Greedy Algorithms

Murat Osmanoglu

Interval Scheduling

- given a set of intervals (I_1, I_2, ... , I_n)
- each interval I_i has a starting time s_i, a finishing time f_i
- your task is to find the largest subset of mutually non-overlapping intervals

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- Suppose there are n meetings requests for a meeting room.
- Each meeting i has a starting time s_i and an ending time t_i.
- We have a constraint : no two meetings can not be scheduled at same time.
- Our goal is to schedule as many meetings as possible

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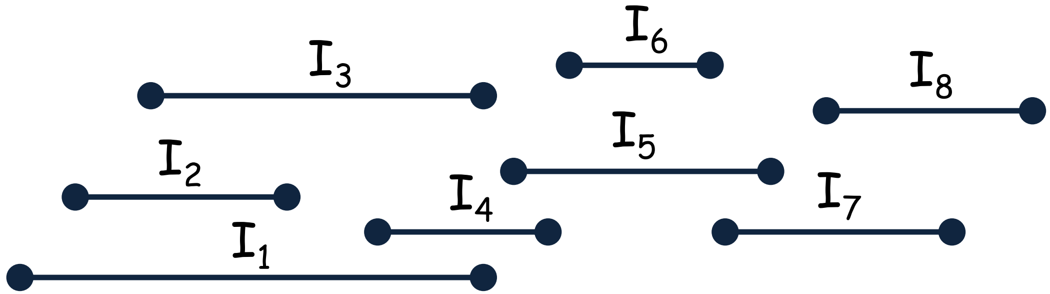

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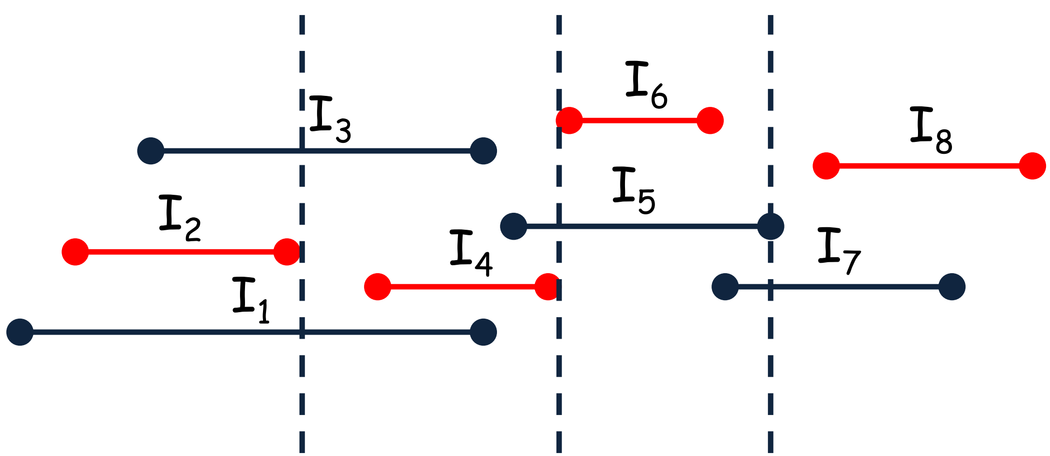
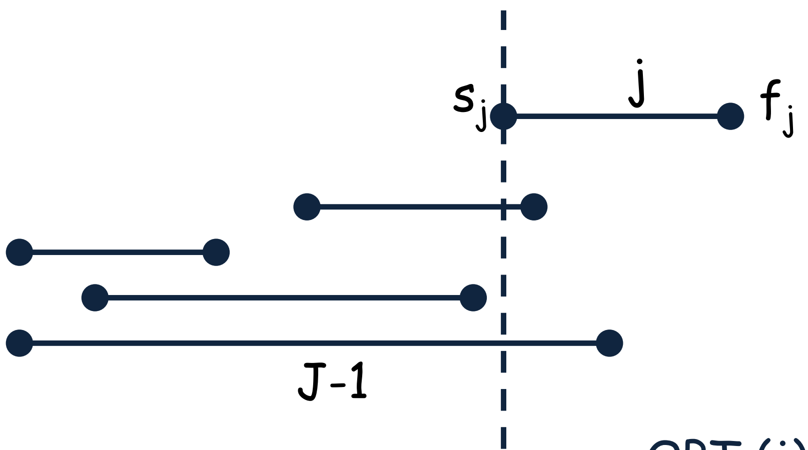


Interval Scheduling

Dynamic Programming Solution



OPT (j) = max { OPT (j-1), 1 + OPT (p(j)) }

Interval Scheduling

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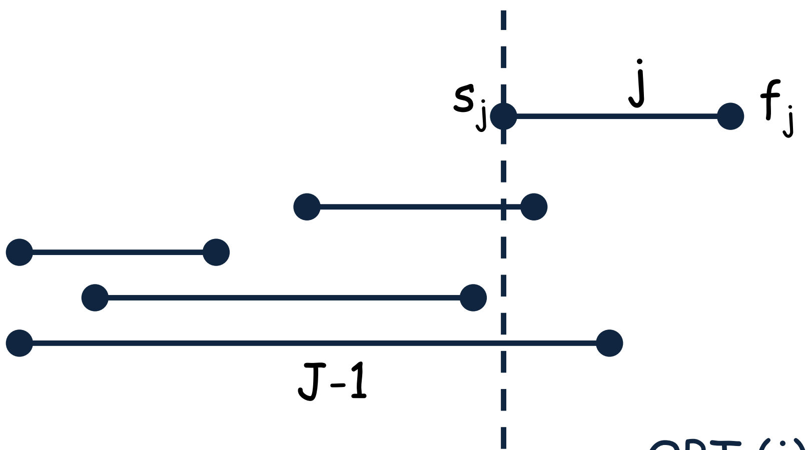

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Can we get a simpler solution?

Interval Scheduling

- solve the problem in myopic fashion

  (don't pay attention the global situaton - don't consider all possible solutions)

- make desicion at each step based on improving local state

  (use greedy approach – pick the one available to you at the moment based on some fixed and simple priority rules)

Interval Scheduling

- What is the best option?
  set the priority rules!

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- choose the first interval as the one having the earliest start time
- remove all intervals not compatible with the chosen one

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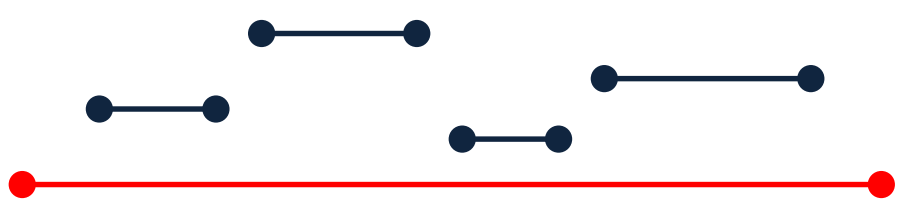

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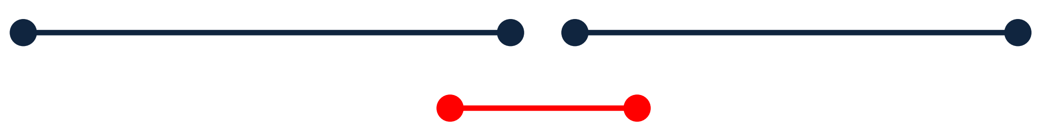

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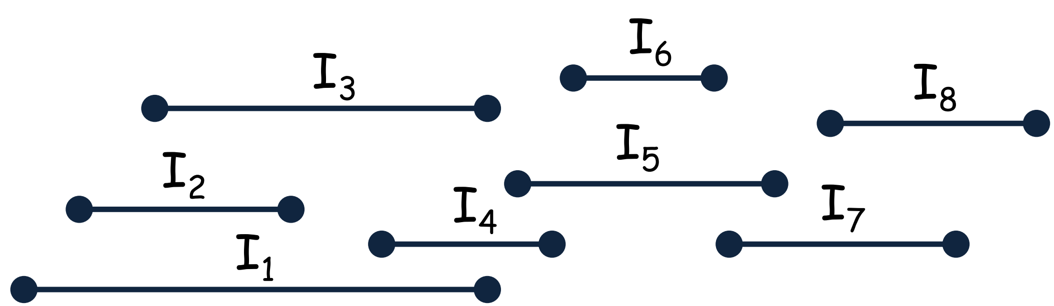

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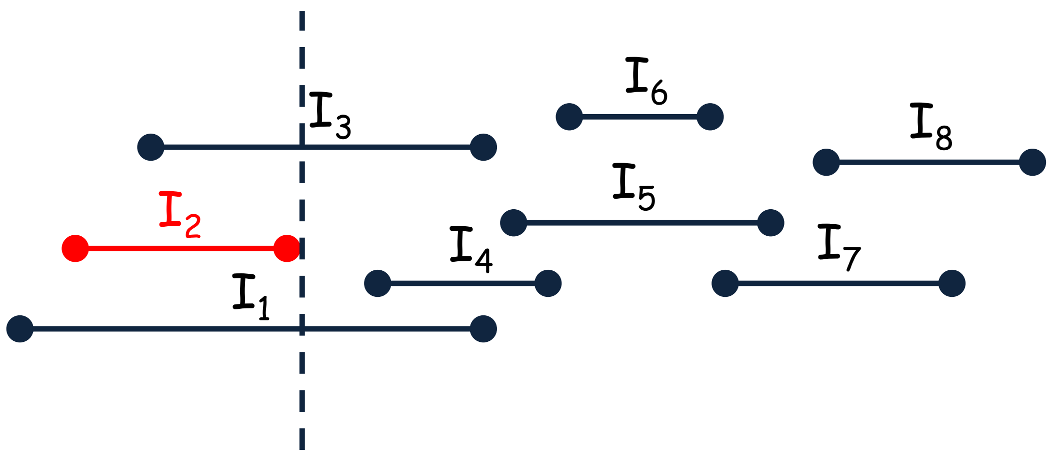

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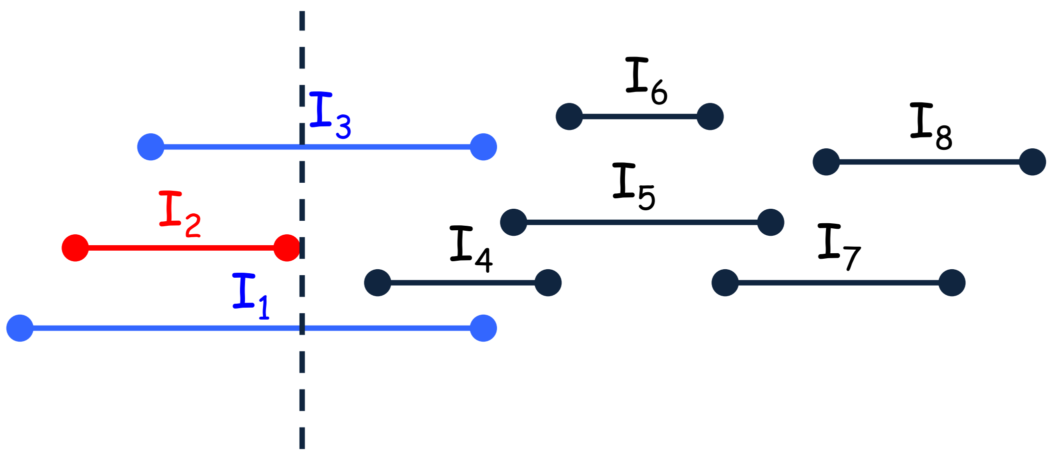

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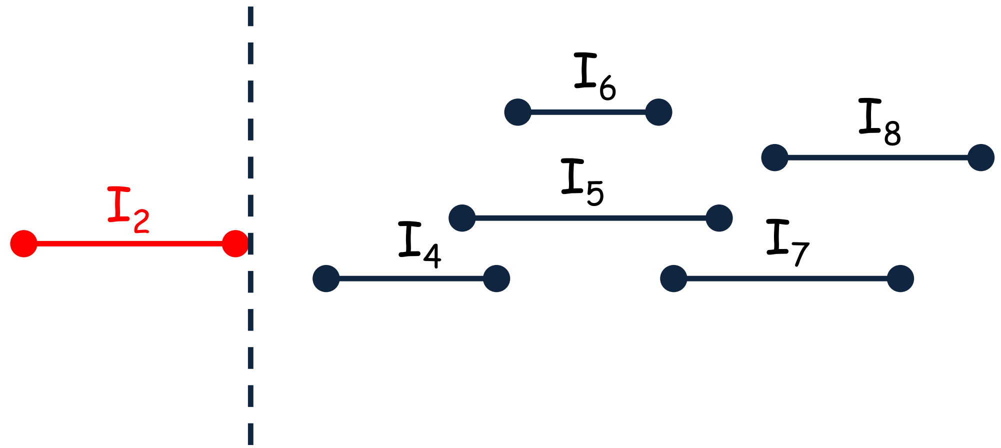

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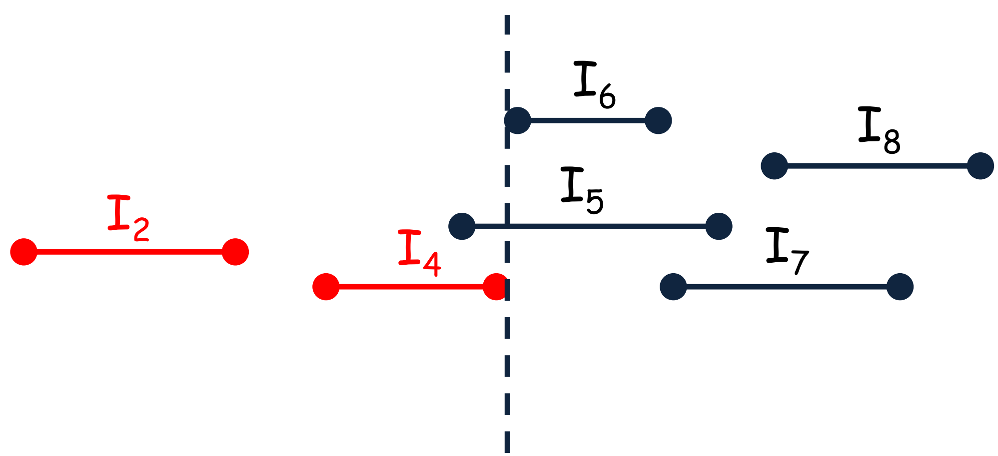

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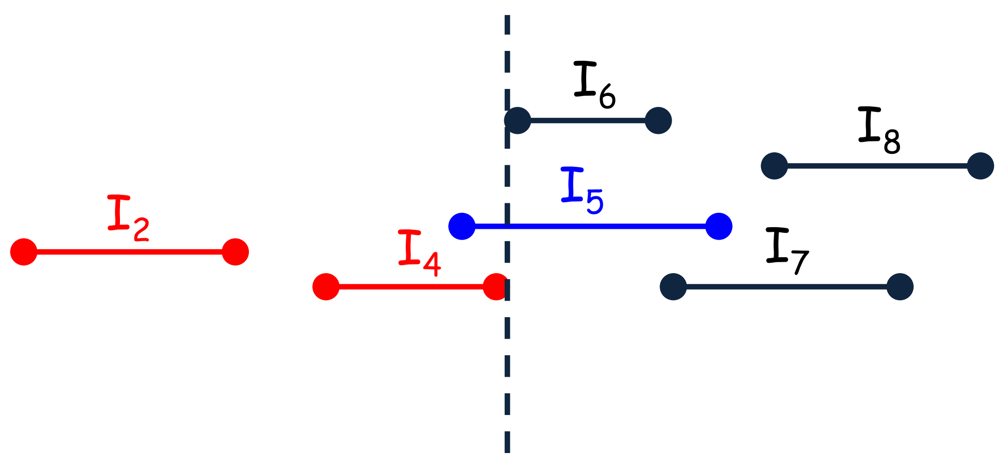

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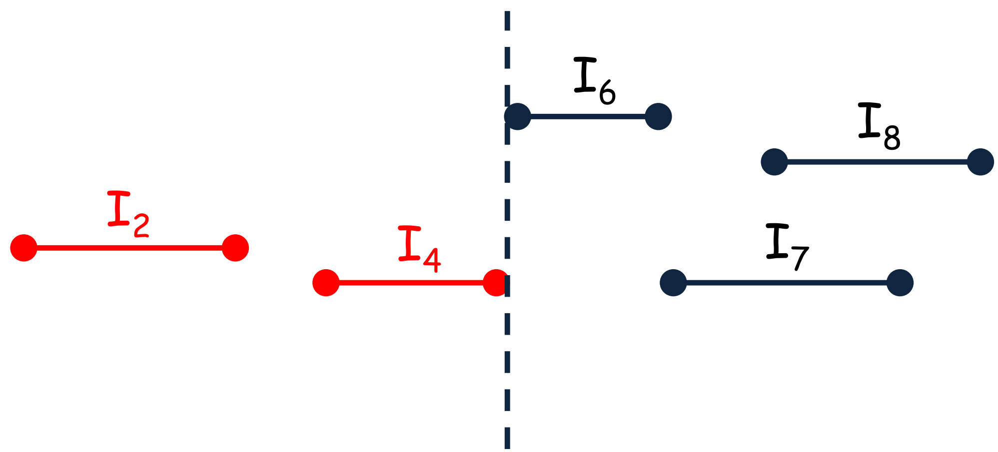

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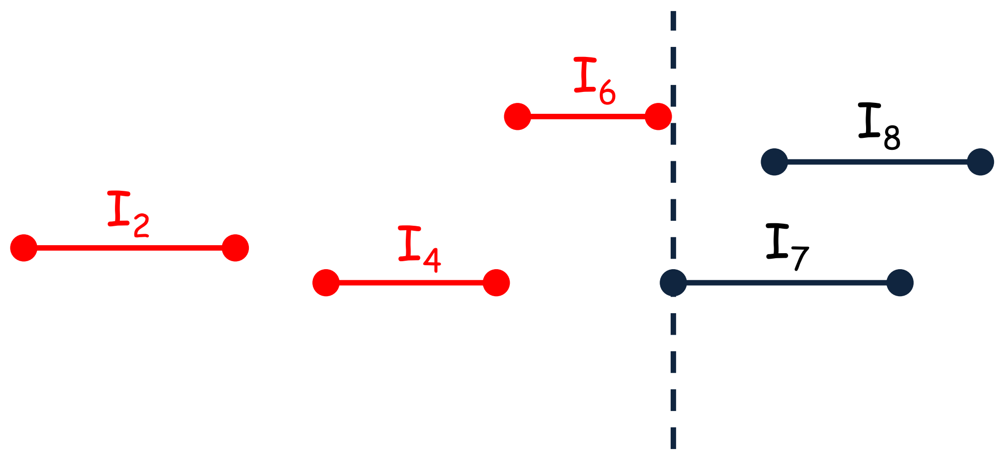

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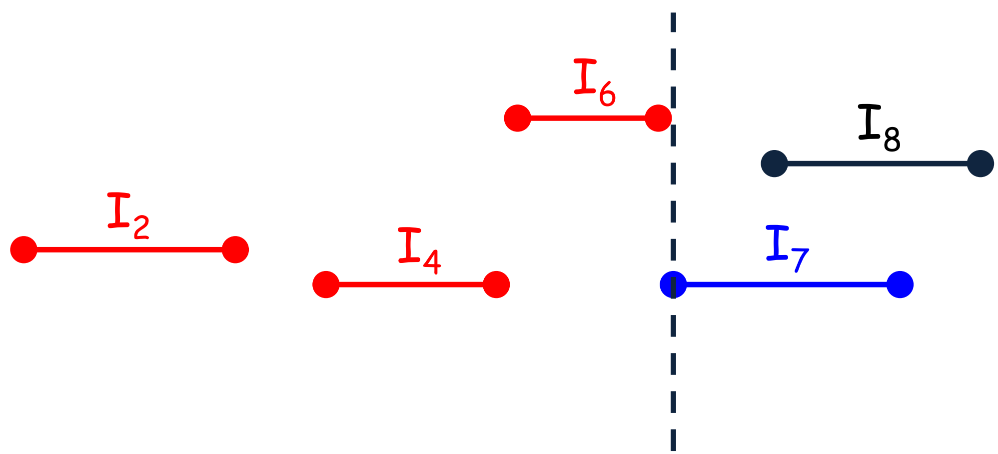

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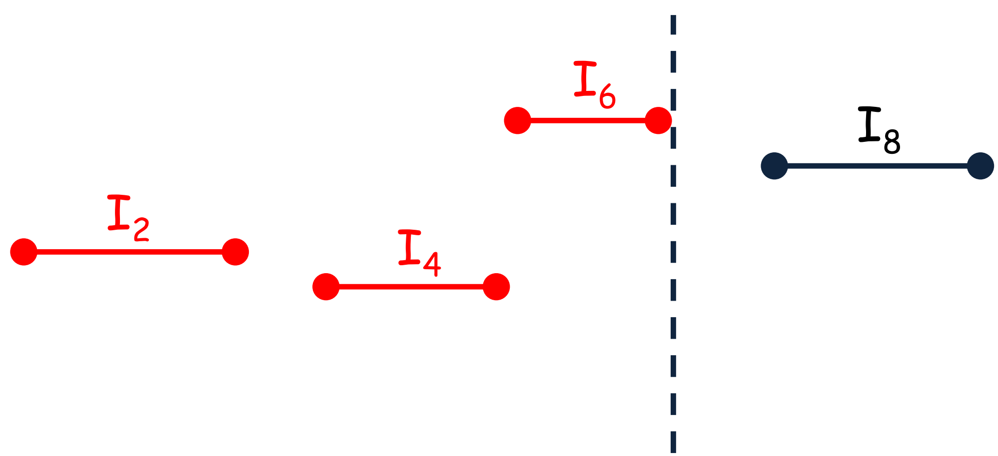

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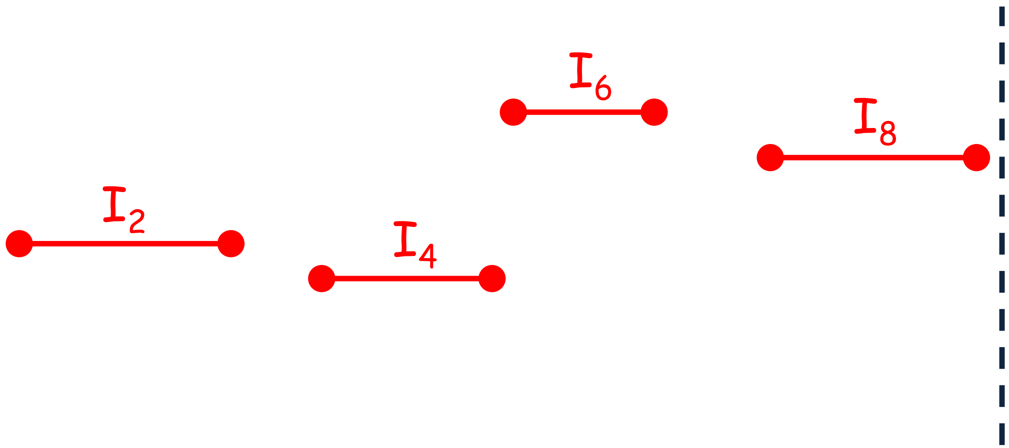

Interval Scheduling

input : n interval (I_1, … , I_n) together with their start time and finish time

--sort intervals according to their finish time ($f_1 \leq f_2 \leq \ldots \leq f_n$)
--initialize an empty set S

```
for (i=1 to n)
    if interval I_i is compatible with S
        S = S ∪ { I_i }
return S
```

Interval Scheduling

Theorem (Greedy-choice property): The interval having earliest finish time (first interval) will be part of some optimal solution set. (Our greedy approach yields us an optimal solution)

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Let I_i^* be the interval in S having earliest finish time.

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Since I_1 has the earliest finish time for all, $f_1 \leq f_i$.

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This is a contradiction!

