Brute Force and Exhaustive Search

Murat Osmanoglu

Brute Force Approach

- a straightforward approach to solving a given problem
(solving a given problem in the most simple, direct, or obvious way)
- directly based on the problem statement and definitions of the concepts involved

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Exponentiation Problem

- Given a nonzero number A and a nonnegative integer n, compute A^n

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Exponentiation Problem

- Given a nonzero number A and a nonnegative integer n, compute A^n
  - $A^n = \underbrace{A.A.A....A}_{\text{n times}}$, thus multiply 1 by A n times to compute the output

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Pluses and Minuses

- Given n consecutive integer from 1 to n, devise an algorithm that puts signs "+" and "-" between them so that the expression obtained is equal 0, or if no such expression exists, returns the message "no solution"

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Pluses and Minuses

- *Given n consecutive integer from 1 to n, devise an algorithm that puts signs "+" and "-"* between them so that the expression obtained is equal 0, or if no such expression exists, returns the message "no solution"
  - write down all possible combinations to verify whether there is such expression ($\approx O(2^n)$)
  - for the input 3, 7, 8, 2, 1, 5;

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  - for the input 3, 7, 8, 2, 1, 5;

3-7-8-2-1-5 = -20, 3-7-8-2-1+5 = -10, 3-7-8-2+1-5 = -18, 3-7-8-2+1+5 = -8, 3-7-8+2-1-5 = -16
3-7-8+2-1+5 = -6, 3-7-8+2+1-5 = -14, 3-7-8+2+1+5 = -4, 3-7+8-2-1-5 = -4, 3-7+8-2-1+5 = 6,
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3+7-8+2+1-5 = 0

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- unlike other design techniques, brute-force can be applied to a very wide variety of problems
- if only a few instances or small-size instances of a problem need to be solved, brute-force can lift the burden of designing more efficient algorithms
- brute-force can serve as a reference point when judging the efficiency of other alternatives

Pluses and

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  - for the input 3, 7, 8, 2, 1, 5;

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…
3+7-8+2+1-5 = 0

Brute Force Approach (Sorting Problem)

- given a sequence of n orderable items $[a_1, a_2, \ldots, a_n]$, reorder the items as $[a_1', a_2', \ldots, a_n']$ such that $a_1' \le a_2' \le \ldots \le a_n'$

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  searching; given a set of n items and a separate item, search whether the set contains the given item

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  closest pair; given a set of n numbers, find the pair of numbers that have the smallest possible difference

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- Why is sorting worth so much attention ?
  - many important techniques have been developed for solving this problem
  - the most studied problem in CS
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  searching; given a set of n items and a separate item, search whether the set contains the given item

  closest pair; given a set of n numbers, find the pair of numbers that have the smallest possible difference

  frequency distribution; given a set of n items, find the frequencies of the items

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(Sorting Problem)

- What would be the most straightforward way of solving sorting problem ?

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Selection Sort

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Selection Sort

- find the smallest element by scanning the sequence from the first to the last, and exchange it with the first element of the sequence

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Selection Sort

- find the smallest element by scanning the sequence from the first to the last, and exchange it with the first element of the sequence
- find the second smallest element by scanning the sequence from the second to the last, and exchange it with the second element of the sequence

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Selection Sort

- find the smallest element by scanning the sequence from the first to the last, and exchange it with the first element of the sequence

- find the second smallest element by scanning the sequence from the second to the last, and exchange it with the second element of the sequence

  ⋮ ⋮ ⋮

- find the i-th smallest element by scanning the sequence from the (i+1)-th to the last, and exchange it with the i-th element of the sequence

Brute Force Approach (Sorting Problem)

SelectionSort([a_1, a_2,…, a_n])

input : a sequence of orderable items
output : sorted sequence in nondecreasing order

```
for i = 1 to n – 1
    min ← i
    for j = i + 1 to n
        if a_j < a_min
            min ← j
    swap a_i and a_min
```

