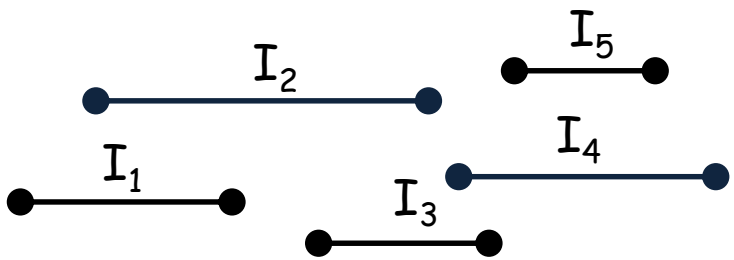

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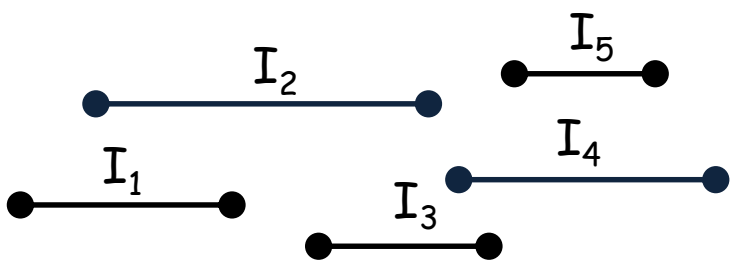


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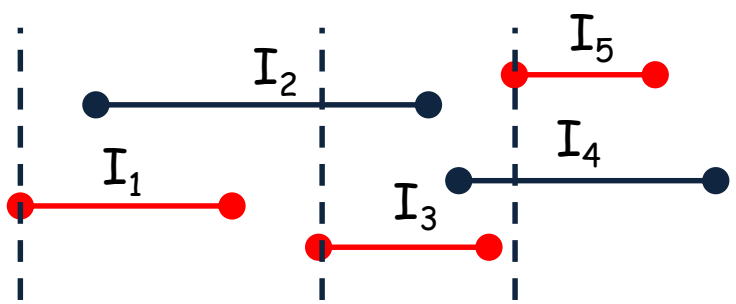


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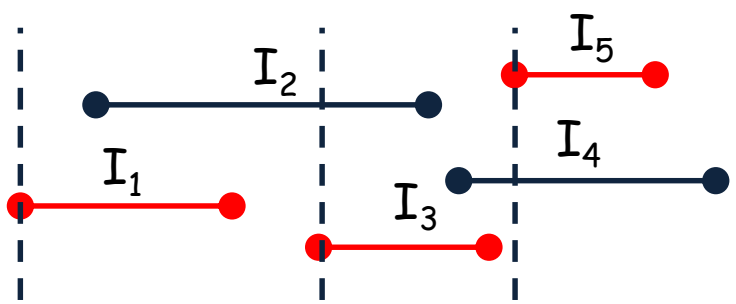


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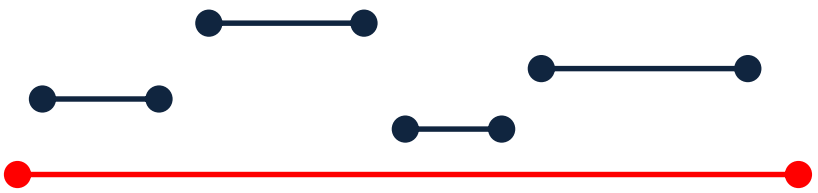

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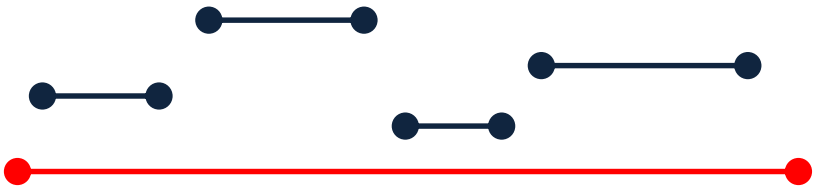

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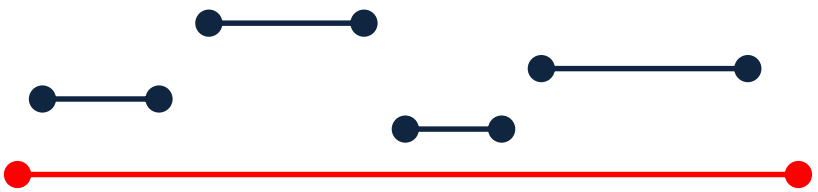

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- how much working memory (typically RAM) required for the algorithm to terminate as a function of input size ?