Greedy Algorithms

- solve the problem by breaking it a sequence of subproblems
- make the best local choice among all feasible one available on that moment (one choice at a time)
  - your choice does not depend on any future choices or any past choices you have made
- prove that the Greedy Choice Property satisfies. A sequence of locally optimal choices yields a global optimal solution

Cashier's Problem

- given a certain amount of money, M cents, and a set of denominations of coins c_1 , ... , c_m
- make change for M cents using a minimum total number of coins

  (each denomination is available in unlimited quantity)

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147 cents

(25,10,5,1)

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2 + 4 + 5 + 32 = 43

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5 + 2 + 0 + 2 = 9

Cashier's Problem

input : an amount of money M
a set of denominations $(c_1 , \ldots , c_n)$

```
sort denominations
c_1 ≥ … ≥ c_n
totalw = M
j=1
k=0
while (j ≤ n)
    if ( c_j ≤ totalw )
        totalw = totalw - c_j
        k = k +1
    else
        j = j +1
return k
```

Cashier's Problem

Theorem (Greedy-choice property): Let (10, 5, 1) be the denomination set. For the amount M, there exists an optimal solution set that contains the largest denomination $c_j \leq M$.

(Our greedy approach yields us an optimal solution)

Proof

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$$M = a.5 + b.1 \quad \text{(total } a + b \text{ coins)}$$

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M can be written as

$$M = a.5 + b.1 = 10 - 10 + a.5 + b.1$$

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This is contradiction.

Cashier's Problem

Will the Greedy Technique give an optimal solution for all denomination set?

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10

6

1

M = 24

Cashier's Problem

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10

6

1

M = 24 2 +

Cashier's Problem

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10

6

1

M = 24 2 + 0 +

Cashier's Problem

Will the Greedy Technique give an optimal solution for all denomination set?

10	6	1
		

M = 24   2 + 0 + 4 = 6

Cashier's Problem

Will the Greedy Technique give an optimal solution for all denomination set?

10 6 1

M = 24 2 + 0 + 4 = 6

4.6 = 24

Cashier's Problem

Will the Greedy Technique give an optimal solution for all denomination set?

10  6  1 

M = 24 2 + 0 + 4 = 6

use dynamic programming

4.6 = 24

Fractional Knapsack

- given n items and a knapsack with the capacity M
- each item i has a weight w_i, and a value p_i
- you are allowed to get a fraction x_i of an item i that yields a profit $x_i.p_i$ where $0 \leq x_i \leq 1$
- your goal is to get a filling that maximizes the profit under the weight constraint M

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w_1=20 P_1=10	w_2=5 p_2=5	w_1=15 p_1=5	w_1=5 p_1=15
pearl	gold	silver	diamonds

M = 25

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3 . 5

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M = 20

3 . 5 + 1 . 5

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M = 15

$3 \,.\, 5 + 1 \,.\, 5 + (1/2) \,.\, 15$

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M = 25

M = 0

3 . 5 + 1 . 5 + (1/2) . 15 = 27.5

Fractional Knapsack

<u>input</u> : n items together with their prices p_i
and weight w_i, and a knapsack with the capacity M

```
sort items according to the ratio (p_i/w_i)
(p_1/w_1) ≤ … ≤ (p_n/w_n)
totalw = M
j=1
while (totalw > 0)
      if ( w_j > totalw )
            add totalw fraction of item j to the knapsack
            totalw = 0
      else
            add item j to the knapsack
            totalw = totalw - w_j
            j = j + 1
return knapsack
```

Fractional Knapsack

Theorem (Greedy-choice property): Let j be the item with the maximum ratio p_i/w_i. There exists an optimal solution that contains item j as much as possible.

(Our greedy approach yields us an optimal solution)

Proof

Fractional Knapsack

Theorem (Greedy-choice property): Let j be the item with the maximum ratio p_i/w_i. There exists an optimal solution that contains item j as much as possible.

(Our greedy approach yields us an optimal solution)

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Since $(p_k / w_k) < (p_j / w_j)$, $U^* > U$.

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This is contradiction!

0/1 Knapsack Problem

- given n items and a knapsack with the capacity M
- each item i has a weight w_i, and a value p_i
- your goal is to get a filling that maximizes the profit under the weight constraint M
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$w_1=10$ $p_1=60$	$w_2=20$ $p_2=100$	$w_3=30$ $p_3=120$	M = 50
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$M = 20 \longleftrightarrow 60 + 100 = 160$

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use dynamic programming

Process Scheduling

- given a computer and n processes $p_1, \ldots, p_n$ such that each of them has a completion time t_i
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$t_1 = 4$ $t_2 = 2$ $t_3 = 5$ $t_4 = 3$

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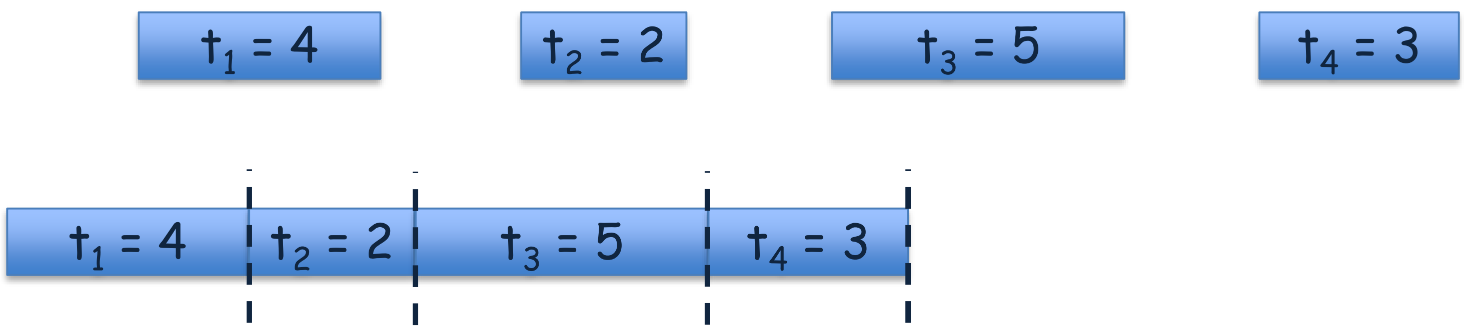

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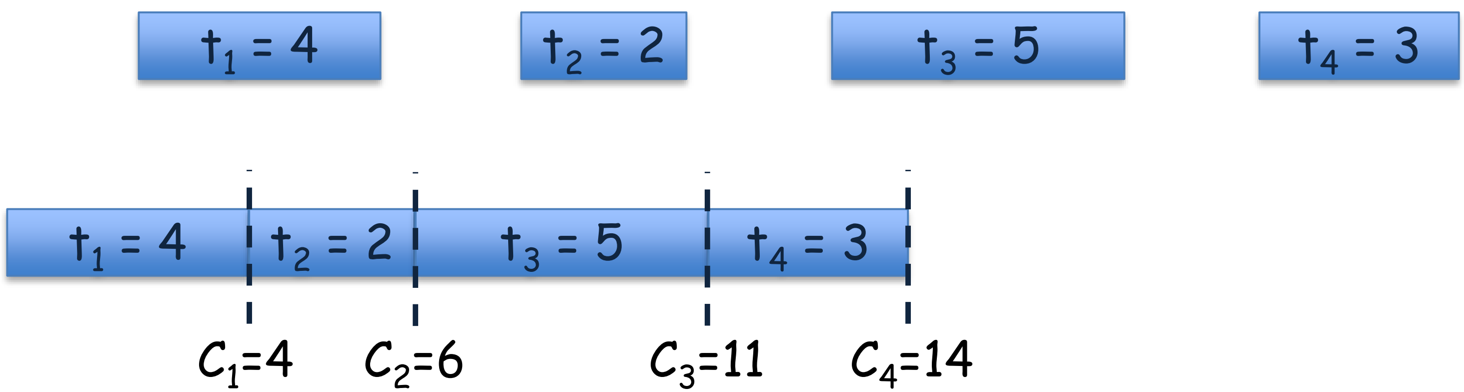

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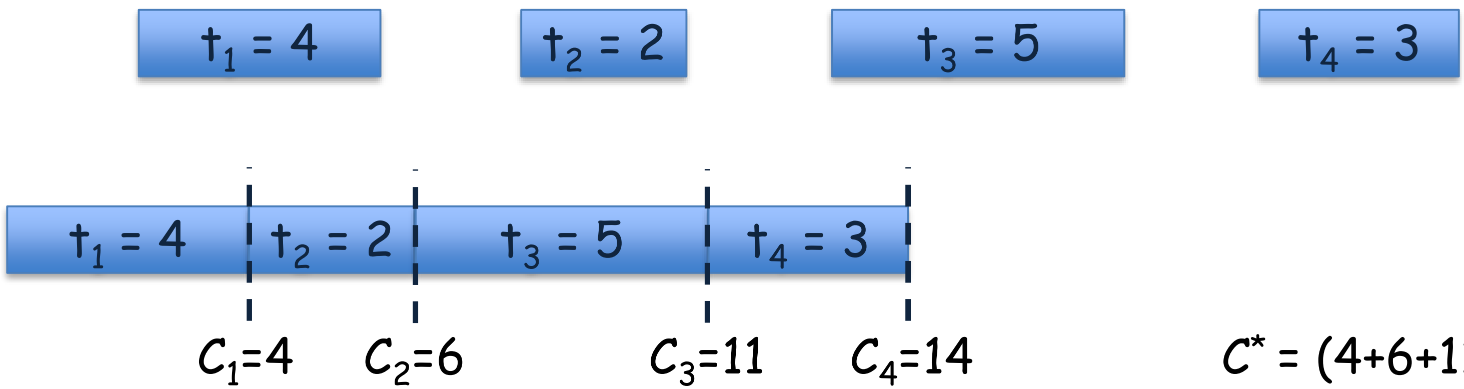

$C^* = (4+6+11+14)/4$

$C^* = 8.75$

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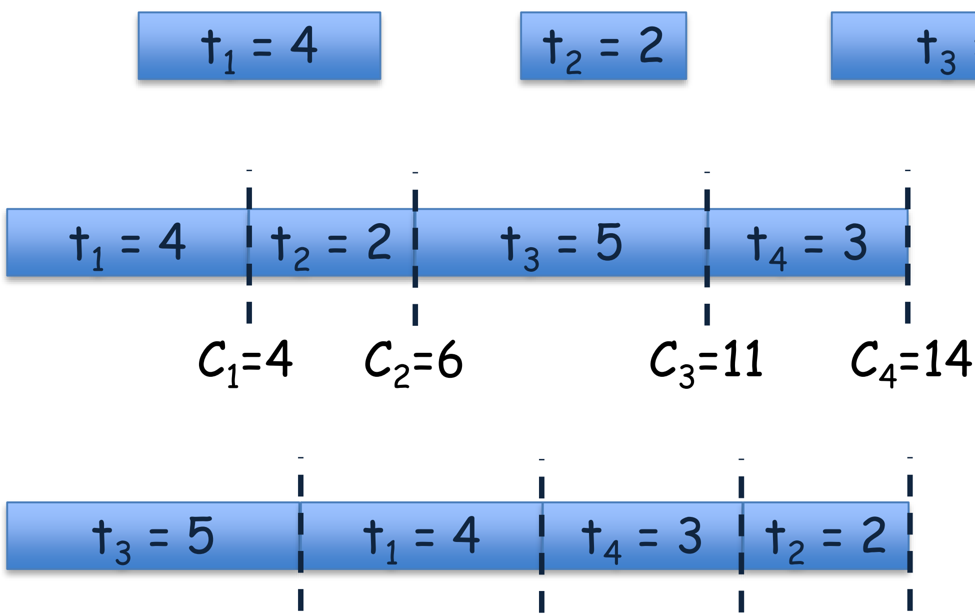

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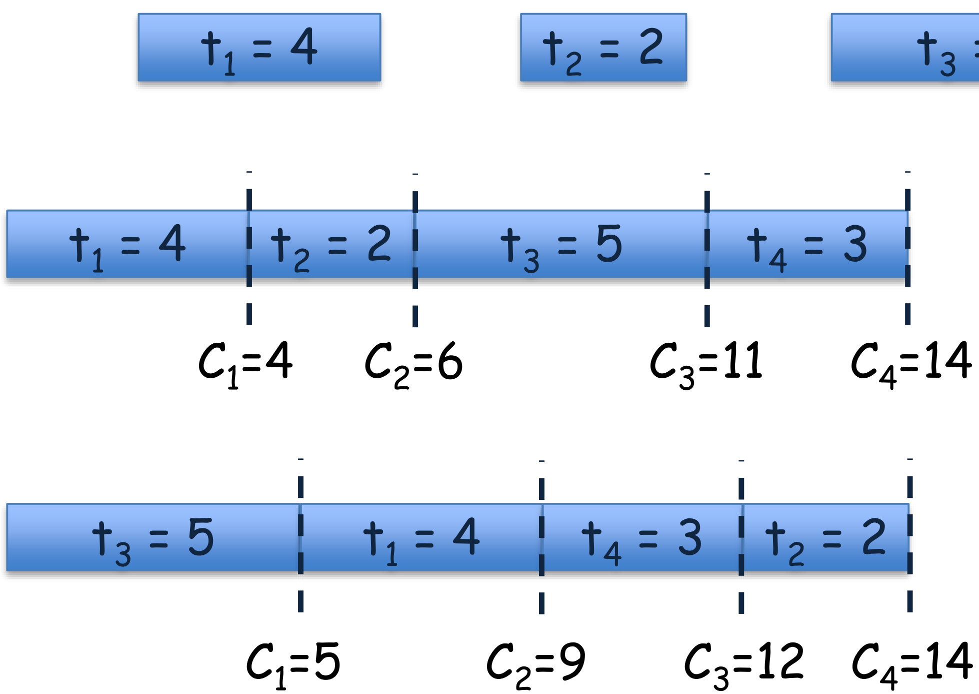

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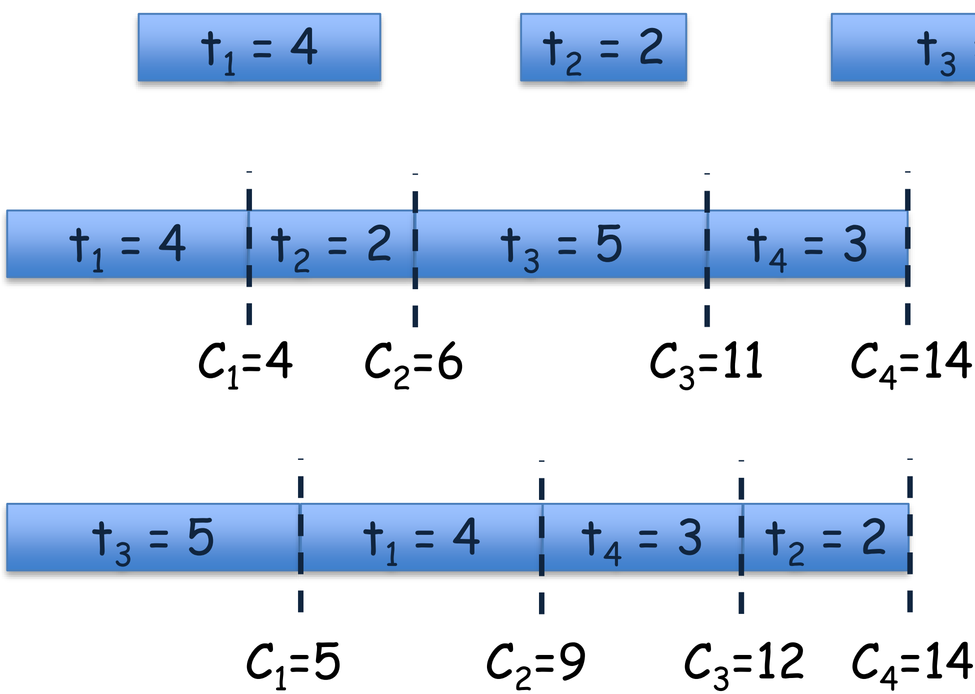

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$C^* = 10$

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this part is constant

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$C_1 = t_1$
$C_2 = t_1 + t_2$

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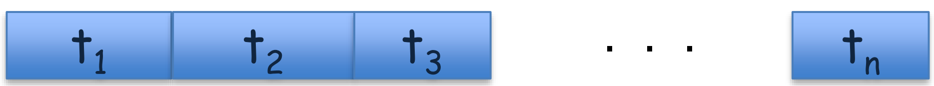

$C_1 = t_1$
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$C_3 = t_1 + t_2 + t_3$
.
.
.
$C_n = t_1 + t_2 + t_3 + . . . + t_n$

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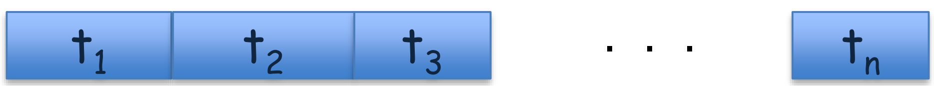

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$$\Sigma_{i=1..n} C_i = n.t_1 + (n-1).t_2 + \ldots + t_n$$

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t_1	t_2	t_3	. . .	t_n

$$
\begin{aligned}
C_1 &= t_1 \\
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small t_1 makes the sum smaller

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Greedy Approach : sort the processes according to the completion time in increasing order

small t_1 makes the sum smaller

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Theorem (Greedy-choice property): Let S be a sequence of processes ordered according to the completion time. Then S is an optimal sequence. (Our greedy approach yields us an optimal solution)

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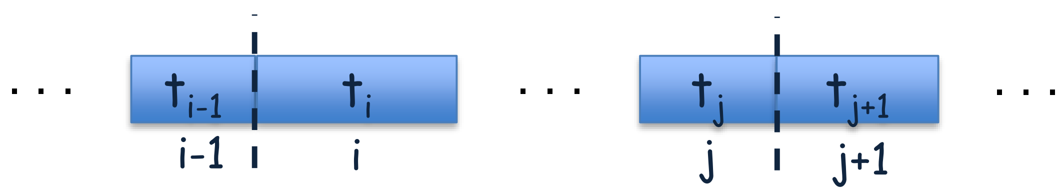

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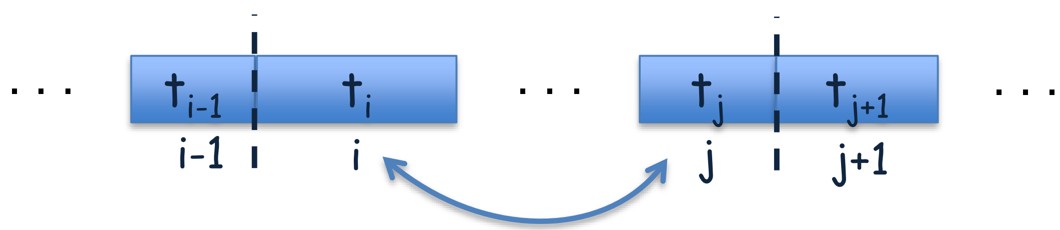

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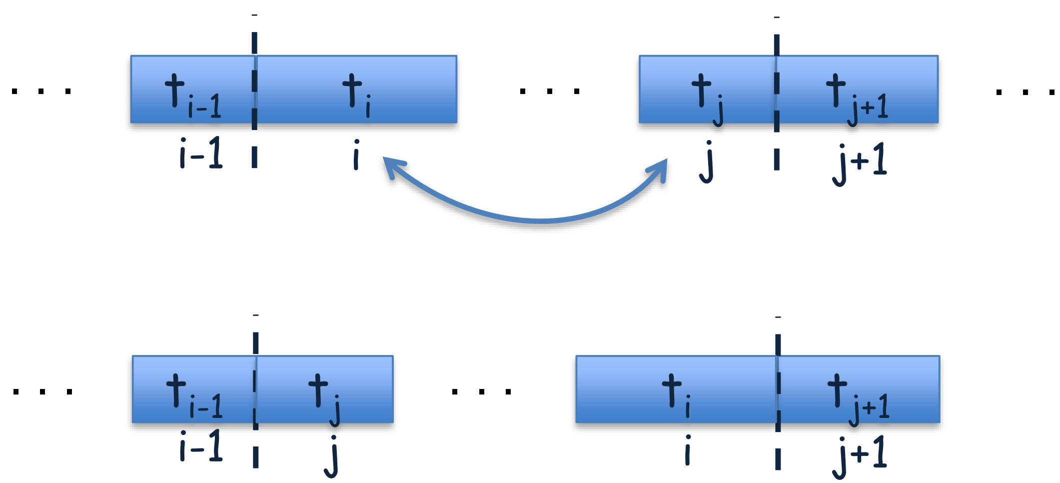

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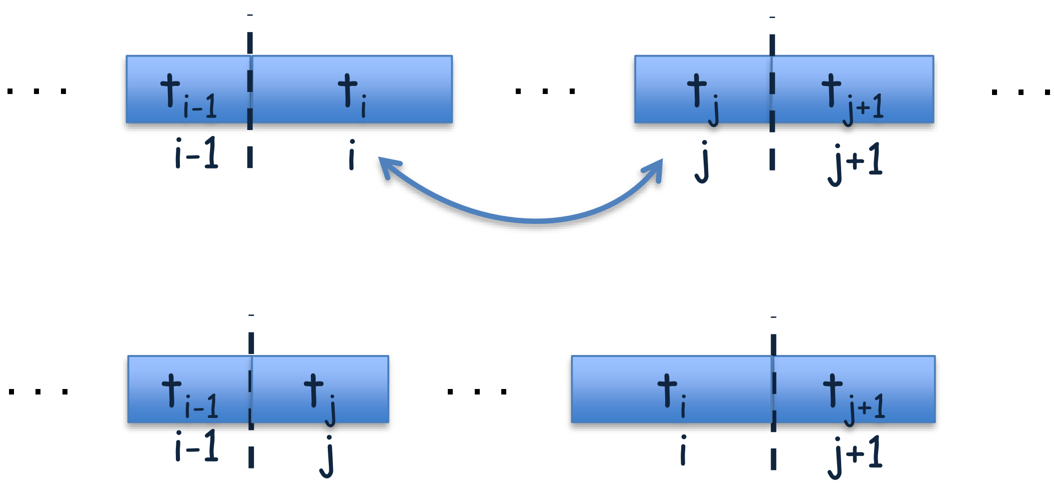

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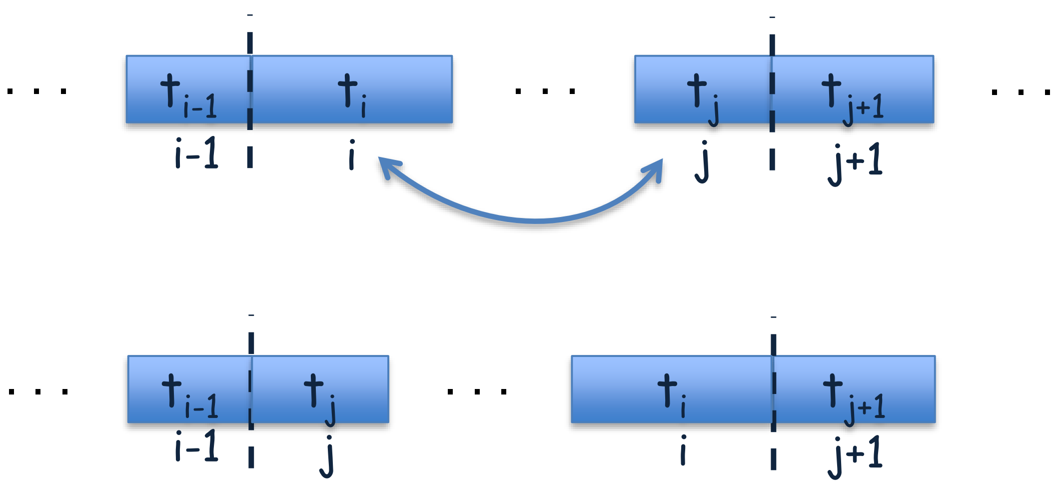