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S E L E C T I O N

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i = 1

min ← 1

S E L E C T I O N

j = 2

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(Sorting Problem)

SelectionSort($[a_1, a_2, \dots, a_n]$)

input : a sequence of orderable items
output : sorted sequence in nondecreasing order

for $i = 1$ to $n - 1$
    min $\leftarrow$ i
    for $j = i + 1$ to n
        if $a_j < a_{min}$
            min $\leftarrow$ j
    swap a_i and a_{min}

i = 1

min $\leftarrow$ 5

S E L E C T I O N

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min ← 2

C E L E S T I O N

j = 3

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input : a sequence of orderable items
output : sorted sequence in nondecreasing order

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    min $\leftarrow$ i
    for $j = i + 1$ to n
        if $a_j < a_{min}$
            min $\leftarrow$ j
    swap a_i and a_{min}

$i = 2$

min $\leftarrow$ 2

C E L E S T I O N

$j = 9$

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$i = 3$

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C E L E S T I O N

$j = 4$

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min ← 4

j = 4

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i = 4

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i = 4

min ← 7

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i = 8

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C E E I L N O S N

j = 9

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j = 9

$$T(n) = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} 1 = \sum_{i=1}^{n-1} [n - (i+1) + 1] = \sum_{i=1}^{n-1} (n - i)$$

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$$T(n) = \sum_{i=1}^{n-1} n - \sum_{i=1}^{n-1} i$$

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$$T(n) = \sum_{i=1}^{n-1} n - \sum_{i=1}^{n-1} i = n(n-1) - n(n-1)/2$$

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$$T(n) = \sum_{i=1}^{n-1} n - \sum_{i=1}^{n-1} i = n(n-1) - n(n-1)/2$$

$$T(n) = n(n-1)/2 \in O(n^2)$$

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Bubble Sort

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- compare the adjacent elements of the sequence from the first to the last, and exchange them if they are out of order in order to bubble up the largest to the last position

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- compare the adjacent elements of the sequence from the first to the last, and exchange them if they are out of order in order to bubble up the largest to the last position
- compare the adjacent elements of the sequence from the first to the second last, and exchange them if they are out of order in order to bubble up the second largest to the second last position

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- compare the adjacent elements of the sequence from the first to the second last, and exchange them if they are out of order in order to bubble up the second largest to the second last position

$\vdots \qquad \vdots \qquad \vdots$

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input : a sequence of orderable items
output : sorted sequence in nondecreasing order

for i = 1 to n – 1
    for j = 1 to n – i
        if $a_{j+1} < a_j$
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SELECTION

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$\underline{i = 1}$

S E L E C T I O N

$j = 1$

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j = 2

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input : a sequence of orderable items
output : sorted sequence in nondecreasing order

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    for $j = 1$ to $n - i$
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            swap a_{j+1} and a_j

$i = 1$

E	L	S	E	C	T	I	O	N

$j = 3$

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    for j = 1 to n – i
        if $a_{j+1} < a_j$
            swap a_{j+1} and a_j

i = 1

E	L	E	S	C	T	I	O	N	

j = 4

Brute Force Approach
(Sorting Problem)

SelectionSort($[a_1, a_2, \ldots, a_n]$)

input : a sequence of orderable items
output : sorted sequence in nondecreasing order

for $i = 1$ to $n - 1$
    for $j = 1$ to $n - i$
        if $a_{j+1} < a_j$
            swap a_{j+1} and a_j

$i = 1$

E L E C S T I O N

$j = 5$

Brute Force Approach
(Sorting Problem)

SelectionSort($[a_1, a_2, \ldots, a_n]$)

input : a sequence of orderable items
output : sorted sequence in nondecreasing order

for $i = 1$ to $n - 1$
  for $j = 1$ to $n - i$
    if $a_{j+1} < a_j$
      swap a_{j+1} and a_j

$i = 1$

E L E C S T I O N

$j = 6$

Brute Force Approach (Sorting Problem)