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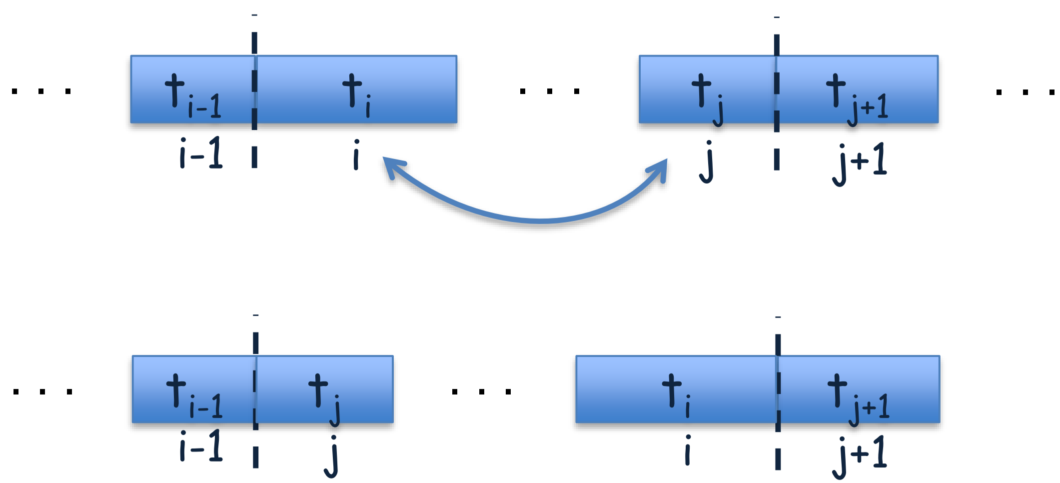

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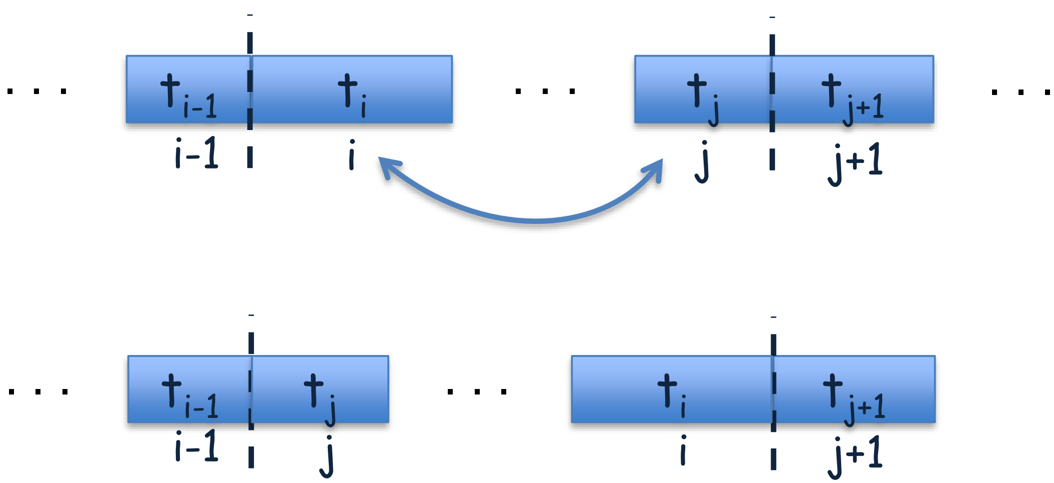

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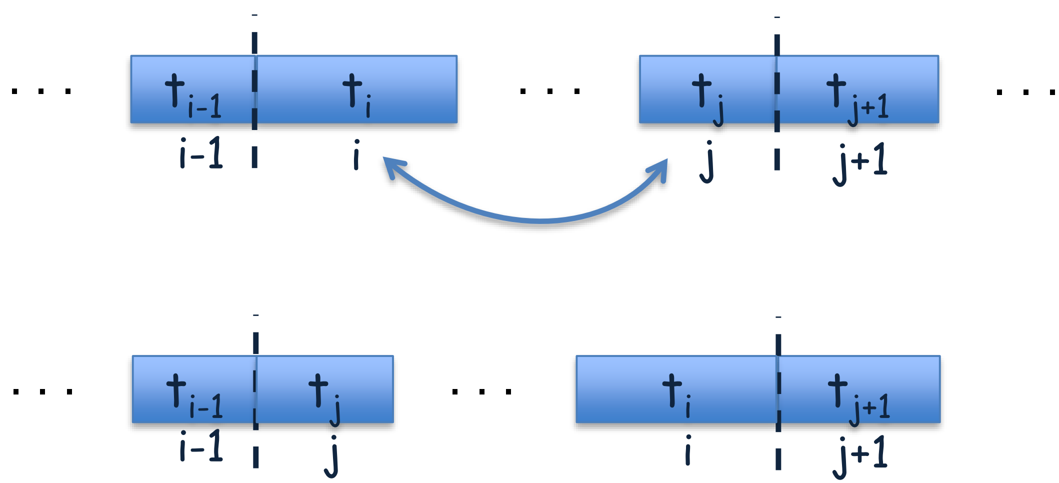

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- $\Sigma_{i=1..n}\, C_i$ decreasing

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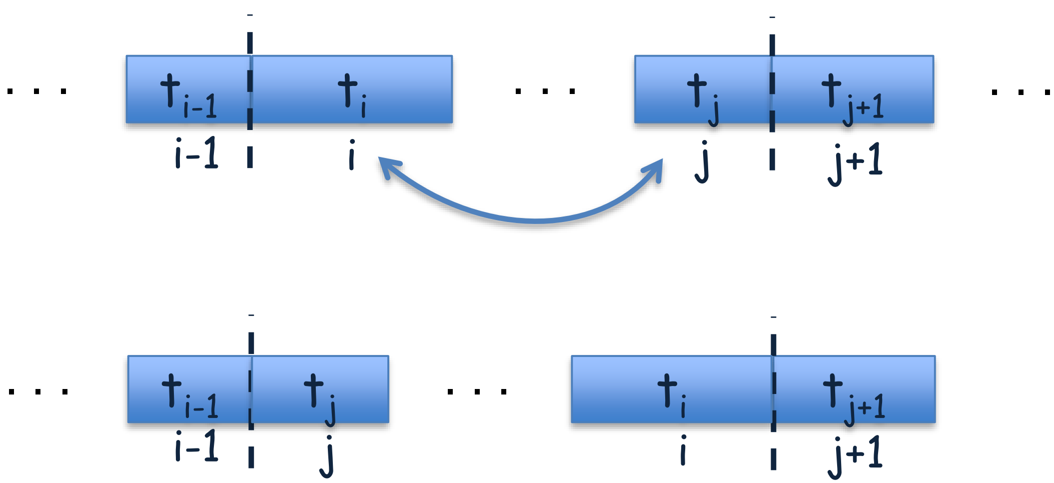

- finishing time of the processes up to i-1 not changing (C_1 , …, C_{i-1} remain same)
- finishing time of the processes after j not changing (C_j , …, C_n remain same)
- Let $\Delta = t_i - t_j$. Then

$$C_i^* = C_i - \Delta$$
$$C_{i+1}^* = C_{i+1} - \Delta$$
$$\vdots$$
$$C_{j-1}^* = C_{j-1} - \Delta$$

this is a contradiction ← • $\Sigma_{i=1..n} C_i$ decreasing

Minimizing Lateness

- given a computer and n processes $p_1, \ldots, p_n$ such that each of them has a processing time t_i and a deadline d_i
- find an optimal order of processes that minimizes the maximum lateness

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	1	**2**	**3**	**4**
t_i	4	2	5	3
d_i	6	4	10	8

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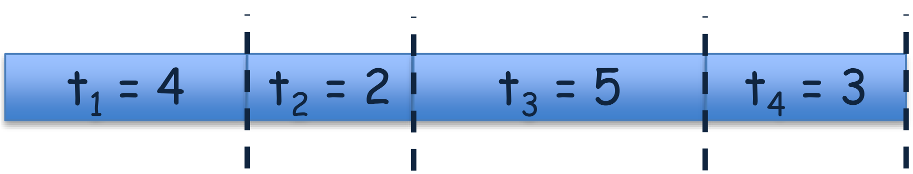

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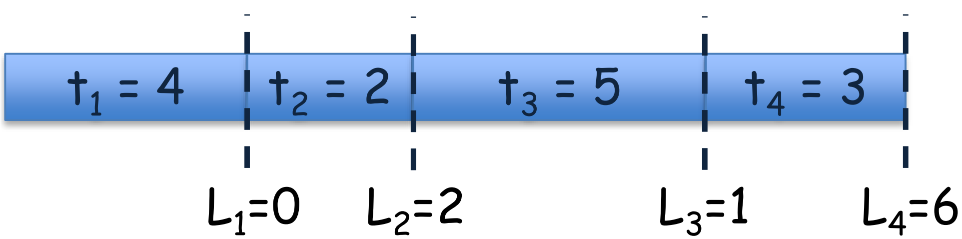

	1	**2**	**3**	**4**
t_i	4	2	5	3
d_i	6	4	10	8

$L = 6$

Minimizing Lateness

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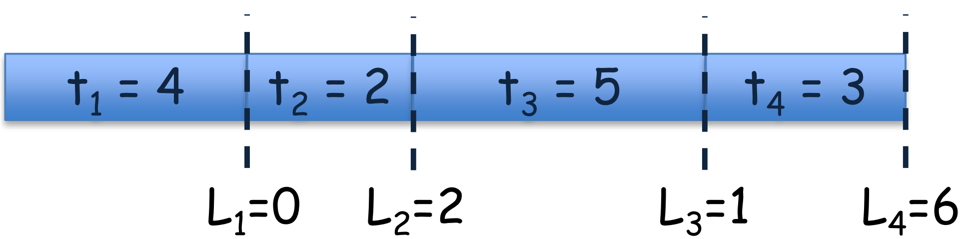

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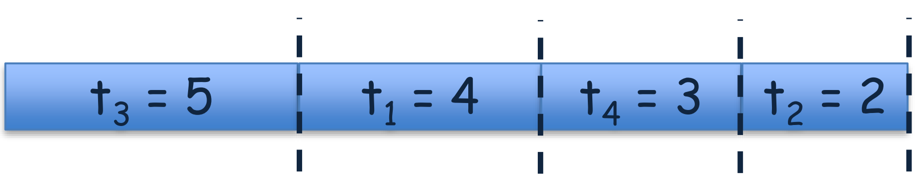

Minimizing Lateness

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	1	2	3	4
t_i	4	2	5	3
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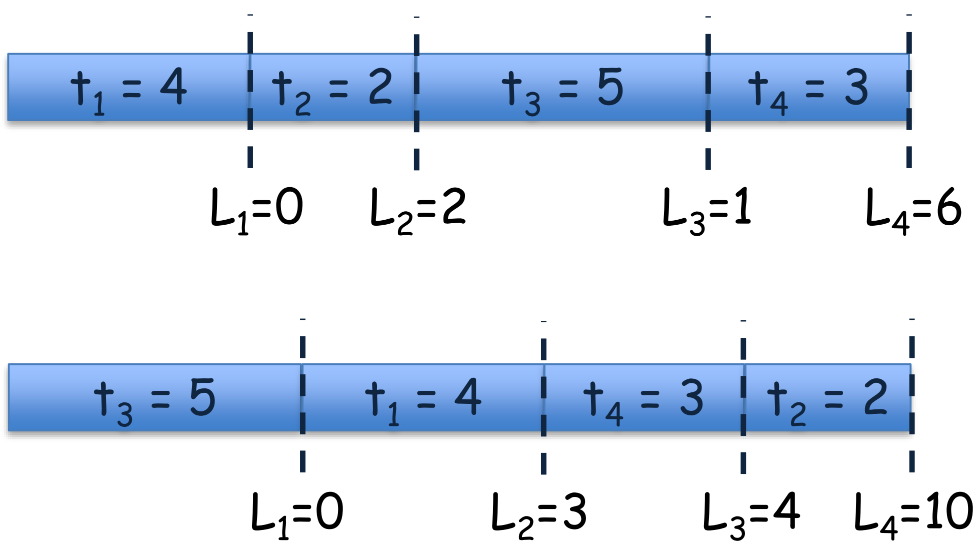

$L = 6$

$L = 10$

Minimizing Lateness

How do we sort the processes ?

- according to their process time t_i

Minimizing Lateness

How do we sort the processes ?

- according to their process time t_i

	1	**2**
t_i	1	5
d_i	12	5

Minimizing Lateness

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	1	**2**
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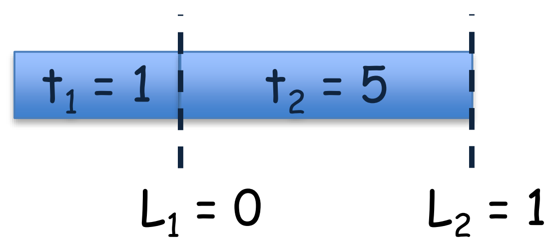


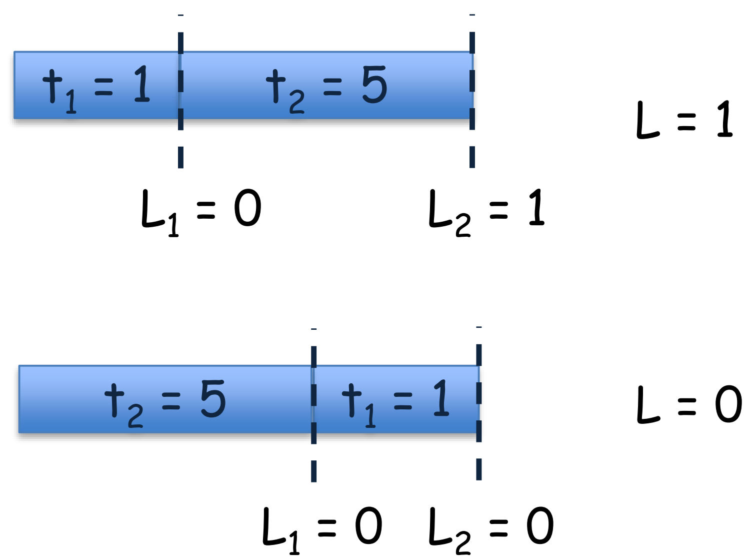
$L = 1$

Minimizing Lateness

How do we sort the processes ?

- according to their process time t_i

	1	**2**
$\mathbf{t_i}$	1	5
$\mathbf{d_i}$	12	5



Minimizing Lateness

How do we sort the processes ?

- according to their slack time $d_i - t_i$

Minimizing Lateness

How do we sort the processes ?

- according to their slack time $d_i - t_i$

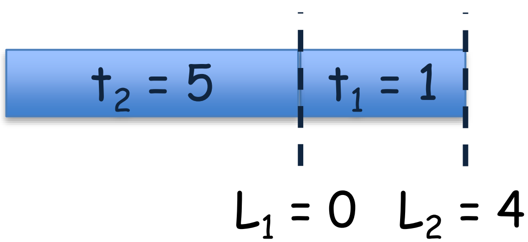
	1	2
t_i	1	5
d_i	2	5

Minimizing Lateness

How do we sort the processes ?

- according to their slack time d_i - t_i

	1	**2**
t_i	1	5
d_i	2	5



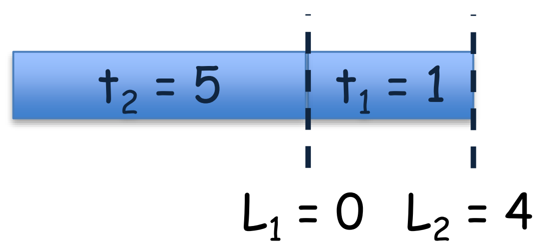
L = 4

Minimizing Lateness

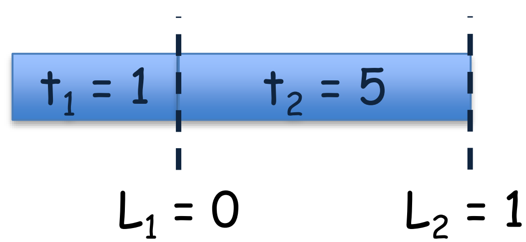
How do we sort the processes ?

- according to their slack time $d_i - t_i$

	1	2
t_i	1	5
d_i	2	5



$L = 4$



$L = 1$

Minimizing Lateness

How do we sort the processes ?

- according to their deadlines d_i

Minimizing Lateness

How do we sort the processes ?

- according to their deadlines d_i

	1	2	3	4
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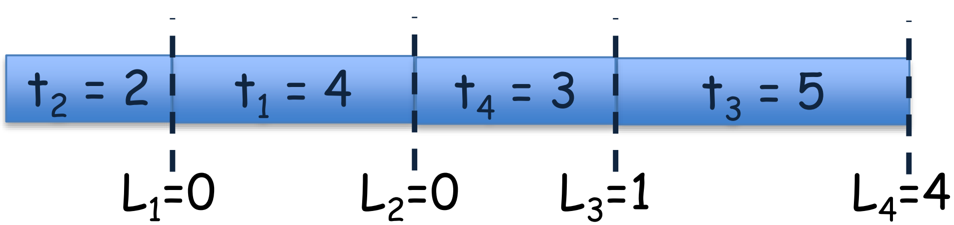

$L = 4$

Minimizing Lateness

Theorem (Greedy-choice property): Let S be a sequence of processes ordered according to the deadline. Then S is an optimal sequence. (Our greedy approach yields us an optimal solution)

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Assume there is an optimal sequence S^* in which the processes have not been sorted. Then there should be indices i and j such that $i < j$ and $d_j < d_i$

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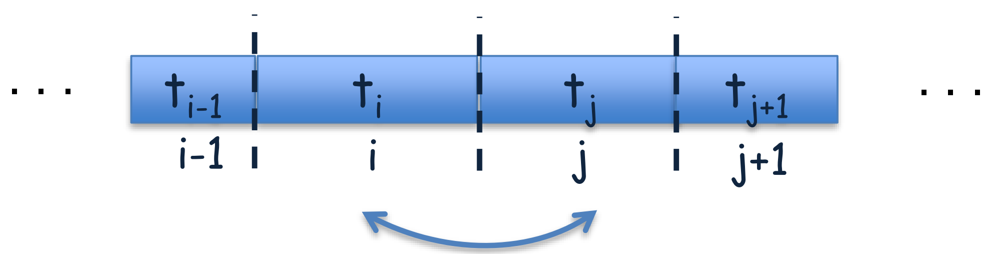

What happens if we swap i and j?

Huffman Coding

Encoding

ABRACADABRA

A B R A C A D A B R A

Huffman Coding

Encoding

ABRACADABRA

A	B	R	A	C	A	D	A	B	R	A
000	001	010	000	011	000	100	000	001	010	000

- Encode them using 3-bit strings

Huffman Coding

Encoding

ABRACADABRA

A	B	R	A	C	A	D	A	B	R	A
000	001	010	000	011	000	100	000	001	010	000

freq

A
B
R
C
D

number of times
the letter appears

- Encode them using 3-bit strings

Huffman Coding

Encoding

ABRACADABRA

A	B	R	A	C	A	D	A	B	R	A
000	001	010	000	011	000	100	000	001	010	000

freq

A	5
B	2
R	2
C	1
D	1

- Encode them using 3-bit strings

Huffman Coding

Encoding

ABRACADABRA

A	B	R	A	C	A	D	A	B	R	A
000	001	010	000	011	000	100	000	001	010	000

	freq	
A	5	000
B	2	001
R	2	010
C	1	011
D	1	100

- Encode them using 3-bit strings

Huffman Coding

Encoding

ABRACADABRA

A	B	R	A	C	A	D	A	B	R	A
000	001	010	000	011	000	100	000	001	010	000

	freq		cost
A	5	000	15
B	2	001	6
R	2	010	6
C	1	011	3
D	1	100	3

- Encode them using 3-bit strings

Huffman Coding

Encoding

ABRACADABRA

A	B	R	A	C	A	D	A	B	R	A
000	001	010	000	011	000	100	000	001	010	000

	freq		cost
A	5	000	15
B	2	001	6
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C	1	011	3
D	1	100	3
			33

- Encode them using 3-bit strings
- Total 33 bits required to encode

Huffman Coding

Encoding

ABRACADABRA

A	B	R	A	C	A	D	A	B	R	A
000	001	010	000	011	000	100	000	001	010	000

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Can we get better encoding?

Huffman Coding

Encoding

ABRACADABRA

A	B	R	A	C	A	D	A	B	R	A
000	001	010	000	011	000	100	000	001	010	000

	freq		cost
A	5	000	15
B	2	001	6
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- Total 33 bits required to encode

Can we get better encoding?

1 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0

Huffman Coding

Encoding

ABRACADABRA

A	B	R	A	C	A	D	A	B	R	A
000	001	010	000	011	000	100	000	001	010	000

	freq		cost
A	5	000	15
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Huffman Coding

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ABRACADABRA

A	B	R	A	C	A	D	A	B	R	A
000	001	010	000	011	000	100	000	001	010	000

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A	5	000	15
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- Total 33 bits required to encode

Can we get better encoding?

1 0 0	0 0 0	0 0 1	0 0 0	0 1 0	0 0 0
D	A	B	A	R	A

Huffman Coding

Encoding

ABRACADABRA

A	B	R	A	C	A	D	A	B	R	A
000	001	010	000	011	000	100	000	001	010	000

	freq		cost
A	5	000	15
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R	2	010	6
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			33

- Encode them using 3-bit strings
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1 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0

D A B A R A

a unique decoding

Huffman Coding

	freq
A	5
B	2
R	2
C	1
D	1

Huffman Coding

	freq
A	5
B	2
R	2
C	1
D	1

- $B(T, \{ f_c \}) = \Sigma f_c . l_c$

Huffman Coding

	freq
A	5
B	2
R	2
C	1
D	1

- $B(T, \{ f_c \}) = \Sigma f_c . l_c$
- try to minimize the function B

Huffman Coding

	freq
A	5
B	2
R	2
C	1
D	1

- $B(T, \{ f_c \}) = \Sigma f_c . l_c$
- try to minimize the function B
- use smaller length encoding for the character having larger frequency

Huffman Coding

	freq	
A	5	0
B	2	1
R	2	01
C	1	10
D	1	11

- $B(T, \{ f_c \}) = \Sigma f_c.l_c$
- try to minimize the function B
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Huffman Coding

	freq		cost
A	5	0	5
B	2	1	2
R	2	01	4
C	1	10	2
D	1	11	2
			15

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Is there any problem for this encoding?

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0 | 0 | 1 1 | 0 1 | 1 0

A A D R C

Huffman Coding

	freq		cost
A	5	0	5
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0 | 0 1 | 1 | 0 1 | 1 0

A R B R C

AADRC

Huffman Coding

	freq		cost
A	5	0	5
B	2	1	2
R	2	01	4
C	1	10	2
D	1	11	2
			15

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Is there any problem for this encoding?

0 0 1 1 0 1 1 0

A A B B A D A

AADRC

ARBRC

Huffman Coding

	freq		cost
A	5	0	5
B	2	1	2
R	2	01	4
C	1	10	2
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0 0 1 1 0 1 1 0

A A BBA D A

AADRC
ARBRC
AABBADA

Huffman Coding

	freq		cost
A	5	0	5
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0 0 1 1 0 1 1 0

A A BBA D A

AADRC
ARBRC
AABBADA
.
.
.

Huffman Coding

	freq		cost
A	5	0	5
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			15

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Is there any problem for this encoding?

0 0 1 1 0 1 1 0

A A BBA D A

decoding is not unique

AADRC

ARBRC

AABBADA

.

.

.

Huffman Coding

	freq		cost
A	5	0	5
B	2	1	2
R	2	01	4
C	1	10	2
D	1	11	2
			15

- $B(T, \{ f_c \}) = \Sigma f_c . l_c$
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Is there any problem for this encoding?