SelectionSort([a_1, a_2,…, a_n])

input : a sequence of orderable items
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for i = 1 to n – 1
  for j = 1 to n – i
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i = 1

E L E C S I T O N

j = 7

Brute Force Approach (Sorting Problem)

SelectionSort([a_1, a_2,…, a_n])

input : a sequence of orderable items
output : sorted sequence in nondecreasing order

for i = 1 to n – 1
    for j = 1 to n – i
        if $a_{j+1} < a_j$
            swap a_{j+1} and a_j

i = 1

E	L	E	C	S	I	O	T	N

j = 8

Brute Force Approach
(Sorting Problem)

SelectionSort([a_1, a_2,…, a_n])

input : a sequence of orderable items
output : sorted sequence in nondecreasing order

for i = 1 to n – 1
    for j = 1 to n – i
        if $a_{j+1} < a_j$
            swap a_{j+1} and a_j

i = 1

E	L	E	C	S	I	O	N	**T**

j = 8

Brute Force Approach
(Sorting Problem)

<u>SelectionSort</u><u>([a_1, a_2,…, a_n])</u>

input : a sequence of orderable items
output : sorted sequence in nondecreasing order

for i = 1 to n – 1
    for j = 1 to n – i
        if $a_{j+1} < a_j$
            swap a_{j+1} and a_j

<u>i = 2</u>

E L E C S I O N <u>T</u>

j = 1

Brute Force Approach
(Sorting Problem)

SelectionSort([a_1, a_2,…, a_n])

input : a sequence of orderable items
output : sorted sequence in nondecreasing order

for $i = 1$ to $n - 1$
    for $j = 1$ to $n - i$
        if $a_{j+1} < a_j$
            swap a_{j+1} and a_j

$i = 2$

E	L	E	C	S	I	O	N	T

$j = 1$

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(Sorting Problem)

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input : a sequence of orderable items
output : sorted sequence in nondecreasing order

for i = 1 to n – 1
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        if $a_{j+1} < a_j$
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i = 2

E L E C S I O N T

j = 2

Brute Force Approach
(Sorting Problem)

SelectionSort([a_1, a_2,…, a_n])

input : a sequence of orderable items
output : sorted sequence in nondecreasing order

for i = 1 to n – 1
    for j = 1 to n – i
        if $a_{j+1} < a_j$
            swap a_{j+1} and a_j

i = 2

E E L C S I O N T

j = 3

Brute Force Approach (Sorting Problem)

SelectionSort($[a_1, a_2, \ldots, a_n]$)

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output : sorted sequence in nondecreasing order

for $i = 1$ to $n - 1$
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    if $a_{j+1} < a_j$
      swap a_{j+1} and a_j

$i = 2$

E E C L S I O N T

$j = 4$

Brute Force Approach (Sorting Problem)

SelectionSort([a_1, a_2,…, a_n])

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for i = 1 to n – 1
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i = 2

E E C L S I O N T

j = 5

Brute Force Approach
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SelectionSort([a_1, a_2,…, a_n])

input : a sequence of orderable items
output : sorted sequence in nondecreasing order

for i = 1 to n – 1
    for j = 1 to n – i
        if $a_{j+1} < a_j$
            swap a_{j+1} and a_j

i = 2

E E C L I S O N T

j = 6

Brute Force Approach
(Sorting Problem)

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i = 2

E E C L I O S N T

j = 7

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C	E	E	I	L	O	N	S	T

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Brute Force Approach
(Sorting Problem)

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i = 8

C E E I L O N S T

j = 1

$$T(n) = \sum_{i=1}^{n-1} \sum_{j=1}^{n-i} 1 = \sum_{i=1}^{n-1} (n-i) = n(n-1)/2 \in O(n^2)$$

Brute Force Approach (Searching Problem)

- given a sequence of n items $[a_1, a_2, \ldots, a_n]$ and a search key K, determine whether the sequence contains the search key K

Brute Force Approach
(Searching Problem)

- given a sequence of n items [a_1, a_2,…, a_n] and a search key K, determine whether the sequence contains the search key K