0 | 0 | 1 | 1 | 0 | 1 1 | 0

A A BBA D A

AADRC
ARBRC
AABBADA
.
.
.

decoding is not unique

to get unique decoding, coding should be 'prefix-free'

Huffman Coding

	freq		cost
A	5	0	5
B	2	1	2
R	2	01	4
C	1	10	2
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			15

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	freq		cost
A	5	0	5
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			15

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to get unique decoding, coding should be 'prefix-free'

Coding is called 'prefix free' if for any i, j; encoding c_i is not prefix of encoding c_j

Huffman Coding

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A	5	0	5
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			15

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encoding of B -1- is prefix of encoding of C -10-

1 0

Huffman Coding

	freq		cost
A	5	0	5
B	2	1	2
R	2	01	4
C	1	10	2
D	1	11	2
			15

- $B(T, \{ f_c \}) = \Sigma f_c . l_c$
- try to minimize the function B
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encoding of B -1- is prefix of encoding of C -10-

1 0

1 0 = C

Huffman Coding

	freq		cost
A	5	0	5
B	2	1	2
R	2	01	4
C	1	10	2
D	1	11	2
			15

- $B(T, \{ f_c \}) = \Sigma f_c . l_c$
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to get unique decoding, coding should be 'prefix-free'

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encoding of B -1- is prefix of encoding of C -10-

1 0

1 0 = C

1 | 0 = BA

Huffman Coding

	freq
A	25
E	18
M	13
B	10
K	7
T	5
U	2
L	1

- If you have a prefix-free code, you can uniquely decode it

Huffman Coding

	freq
A	25
E	18
M	13
B	10
K	7
T	5
U	2
L	1

- If you have a prefix-free code, you can uniquely decode it
- encoding for each char ends with '0'
- use different length encoding for each char

Huffman Coding

	freq	
A	25	0
E	18	10
M	13	110
B	10	1110
K	7	11110
T	5	111110
U	2	1111110
L	1	11111110

- If you have a prefix-free code, you can uniquely decode it
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Huffman Coding

	freq		cost
A	25	0	25
E	18	10	36
M	13	110	39
B	10	1110	40
K	7	11110	35
T	5	111110	30
U	2	1111110	14
L	1	11111110	8
			227

- If you have a prefix-free code, you can uniquely decode it
- encoding for each char ends with '0'
- use different length encoding for each char

Huffman Coding

	freq		cost
A	25	0	25
E	18	10	36
M	13	110	39
B	10	1110	40
K	7	11110	35
T	5	111110	30
U	2	1111110	14
L	1	11111110	8
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1 1 0 0 1 1 1 1 0 0 1 1 1 1 1 1 1 0 1 0

Huffman Coding

	freq		cost
A	25	0	25
E	18	10	36
M	13	110	39
B	10	1110	40
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1 1 0 | 0 | 1 1 1 1 0 | 0 | 1 1 1 1 1 1 1 0 | 1 0

M A K A L E

Huffman Coding

	freq		cost
A	25	0	25
E	18	10	36
M	13	110	39
B	10	1110	40
K	7	11110	35
T	5	111110	30
U	2	1111110	14
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1 1 0 0 1 1 1 1 0 0 1 1 1 1 1 1 1 0 1 0

M A K A L E

transform this encoding to a binary tree

Huffman Coding

	freq		cost
A	25	0	25
E	18	10	36
M	13	110	39
B	10	1110	40
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T	5	111110	30
U	2	1111110	14
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- for '1', create a left child
  for '0', create a right child

transform this encoding to a binary tree

Huffman Coding

	freq		cost
A	25	0	25
E	18	10	36
M	13	110	39
B	10	1110	40
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			227

- for '1', create a left child
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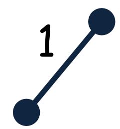

transform this encoding to a binary tree

Huffman Coding

	freq		cost
A	25	0	25
E	18	10	36
M	13	110	39
B	10	1110	40
K	7	11110	35
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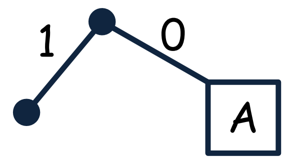

transform this encoding to a binary tree

Huffman Coding

	freq		cost
A	25	0	25
E	18	10	36
M	13	110	39
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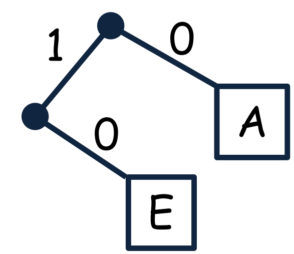

transform this encoding to a binary tree

Huffman Coding

	freq		cost
A	25	0	25
E	18	10	36
M	13	110	39
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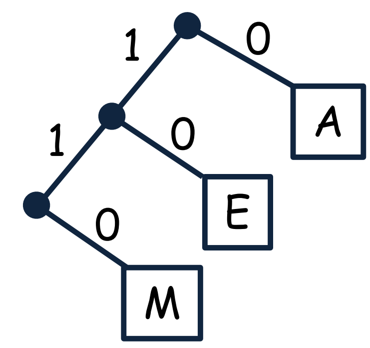

transform this encoding to a binary tree

Huffman Coding

	freq		cost
A	25	0	25
E	18	10	36
M	13	110	39
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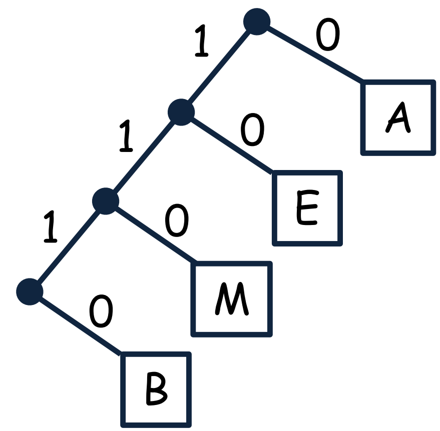

transform this encoding to a binary tree

Huffman Coding

	freq		cost
A	25	0	25
E	18	10	36
M	13	110	39
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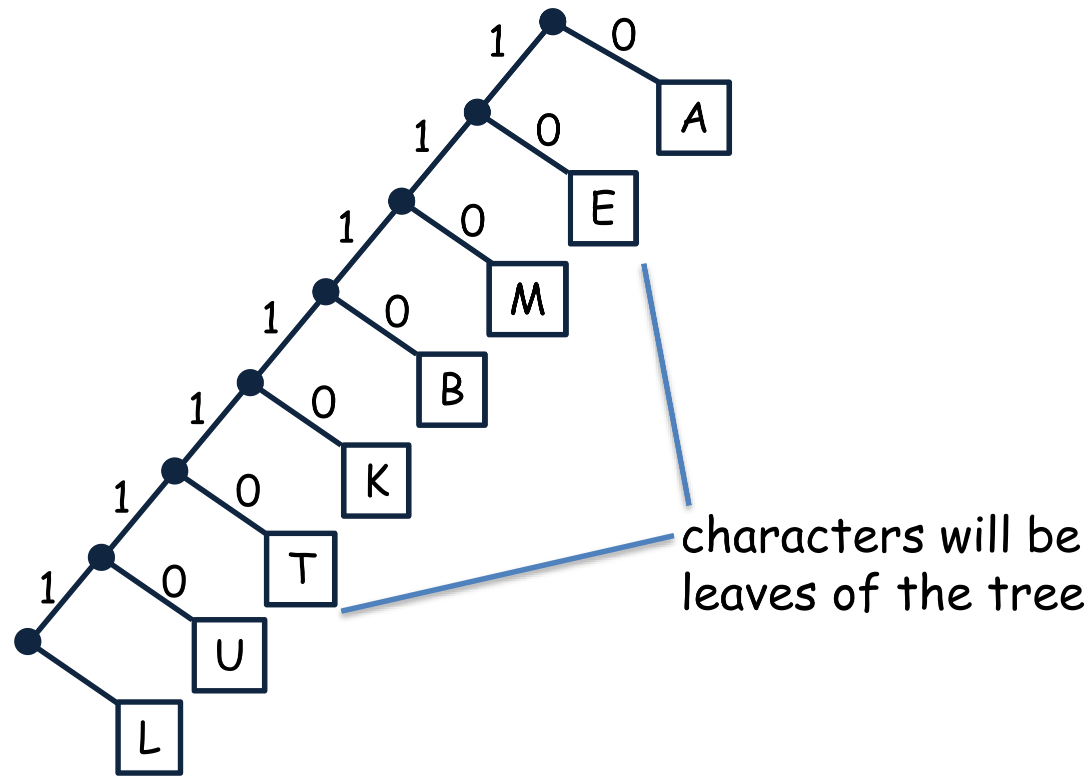

transform this encoding to a binary tree

Huffman Coding

	freq		cost
A	25	0	25
E	18	10	36
M	13	110	39
B	10	1110	40
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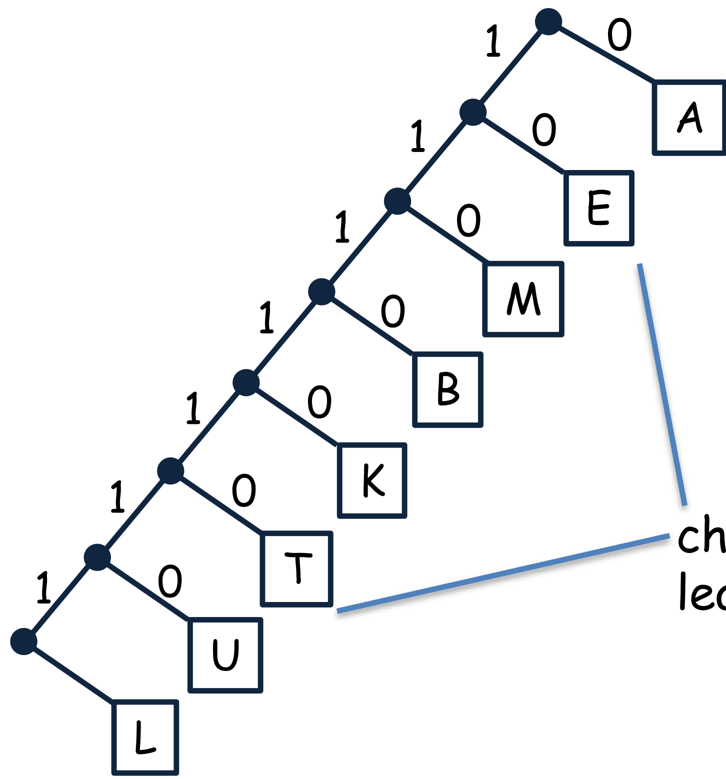

characters will be leaves of the tree

to get an optimal encoding, create an optimal tree

transform this encoding to a binary tree

Huffman Coding

	freq		cost
A	25	0	25
E	18	10	36
M	13	110	39
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			227

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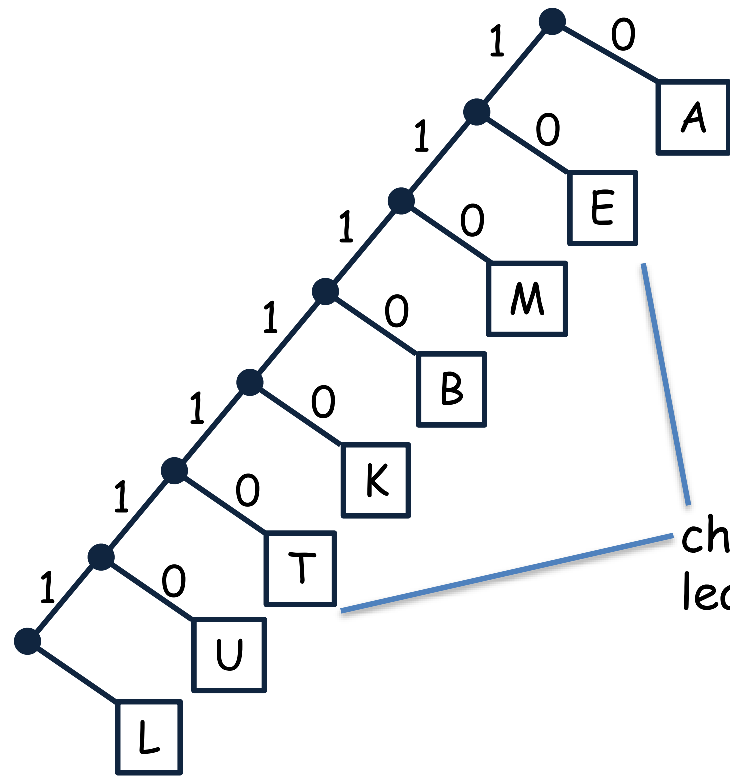

characters will be leaves of the tree

to get an optimal encoding, create an optimal tree

optimal encoding = optimal tree

transform this encoding to a binary tree

Huffman Coding

Lemma : Optimal tree is full.
(every node has either two children or no child)

Proof :

Huffman Coding

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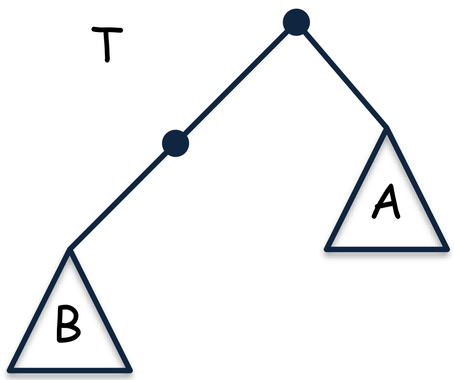

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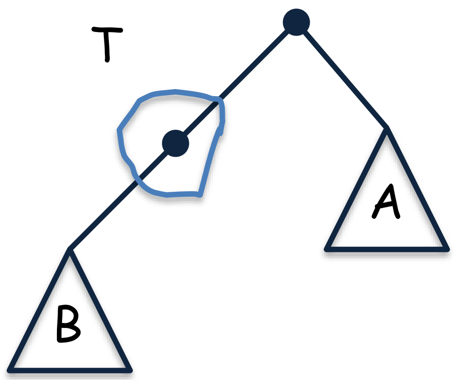

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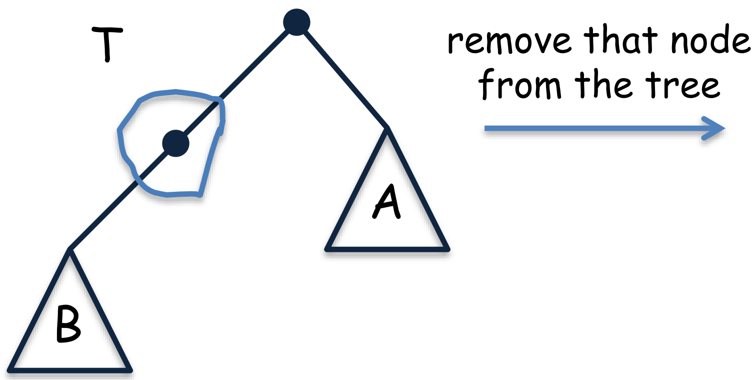

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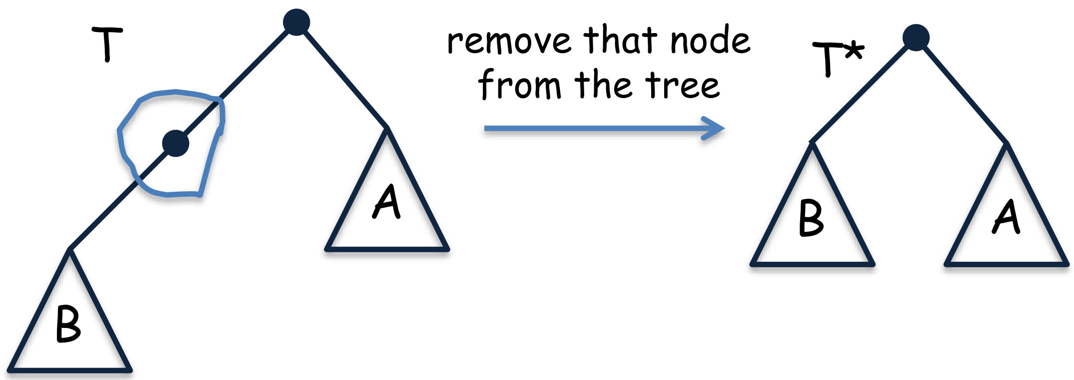

Huffman Coding

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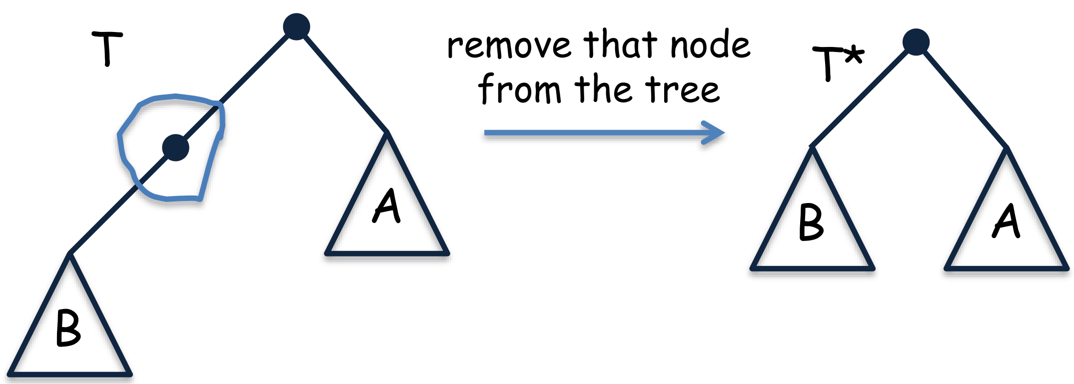

Because all characters in subtree B of T* have encodings 1 bit shorter than encodings in subtree B of T,

$$B(T) > B(T^*)$$

Huffman Coding

- sort all frequencies in decreasing order

Huffman Coding

- sort all frequencies in decreasing order

25 18 13 10 7 5 2 1

Huffman Coding

- sort all frequencies in decreasing order
- start with lowest two frequencies

25	18	13	10	7	5	2	1

Huffman Coding

- sort all frequencies in decreasing order
- start with lowest two frequencies

  combine them in one one, rearrange to preserve the order

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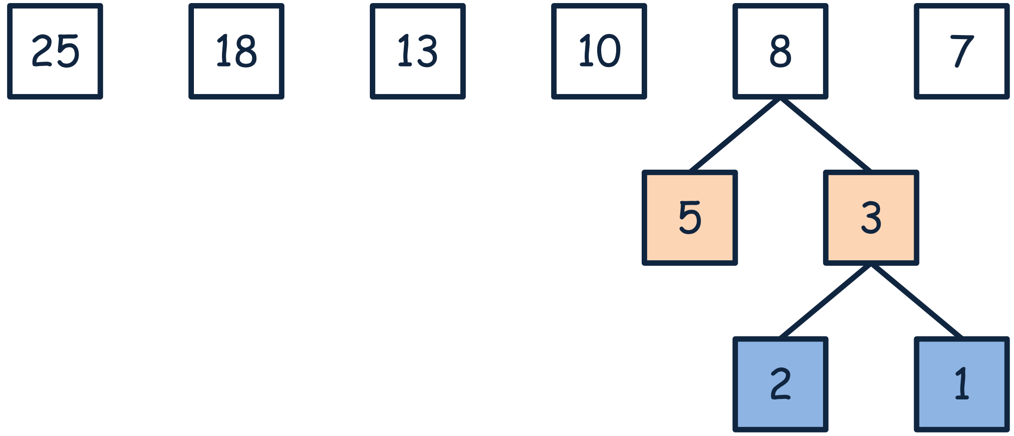

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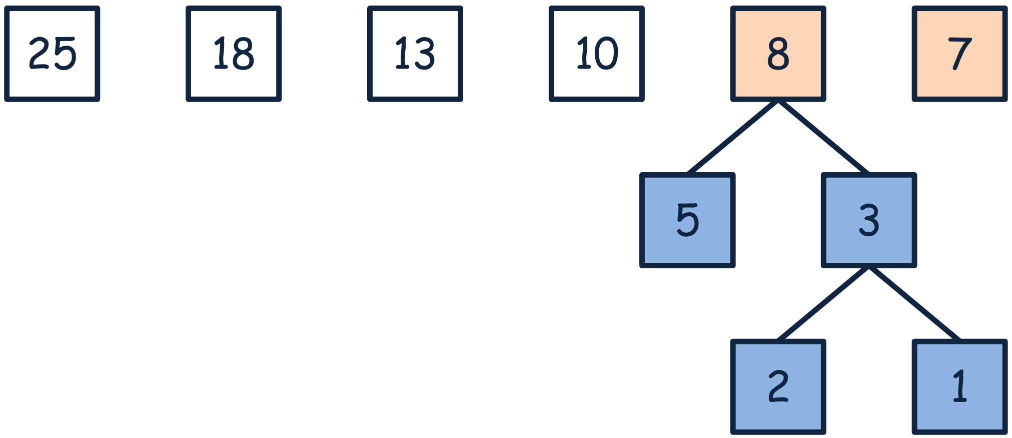

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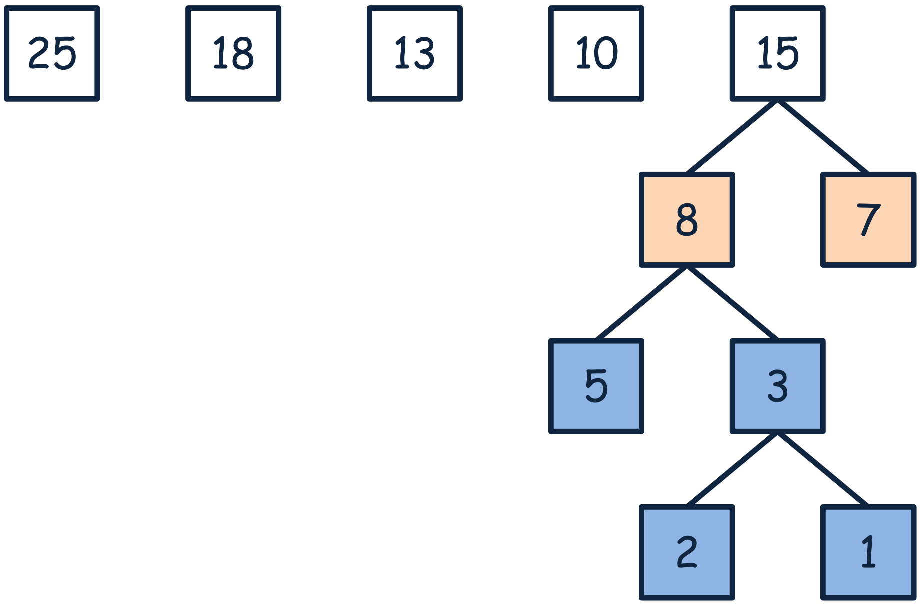

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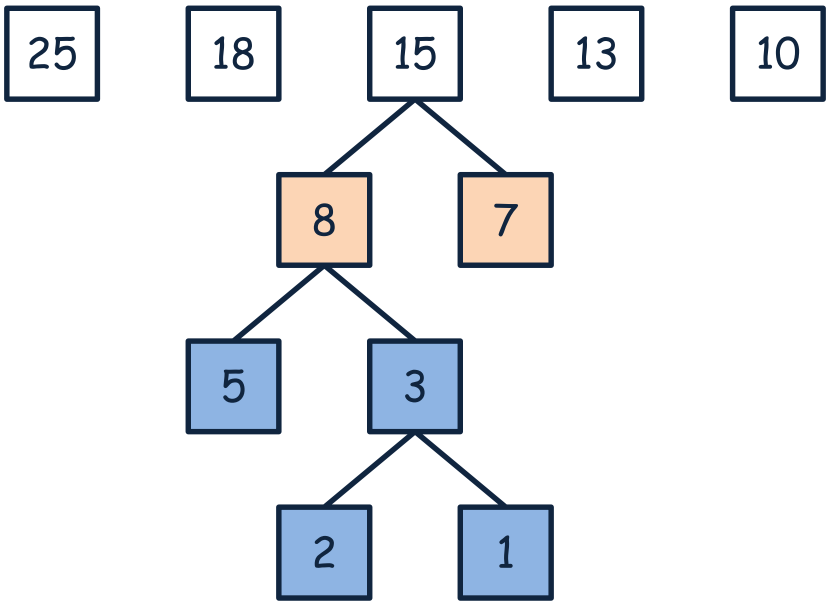

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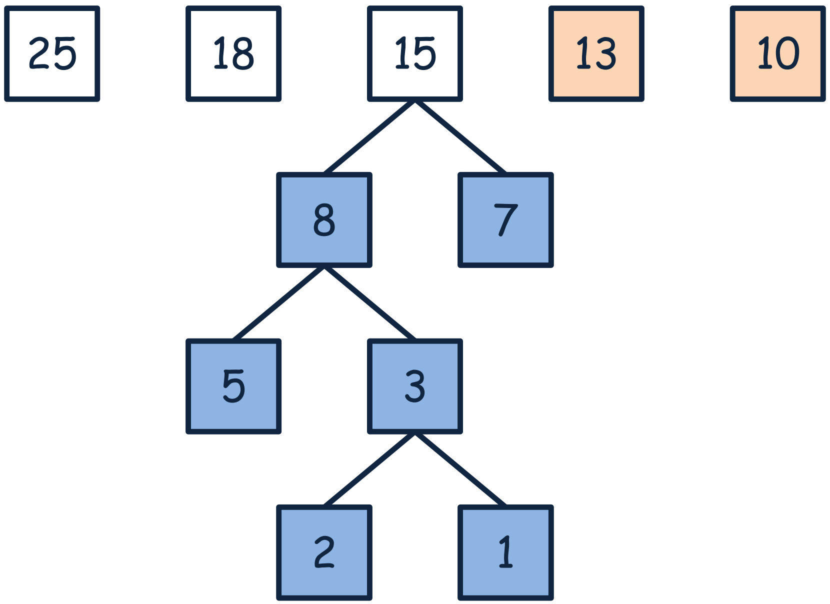

Huffman Coding

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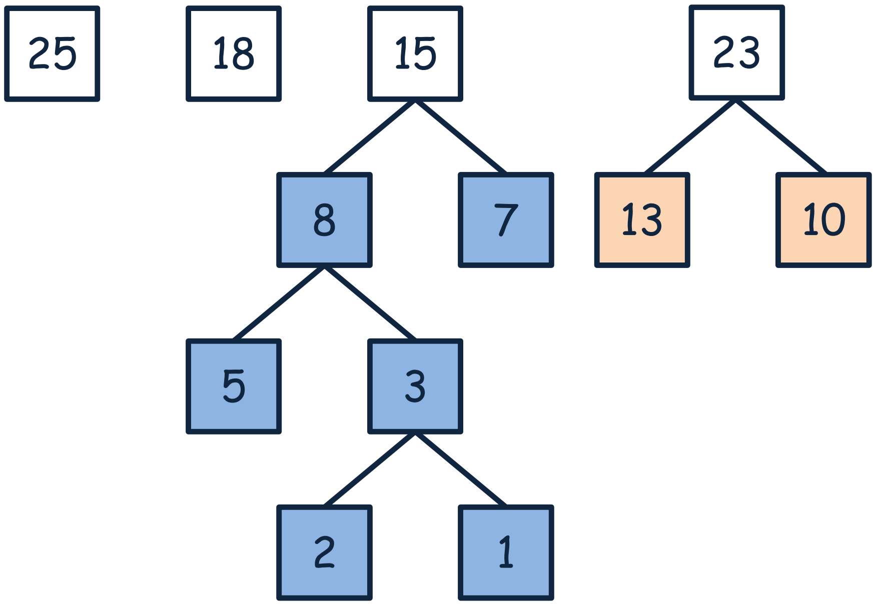

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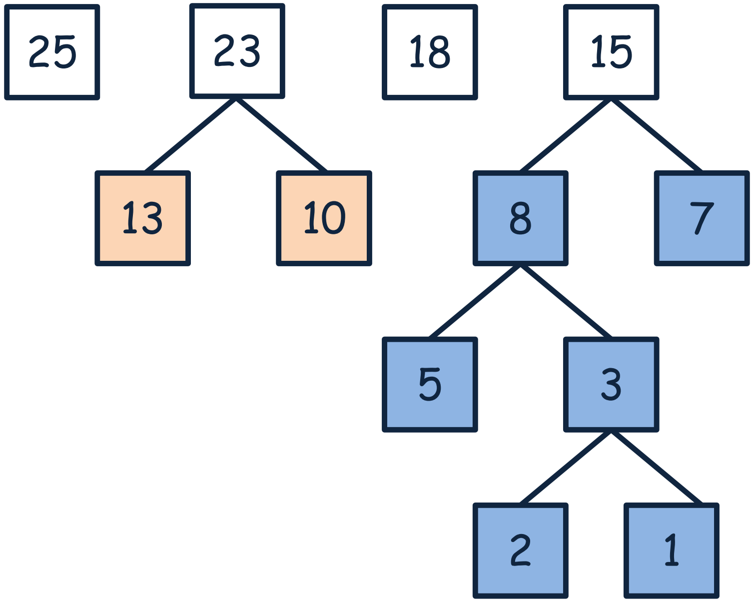

Huffman Coding

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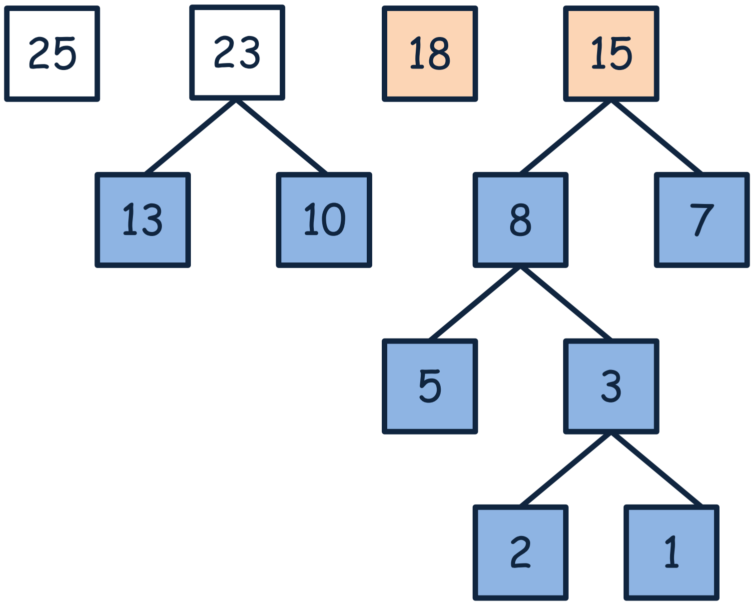

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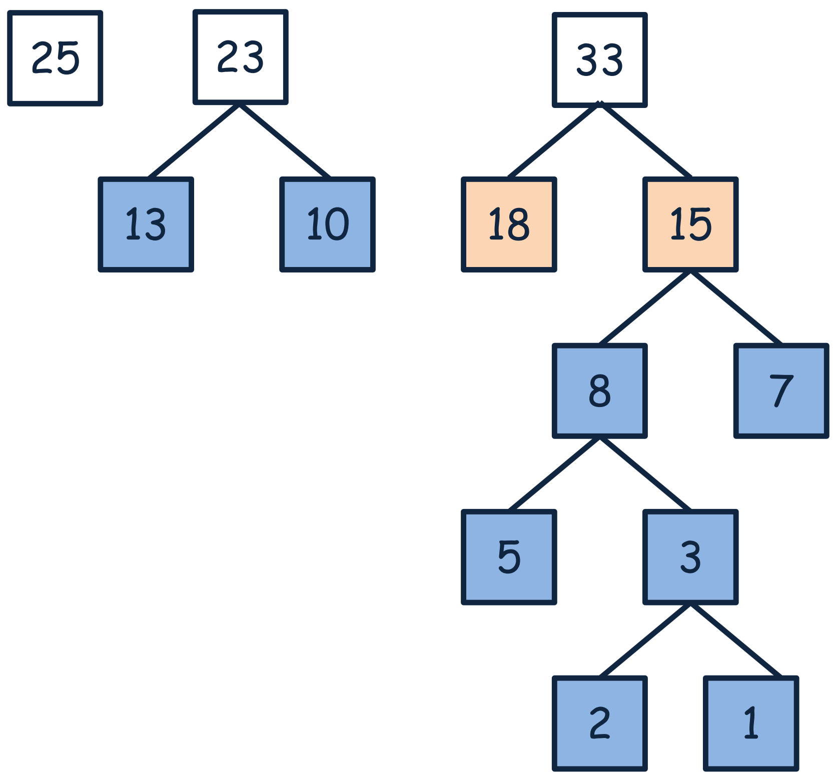

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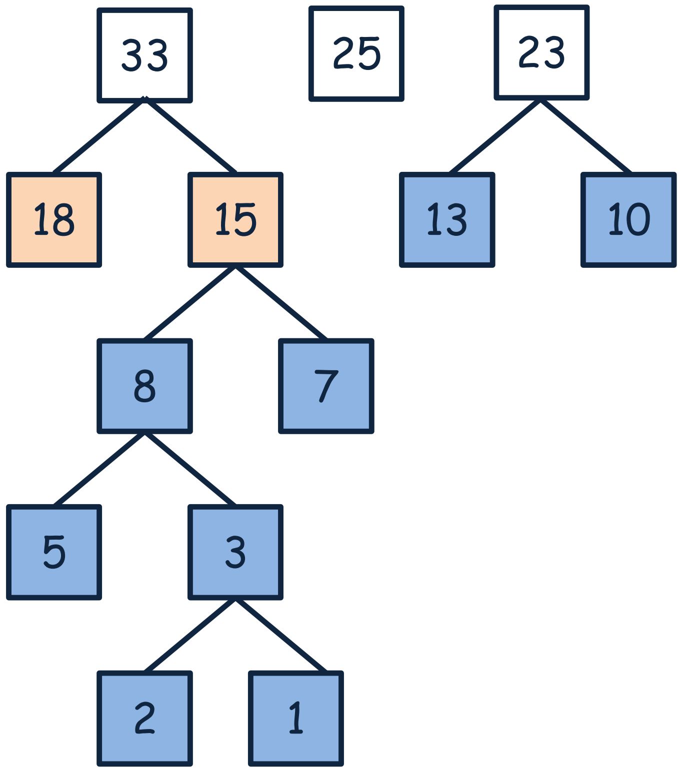

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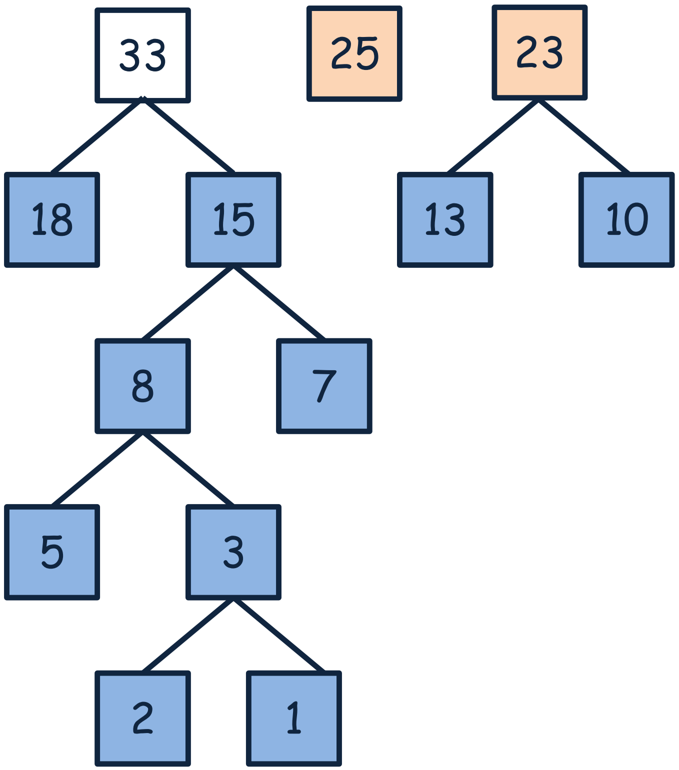

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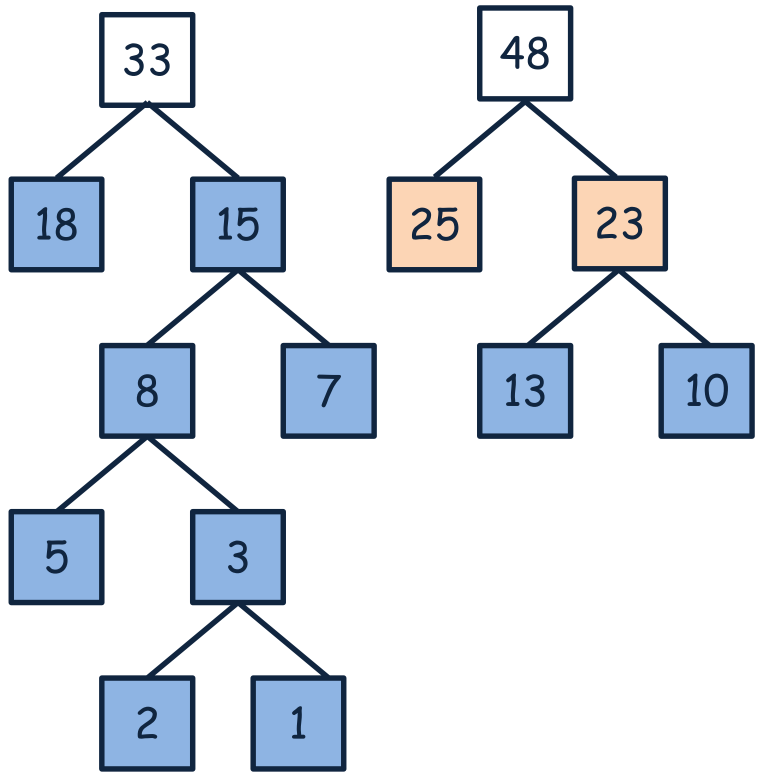

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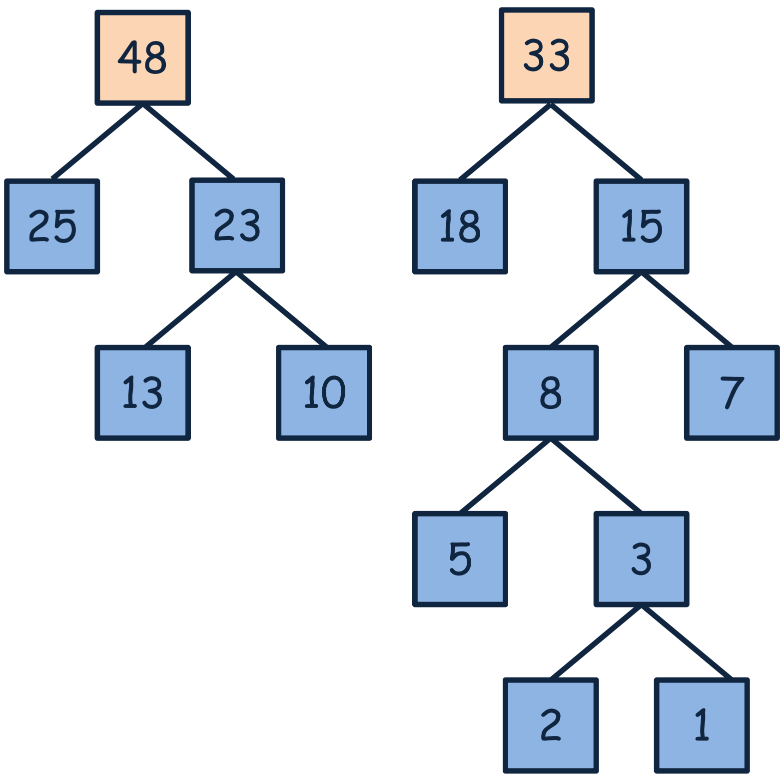

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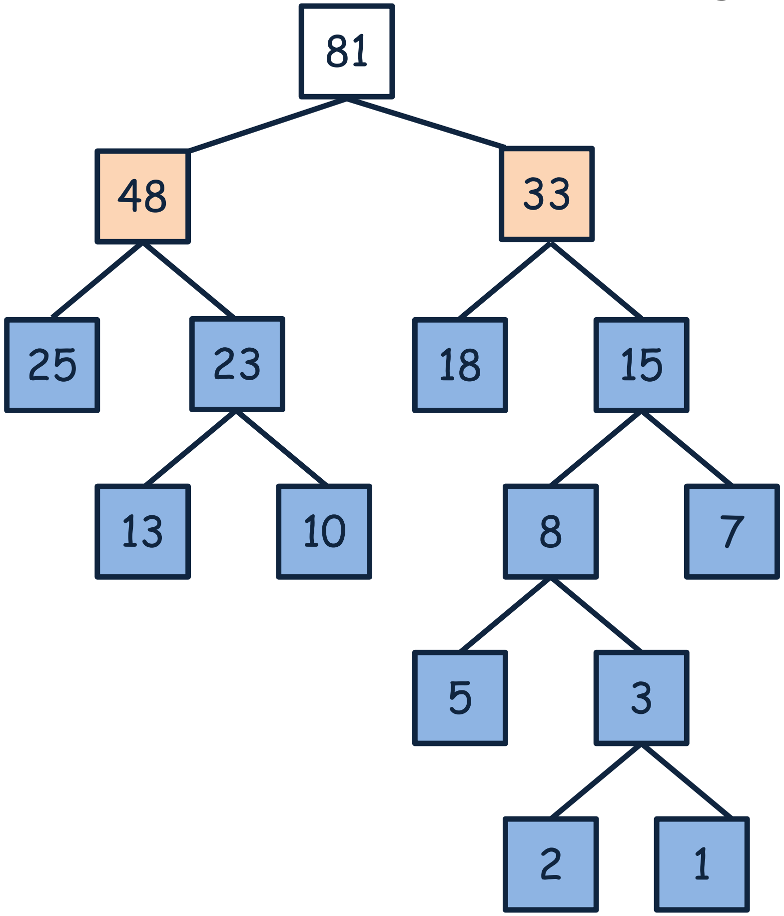

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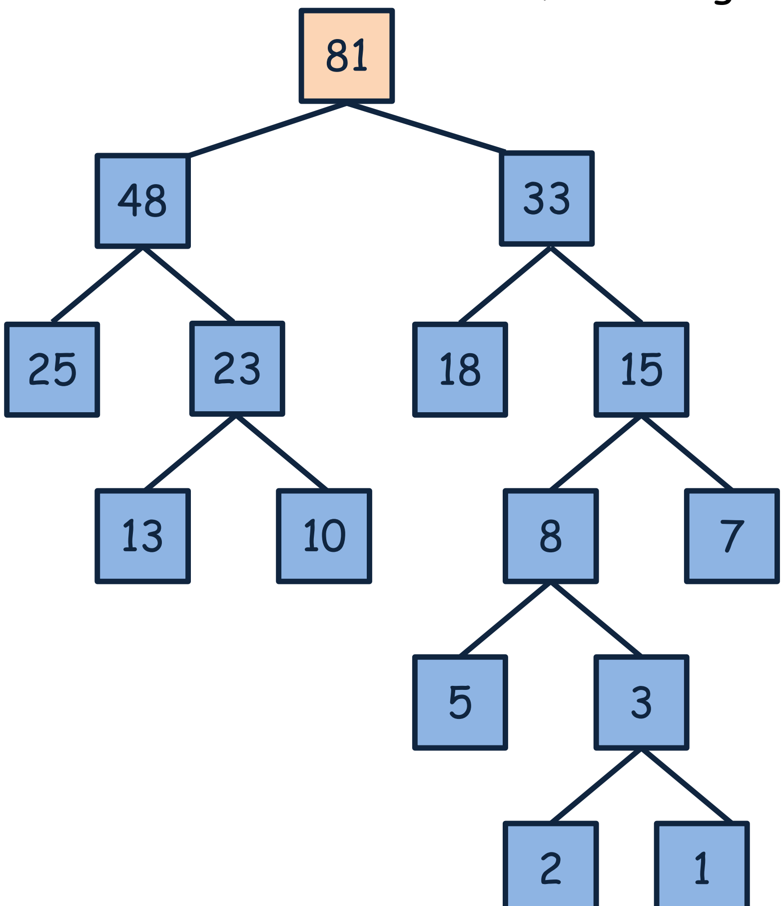

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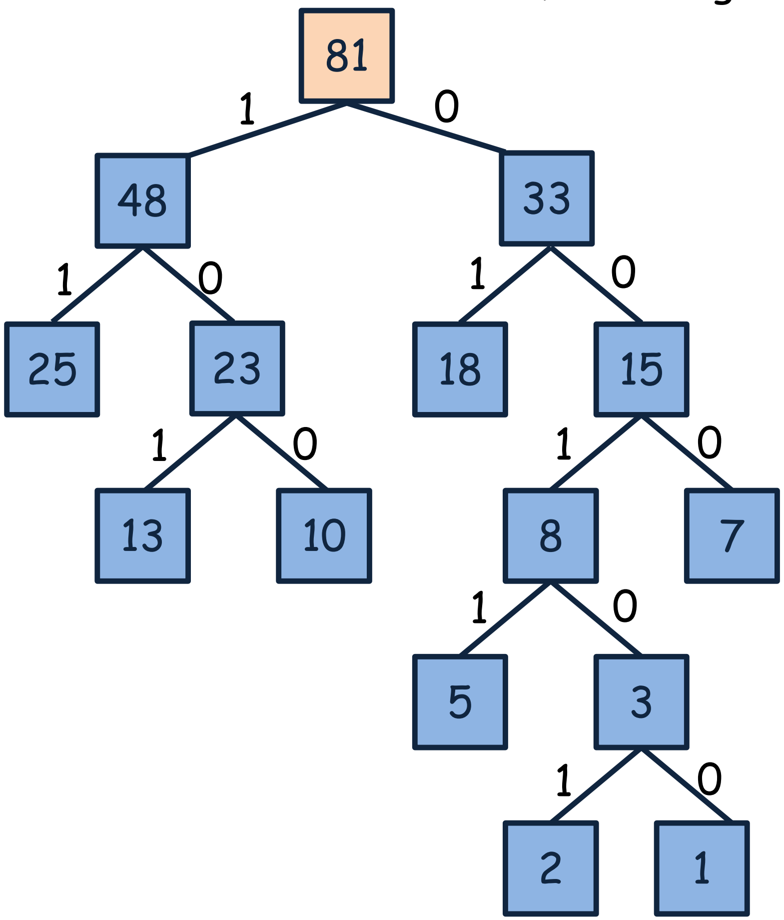