Linear Search

Brute Force Approach (Searching Problem)

- given a sequence of n items [a_1, a_2,…, a_n] and a search key K, determine whether the sequence contains the search key K

Linear Search

- simply compares the items of the sequence with the search key one by one, or terminates without finding a match

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- given a sequence of n items $[a_1, a_2,\ldots, a_n]$ and a search key K, determine whether the sequence contains the search key K

Linear Search

- simply compares the items of the sequence with the search key one by one, or terminates without finding a match

LinearSearch($[a_1, a_2,\ldots, a_n]$;K)

input : a sequence of n items
output : the index of the element that is equal to K, or 0 if no such element is found

```
for i = 1 to n
     if a_i = K
        return i
return 0
```

Brute Force Approach (Searching Problem)

- given a sequence of n items [a_1, a_2,…, a_n] and a search key K, determine whether the sequence contains the search key K

Linear Search

- simply compares the items of the sequence with the search key one by one, or terminates without finding a match

LinearSearch([a_1, a_2,…, a_n];K)

input : a sequence of n items
output : the index of the element that is equal to K, or 0 if no such element is found

for i = 1 to n
    if a_i = K
        return i
return 0

$$T(n) = \sum_{i=1}^{n} 1 = n \in O(n)$$

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(Searching Problem)

String Matching

- Given a string of n characters (text) and a string of m characters (pattern), determine whether the text has a substring that matches the pattern

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 (the characters in the matching substring must be successive)

text   : FEDERICO_FELLINI
pattern : FEL

Brute Force Approach (Searching Problem)

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  (the characters in the matching substring must be successive)

- find all the substrings of length m in the text, and check whether any of them matches the pattern

  text : FEDERICO_FELLINI
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  text : FEDERICO_FELLINI
  pattern : FEL

  FED, EDE, DER, ERI, RIC, ICO, CO_, O_F, _FE, FEL, ELL, LLI, LIN, INI

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  text : FEDERICO_FELLINI
  pattern : FEL

  FED, EDE, DER, ERI, RIC, ICO, CO_, O_F, _FE, FEL, ELL, LLI, LIN, INI

  output : 10

Brute Force Approach
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StringMatching($T = t_1t_2\ldots t_n$, $P = p_1p_2\ldots p_m$)

input : a text with n characters and a pattern with m characters

output : outputs the index of the leftmost character of the first matching substring, or 0 if no such substring exists

for $i = 1$ to $n - m + 1$
    $j \leftarrow 1$
    while $j < m + 1$ and $p_j = t_{i+j-1}$
        $j \leftarrow j+1$
    if $j = m + 1$
        return i
return 0

FEDERICO_FELLINI $n = 16$

FEL $m = 3$

Brute Force Approach
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```
for i = 1 to n – m + 1
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        j ← j+1
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```

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    $j \leftarrow j+1$
  if $j = m + 1$
    return i
return 0

FEDERICO_FELLINI $n = 16$

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$i = 1$

$j = 2$

Brute Force Approach (Searching Problem)

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i = 1

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input : a text with n characters and a pattern with m characters

output : outputs the index of the leftmost character of the first matching substring, or 0 if no such substring exists

```
for i = 1 to n – m + 1
    j ← 1
    while j < m + 1 and p_j = t_{i+j-1}
        j ← j+1
    if j = m + 1
        return i
return 0
```

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        return i
return 0

FEDERICO_FELLINI $n = 16$

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$i = 10$

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Brute Force Approach (Searching Problem)

String Matching

- Given a string of n characters (text) and a string of m characters (pattern), determine whether the text has a substring that matches the pattern
- find all the substrings of length m in the text, and check whether any of them matches the pattern