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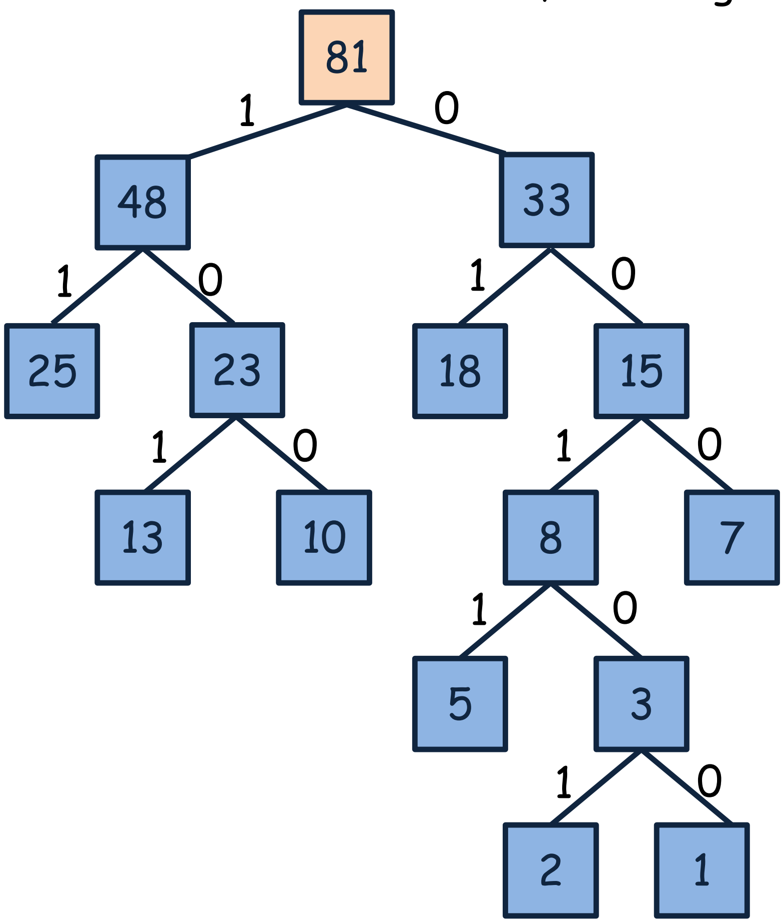
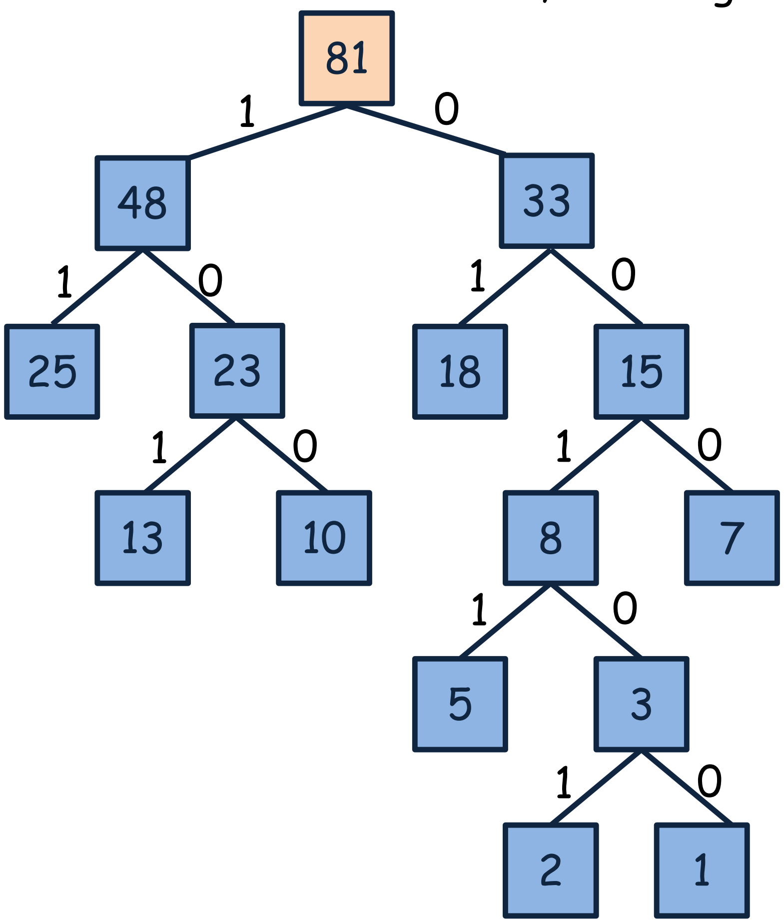


	freq
A	25
E	18
M	13
B	10
K	7
T	5
U	2
L	1

Huffman Coding

- sort all frequencies in decreasing order
- start with lowest two frequencies

  combine them in one one, rearrange to preserve the order

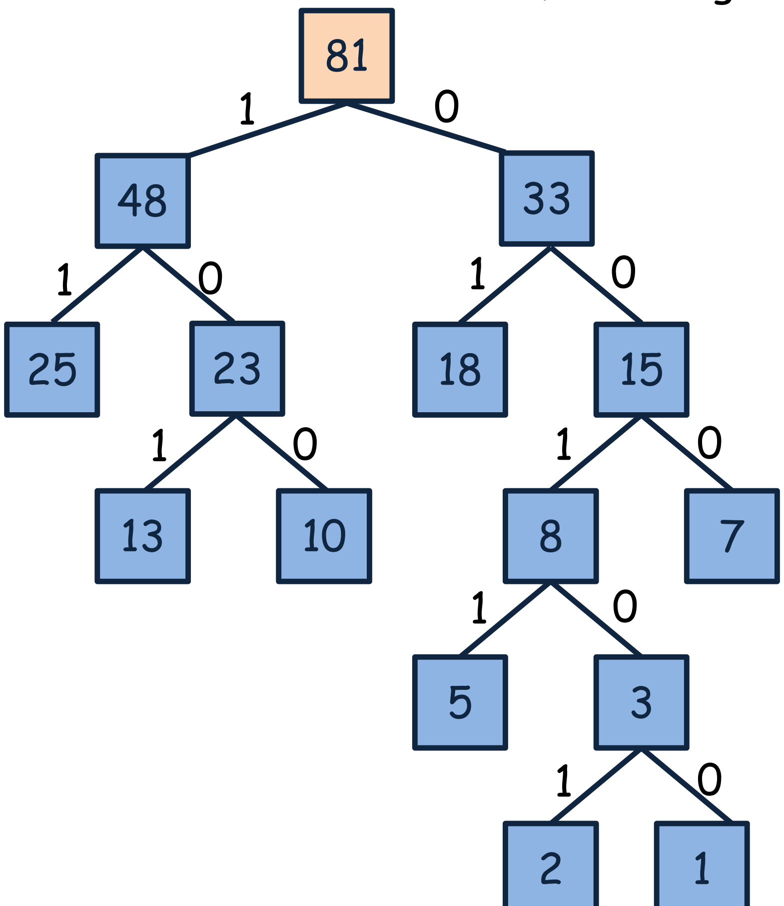


	freq	
A	25	11
E	18	01
M	13	101
B	10	100
K	7	000
T	5	0011
U	2	00101
L	1	00100

Huffman Coding

- sort all frequencies in decreasing order
- start with lowest two frequencies

combine them in one one, rearrange to preserve the order



	freq		cost
A	25	11	50
E	18	01	36
M	13	101	39
B	10	100	30
K	7	000	21
T	5	0011	20
U	2	00101	10
L	1	00100	5

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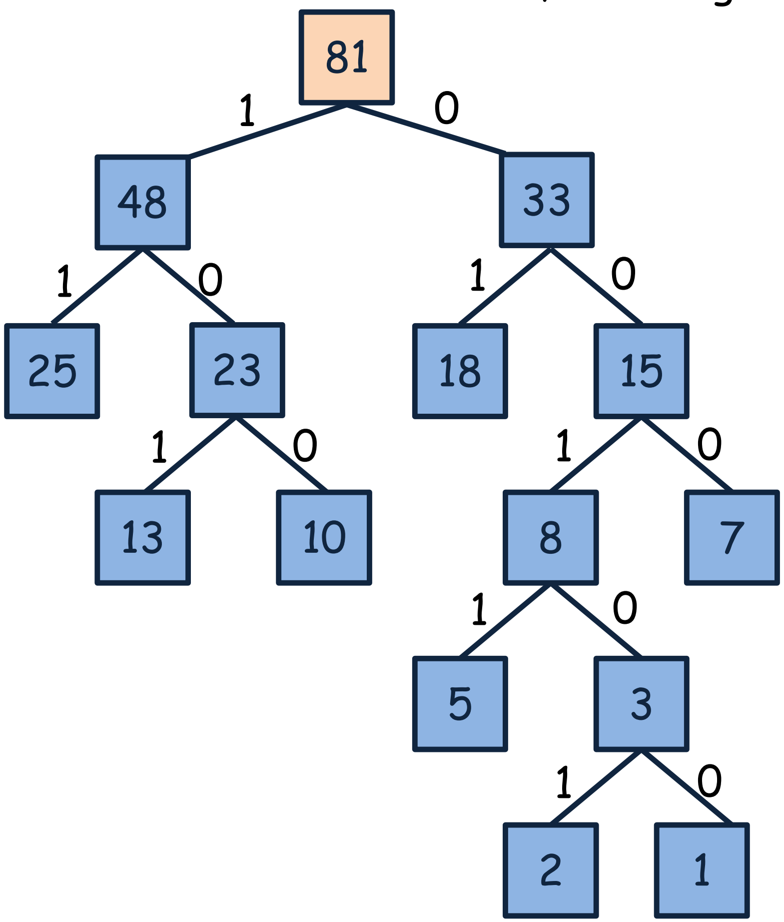
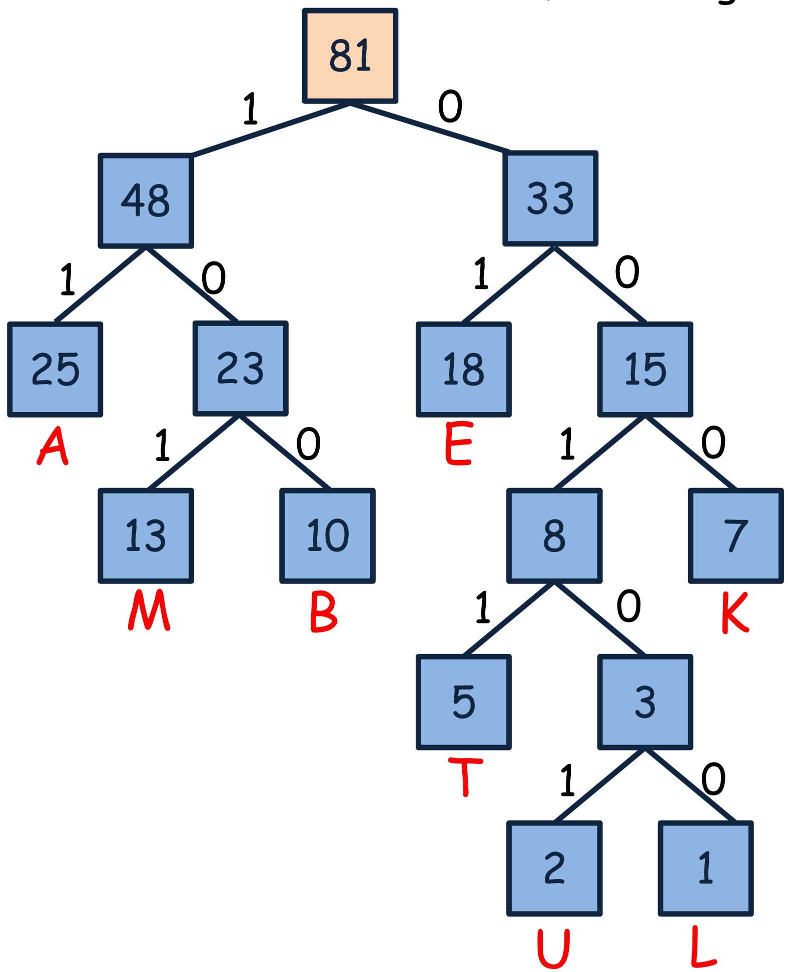


	freq		cost
A	25	11	50
E	18	01	36
M	13	101	39
B	10	100	30
K	7	000	21
T	5	0011	20
U	2	00101	10
L	1	00100	5
			211

Huffman Coding

- sort all frequencies in decreasing order
- start with lowest two frequencies

combine them in one one, rearrange to preserve the order



	freq		cost
A	25	11	50
E	18	01	36
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B	10	100	30
K	7	000	21
T	5	0011	20
U	2	00101	10
L	1	00100	5
			211

Huffman Coding

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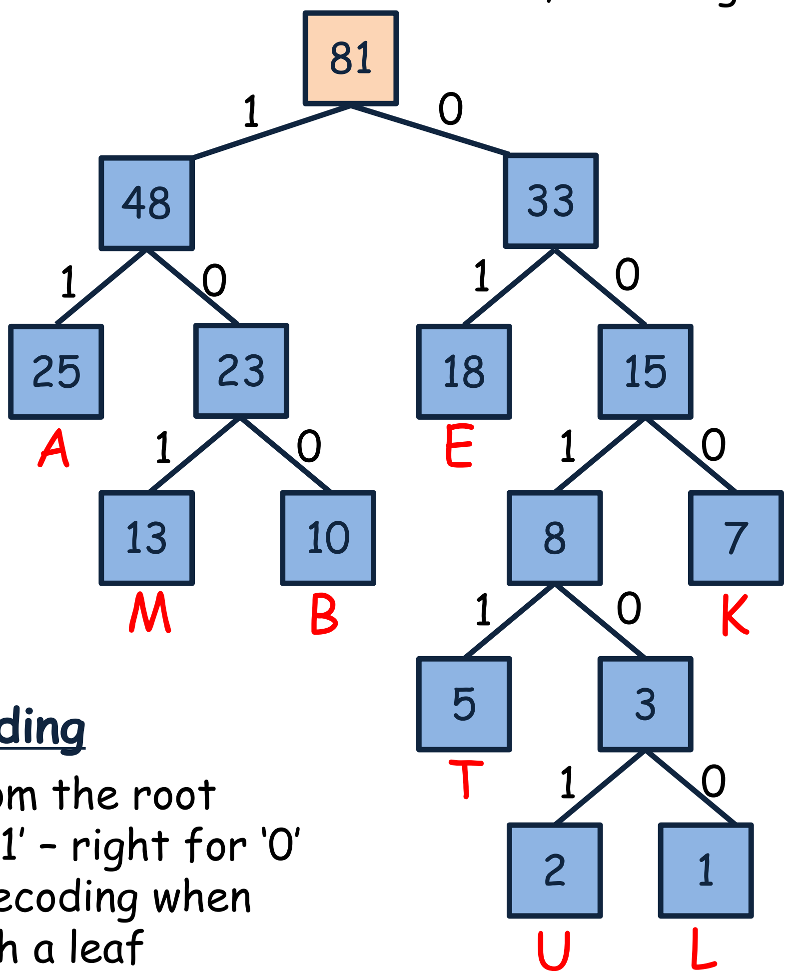

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A	25	11	50
E	18	01	36
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B	10	100	30
K	7	000	21
T	5	0011	20
U	2	00101	10
L	1	00100	5
			211

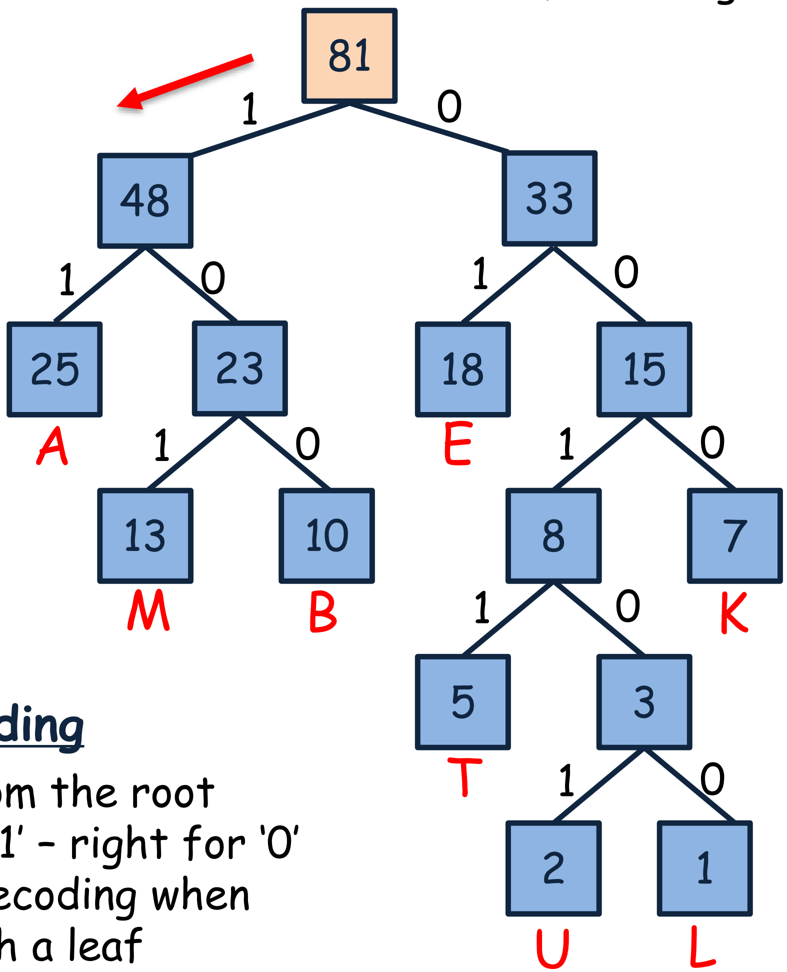
decoding

- start from the root
- left for '1' – right for '0'
- end of decoding when you reach a leaf

Huffman Coding

- sort all frequencies in decreasing order
- start with lowest two frequencies

combine them in one one, rearrange to preserve the order



	freq		cost
A	25	11	50
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U	2	00101	10
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decoding

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- left for '1' – right for '0'
- end of decoding when you reach a leaf

1 0 0 0 1 0 0 0

Huffman Coding

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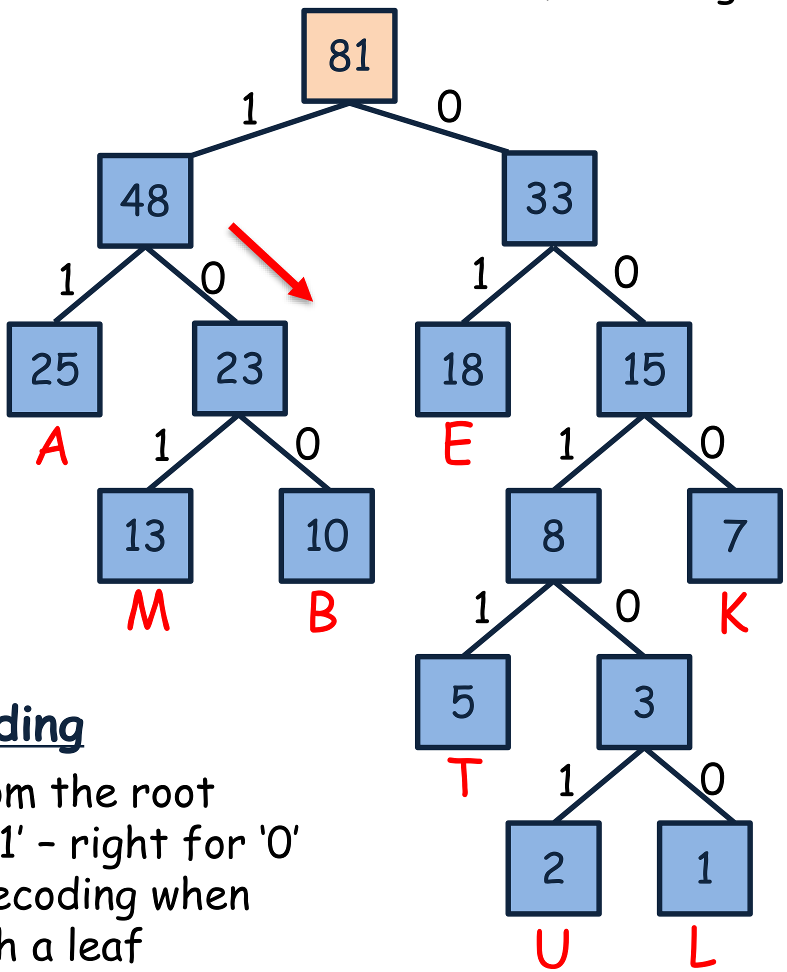
combine them in one one, rearrange to preserve the order



	freq		cost
A	25	11	50
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M	13	101	39
B	10	100	30
K	7	000	21
T	5	0011	20
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L	1	00100	5
			211

1 0 0 0 1 0 0 0

decoding

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Huffman Coding

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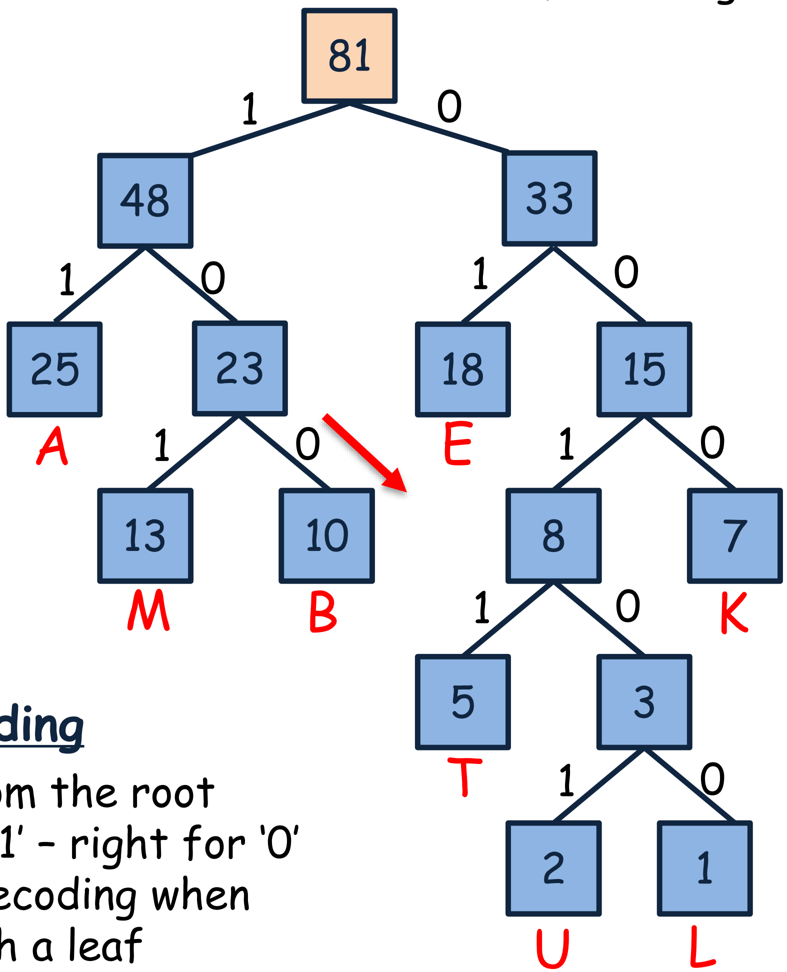

	freq		cost
A	25	11	50
E	18	01	36
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K	7	000	21
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Huffman Coding

- sort all frequencies in decreasing order
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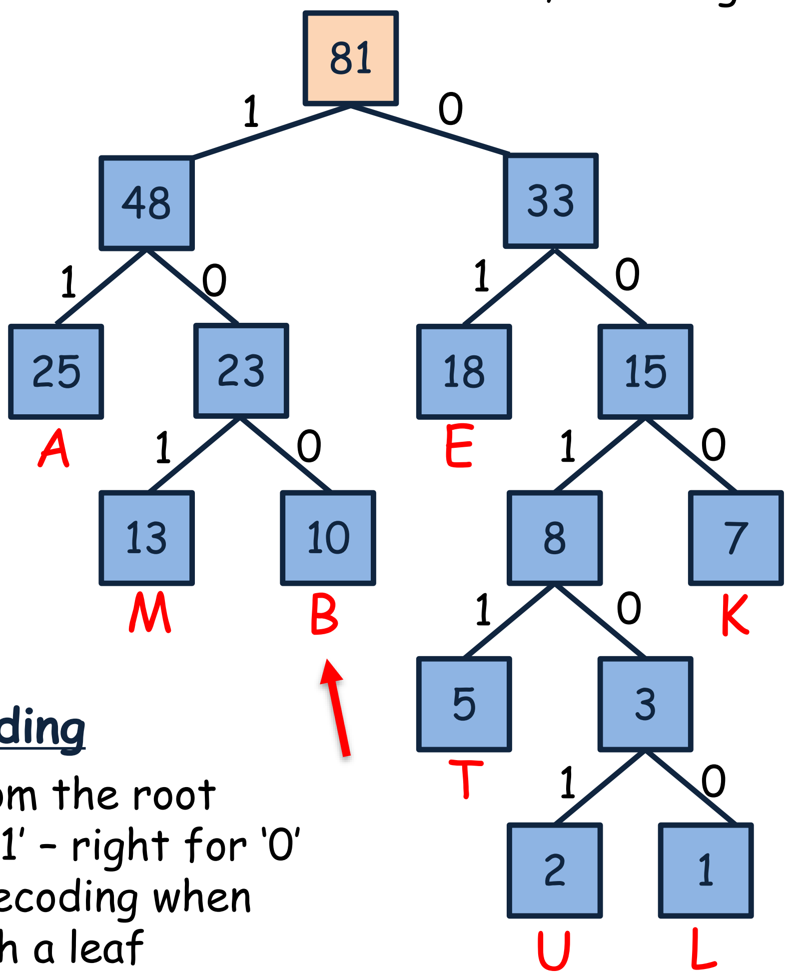
  combine them in one one, rearrange to preserve the order



	freq		cost
A	25	11	50
E	18	01	36
M	13	101	39
B	10	100	30
K	7	000	21
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U	2	00101	10
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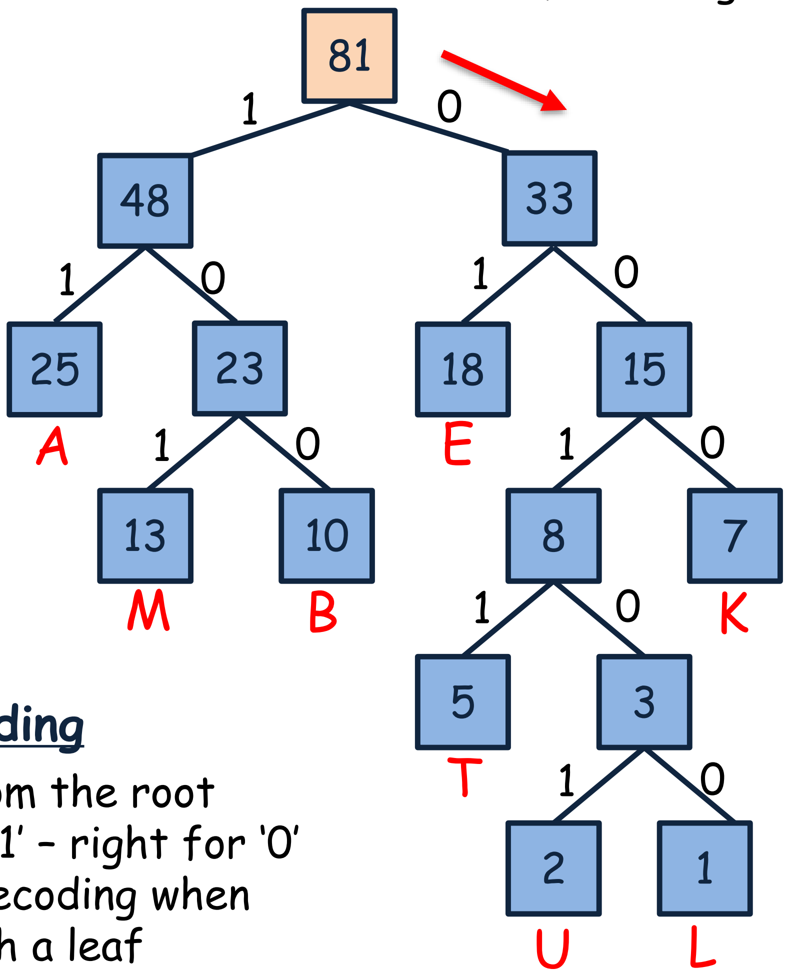

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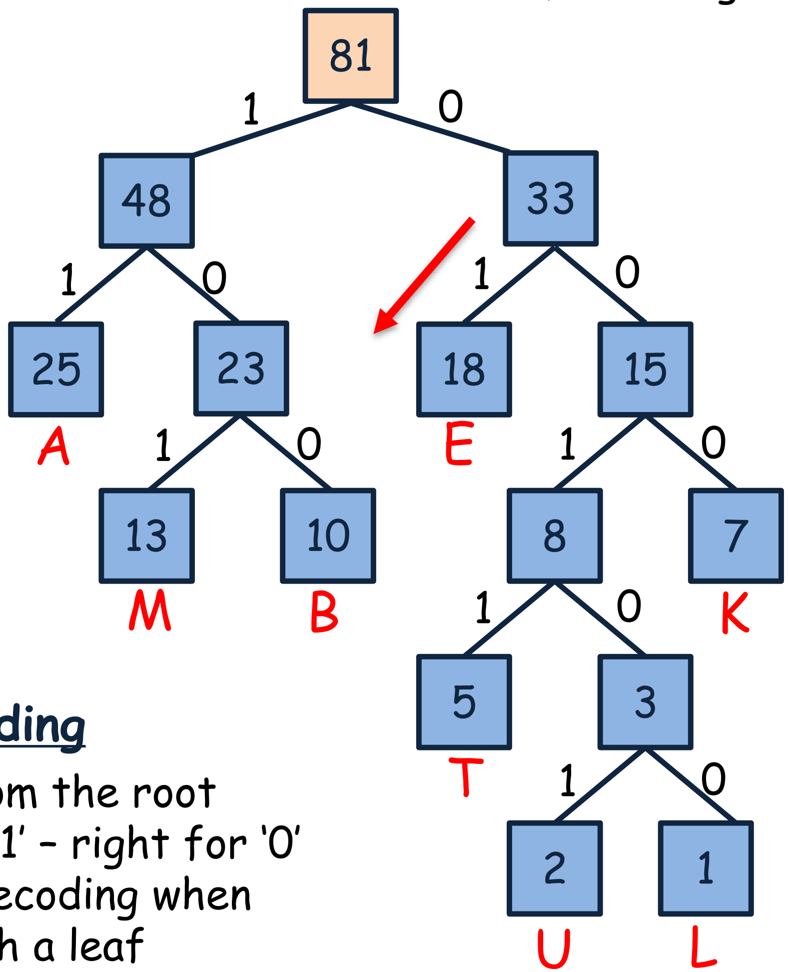

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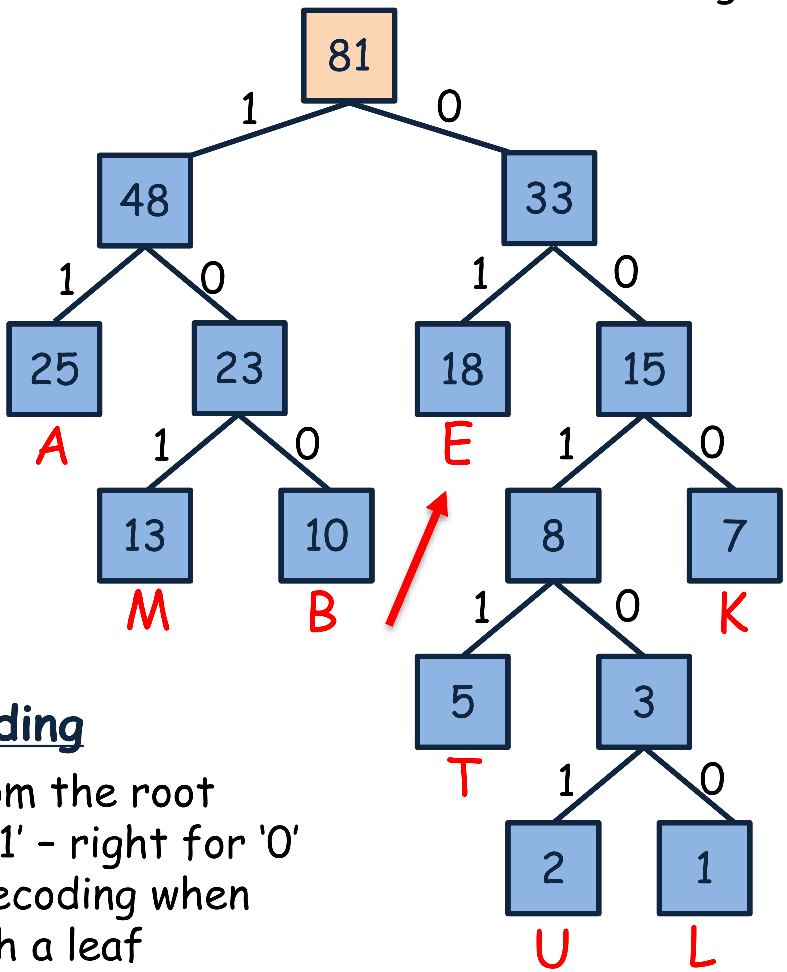

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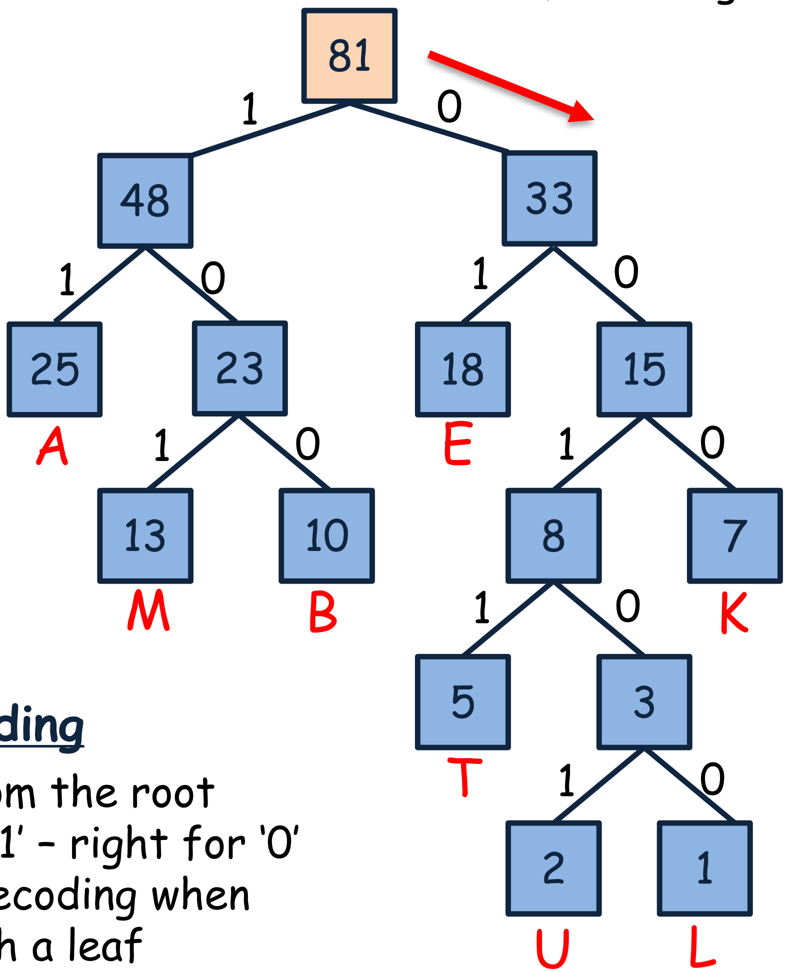

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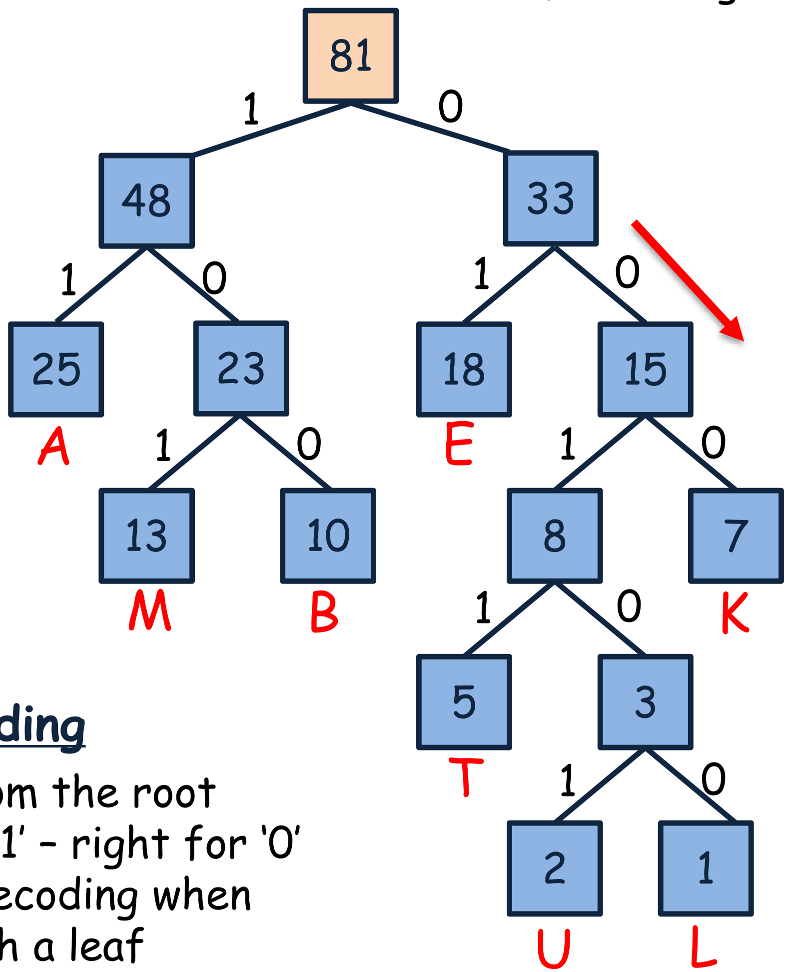

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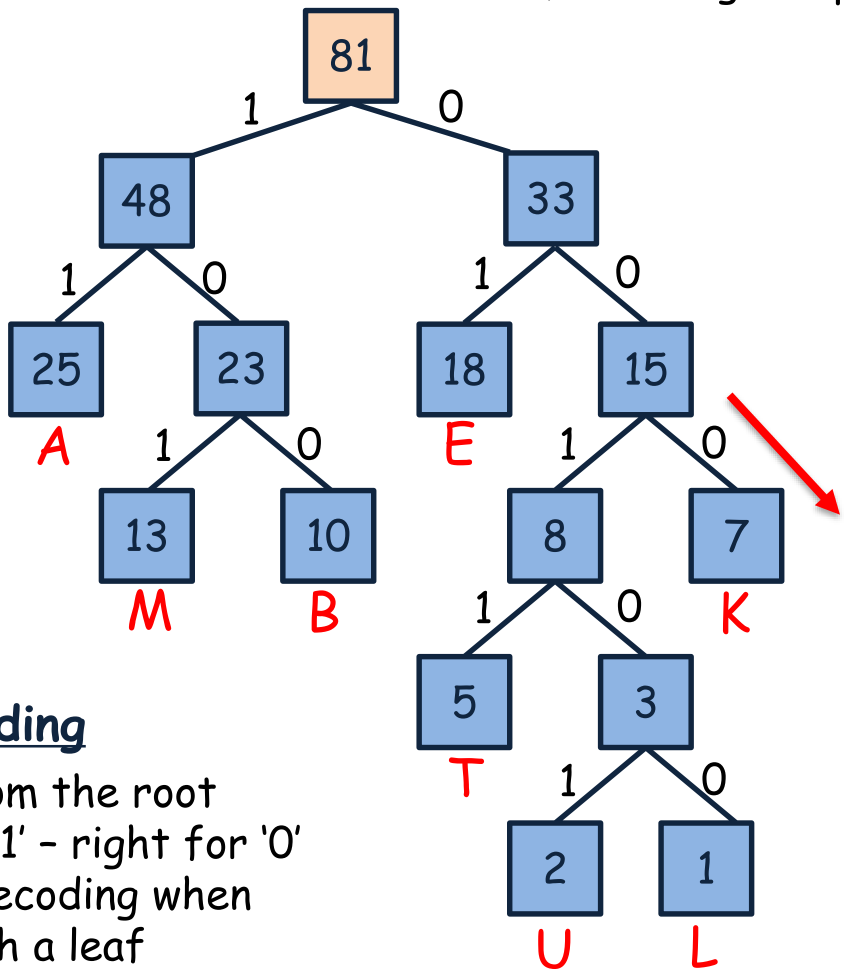

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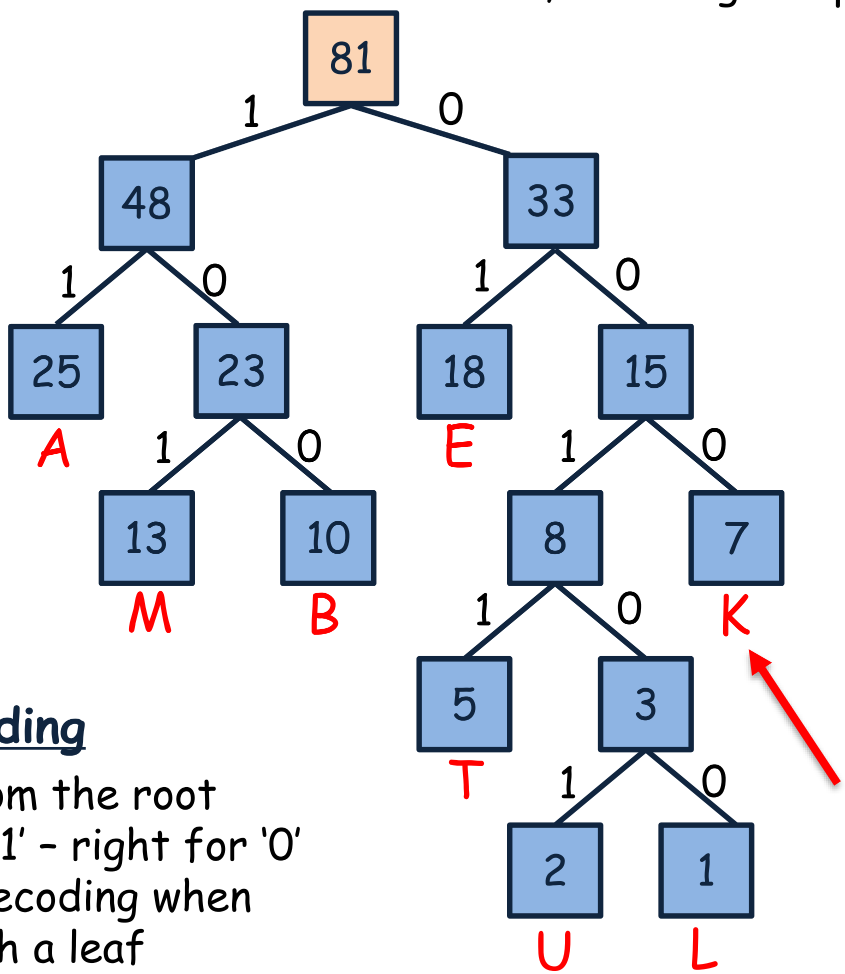

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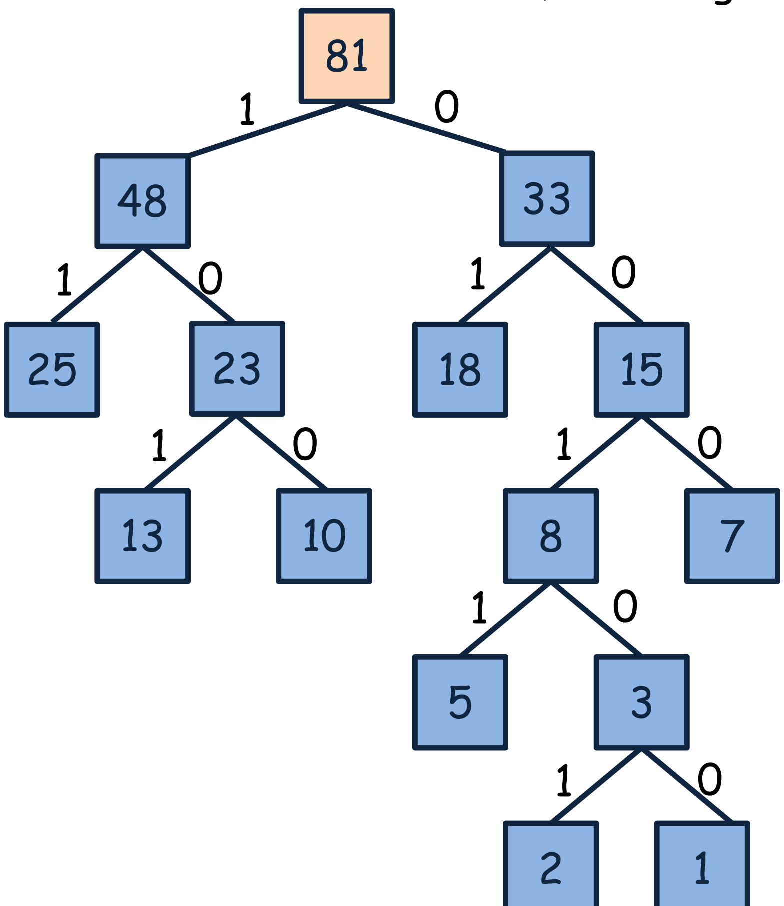

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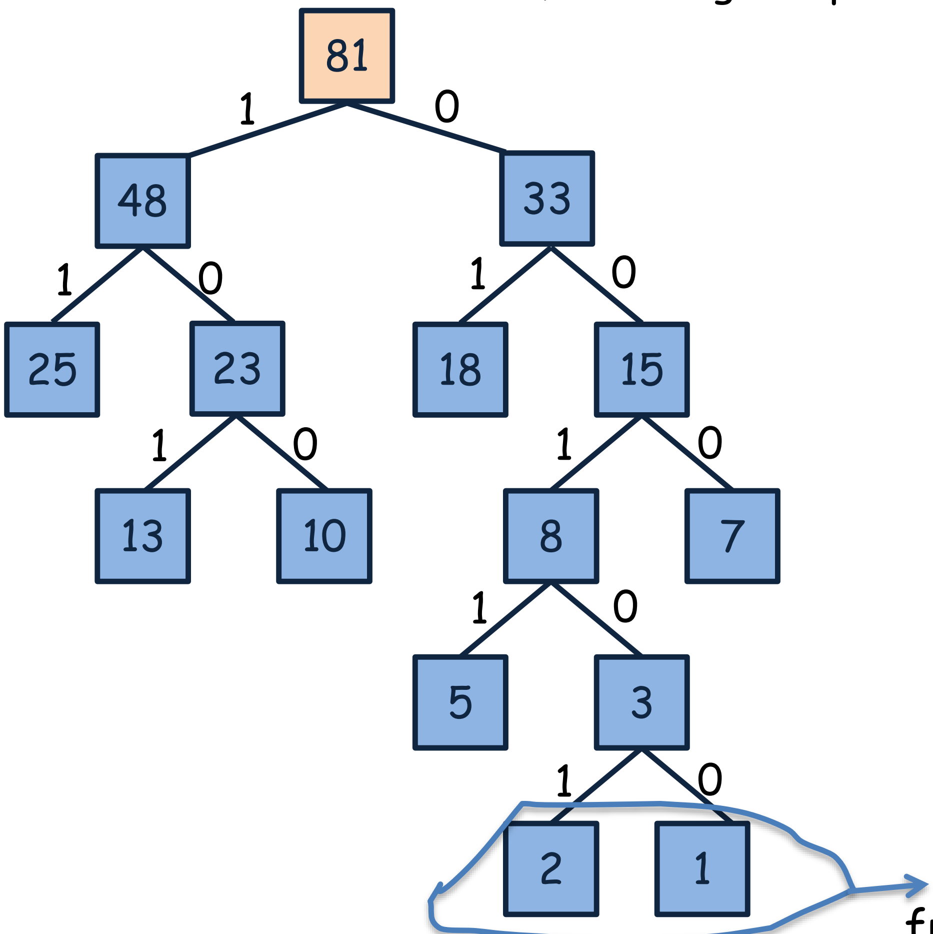

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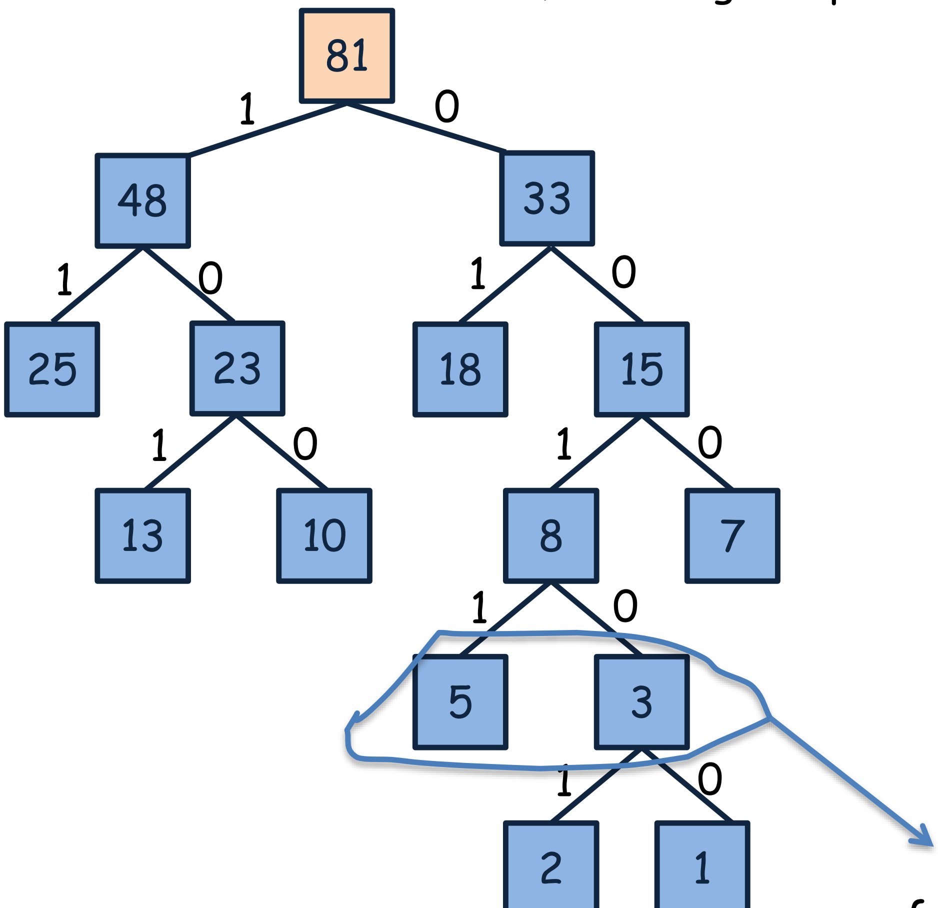