StringMatching($T = t_1t_2...t_n$, $P = p_1p_2...p_m$)

input : a text with n characters and a pattern with m characters

output : outputs the index of the leftmost character of the first matching substring, or 0 if no such substring exists

```
for i = 1 to n – m + 1
    j ← 1
    while j < m + 1 and p_j = t_{i+j-1}
        j ← j+1
    if j = m + 1
        return i
return 0
```

$$T(n) = (n - m + 1).m = O(nm)$$

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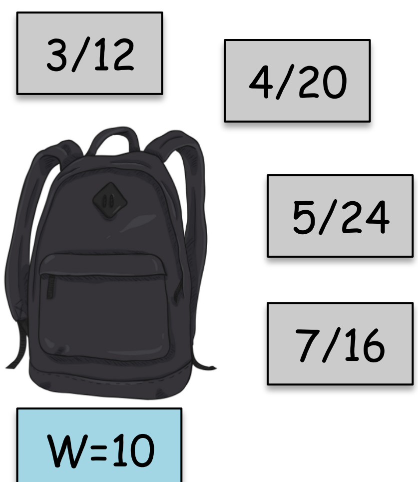

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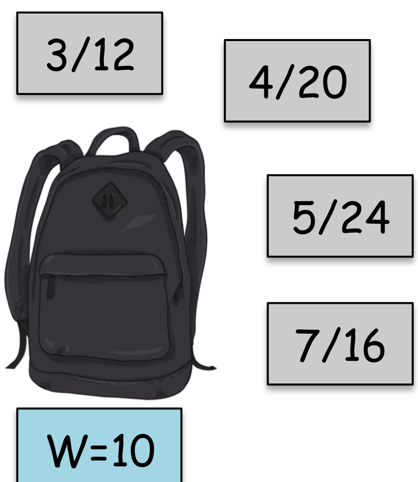

subset	total weight	total value
{}	0	0
{1}	3	12
{2}	4	20
{3}	5	24
{4}	7	16
{1,2}	7	32
{1,3}	8	36
{1,4}	10	28

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{}	0	0	{2,3}	9	44
{1}	3	12	{2,4}	11	no
{2}	4	20	{3,4}	12	no
{3}	5	24	{1,2,3}	12	no
{4}	7	16	{1,2,4}	14	no
{1,2}	7	32	{1,3,4}	15	no
{1,3}	8	36	{2,3,4}	16	no
{1,4}	10	28	{1,2,3,4}	19	no

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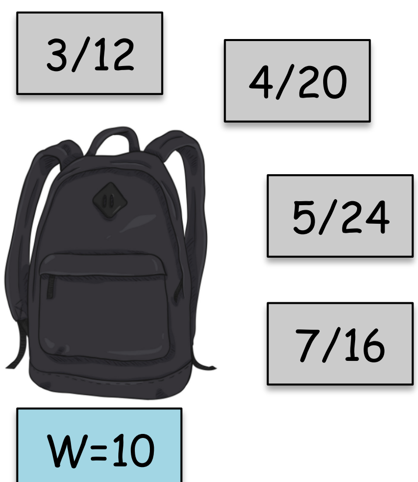

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(Graph Traversal)

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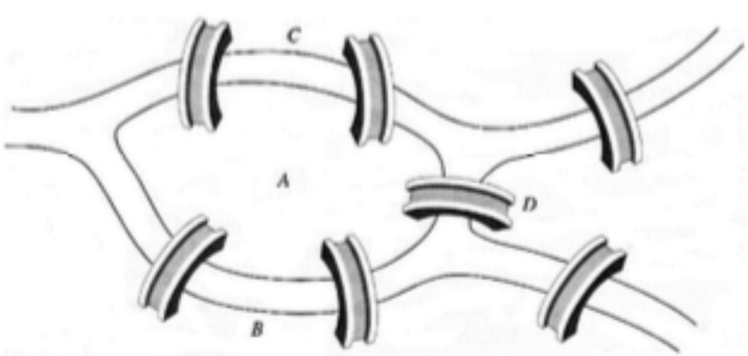
Graph Theory

- Königsberg was a city in Germany in 18th century. There was a river Pregel that divide the city into four distinct regions



Is it possible to take a walk around the city that passes each bridge exactly once ?