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Theorem (Greedy-choice property): Let x and y be twe symbols with the smallest frequencies f_x and f_y. There exists an optimal tree where x and y are siblings with the highest depth. (Our greedy approach yields us an optimal solution)

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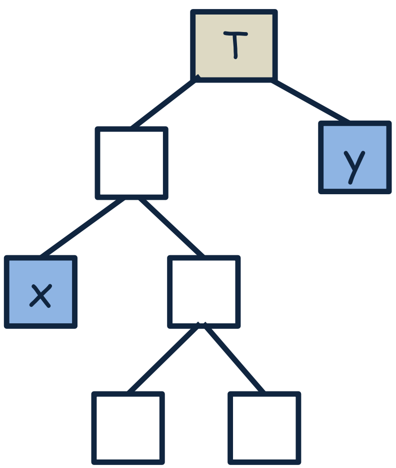

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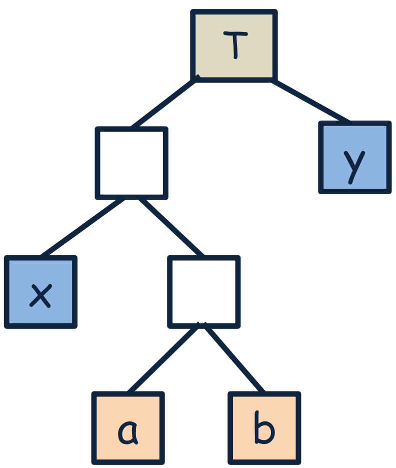

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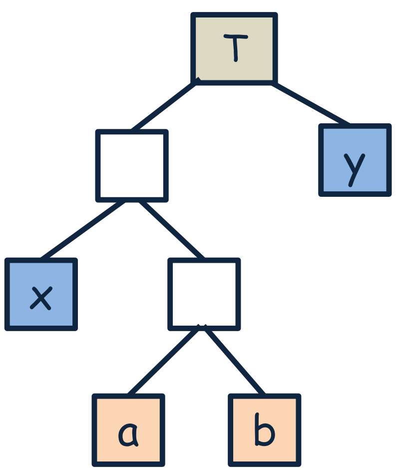

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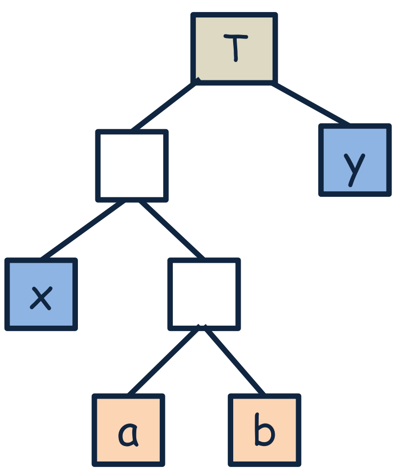

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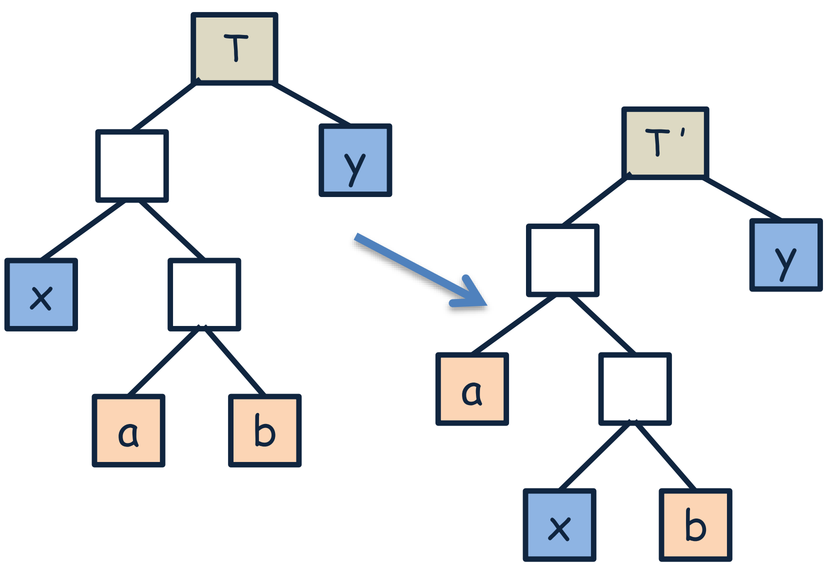

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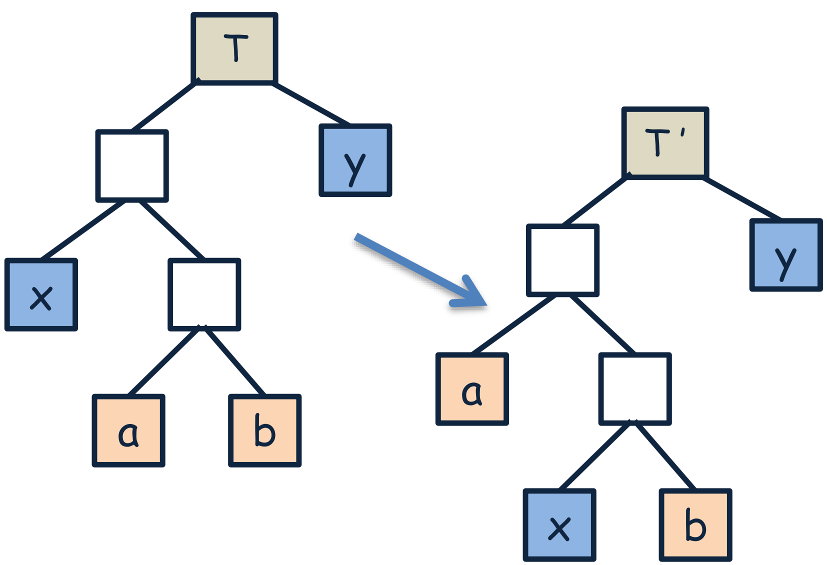

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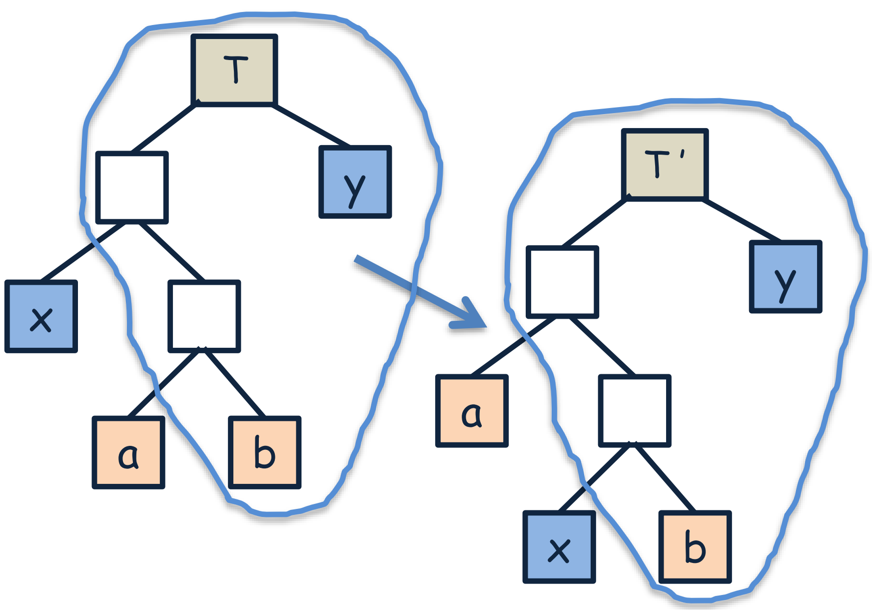

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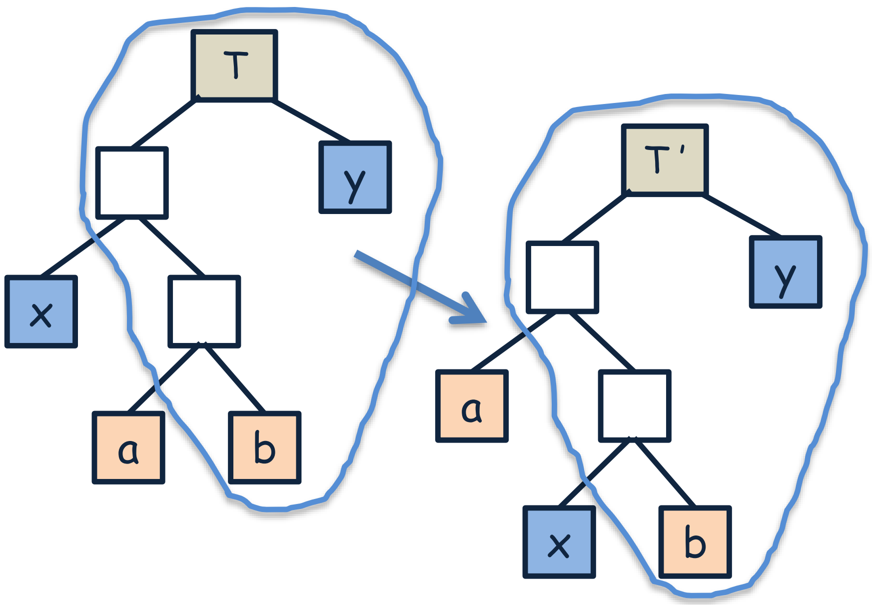

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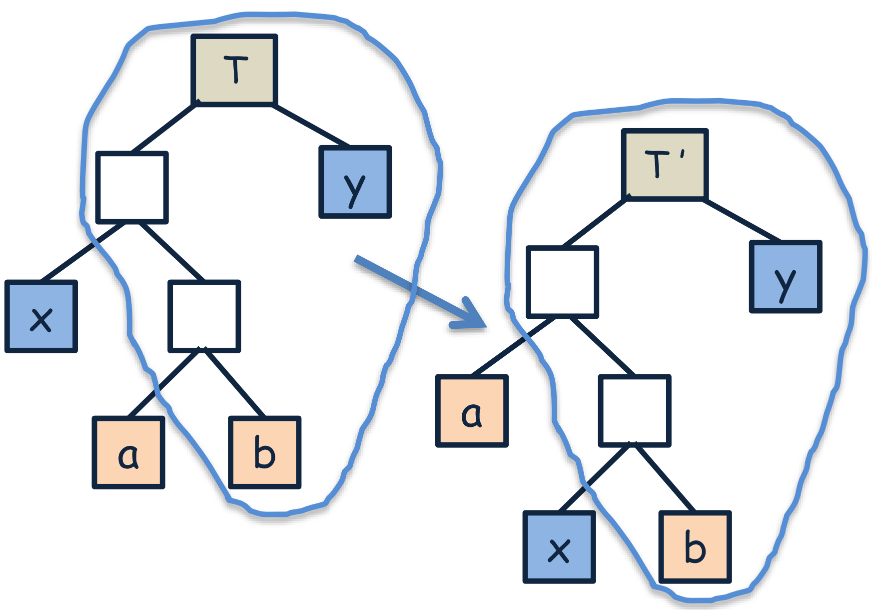

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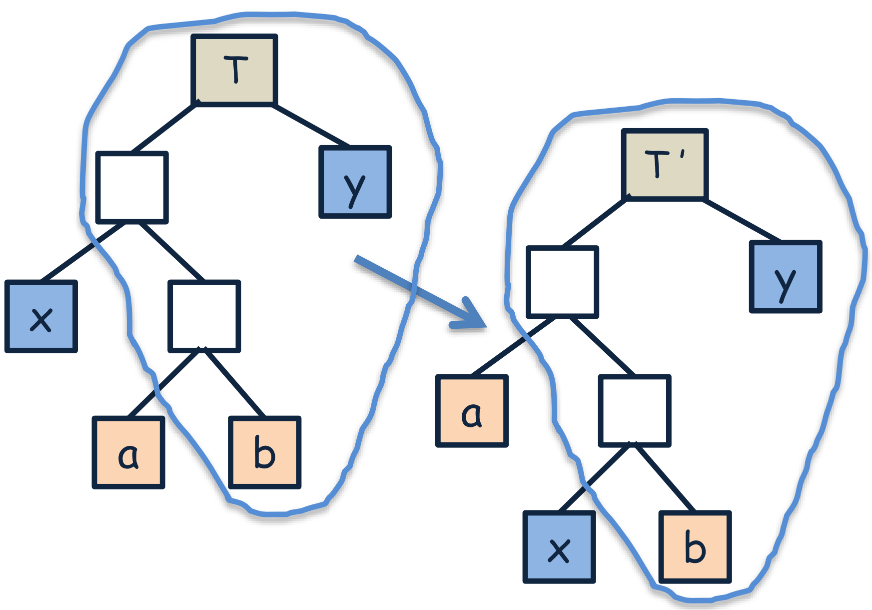

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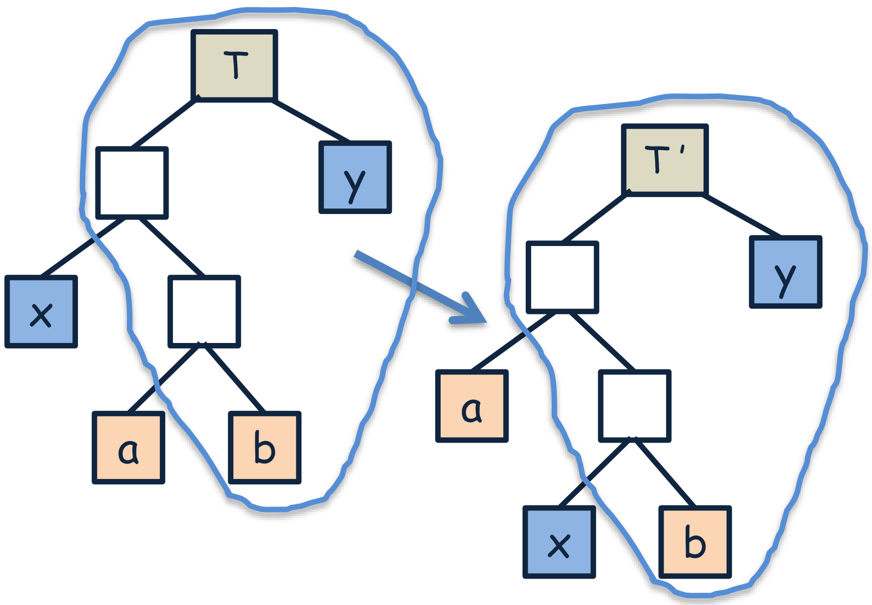

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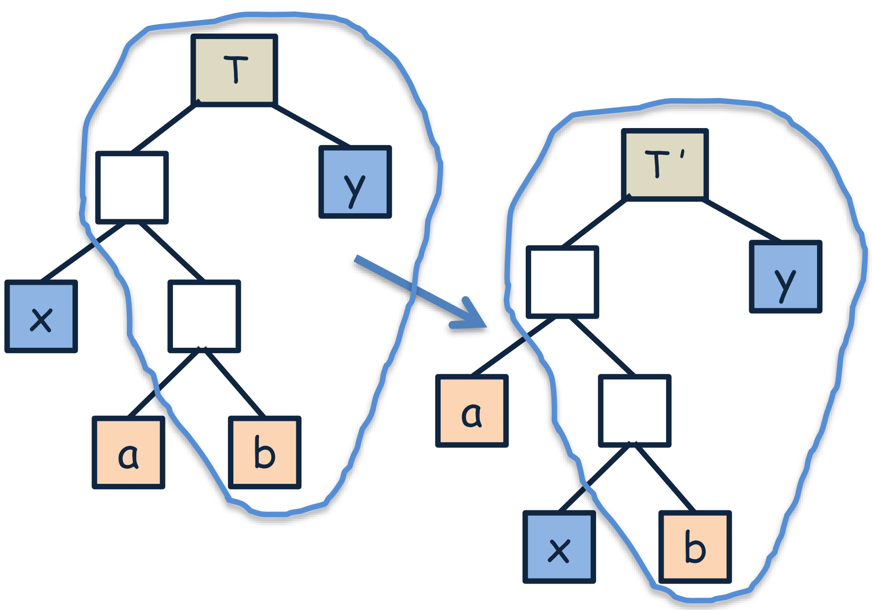

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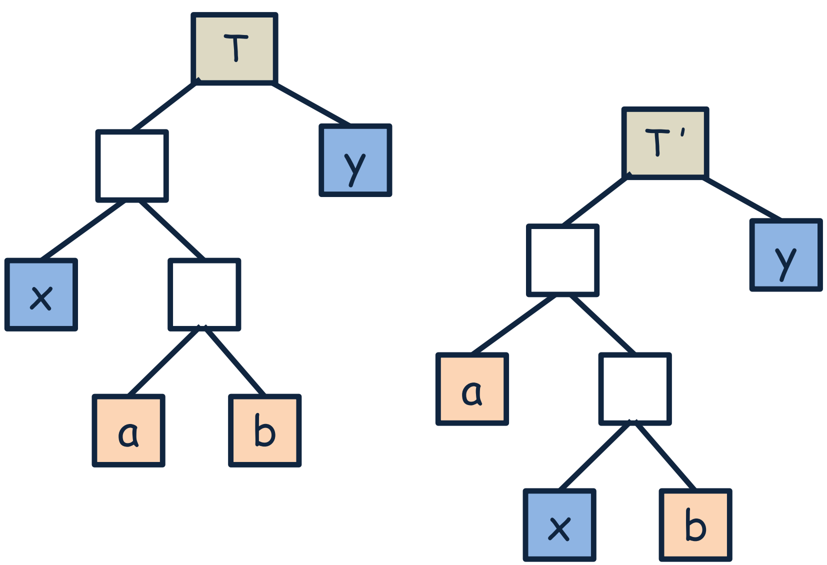

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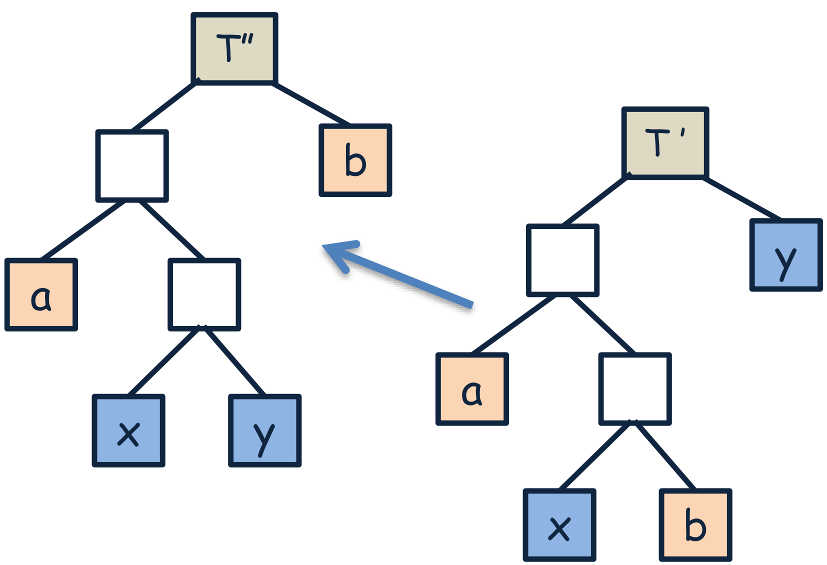

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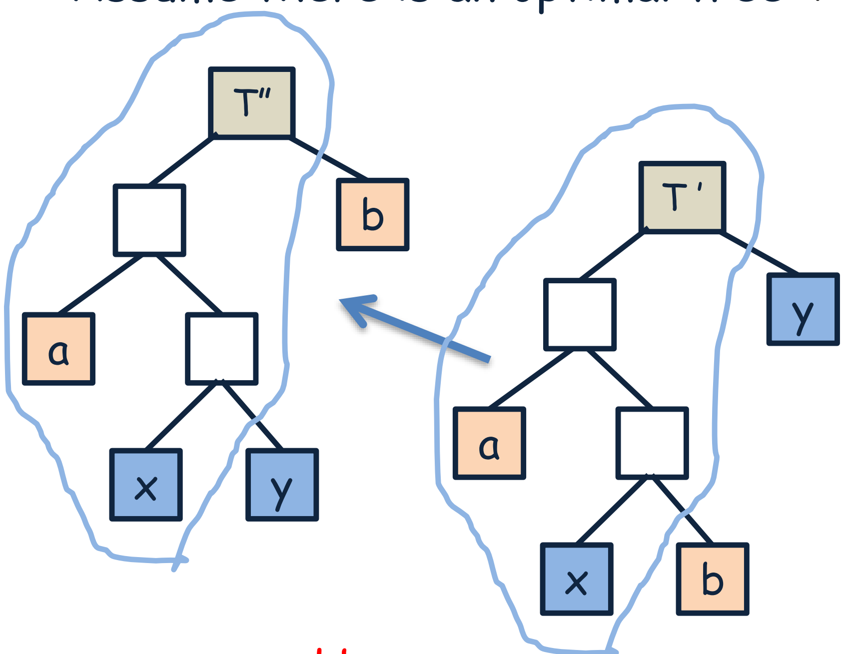

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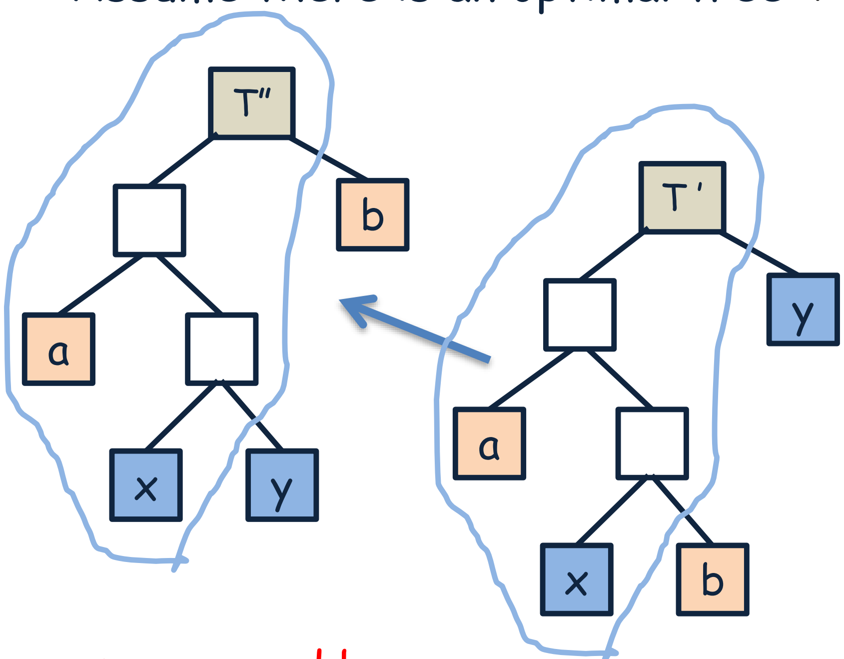

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- $B(T') - B(T'') \geq 0$

this is a contradiction

Greedy Algorithms

- solve the problem by breaking it a sequence of subproblems

- make the best local choice among all feasible one available on that moment (one choice at a time)
    - your choice does not depend on any future choices or any past choices you have made

- prove that the Greedy Choice Property satisfies. A sequence of locally optimal choices yields a global optimal solution