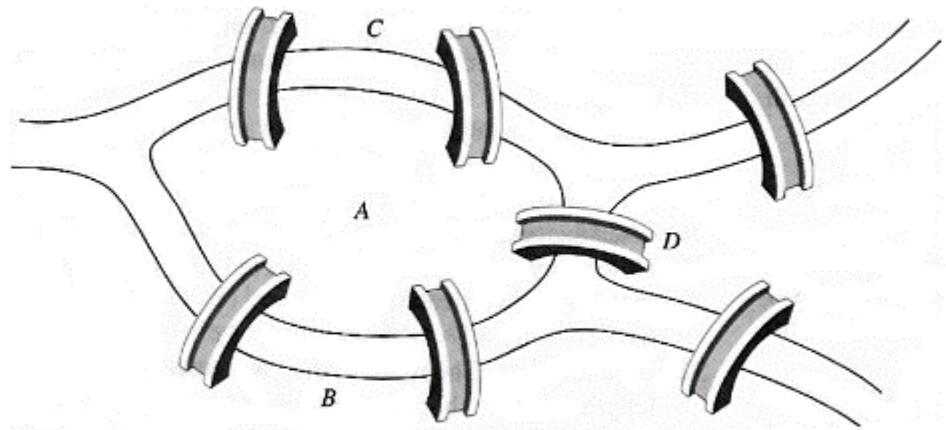
Brute Force and Exhaustive Search II

Murat Osmanoglu

Graph Theory



- Königsberg was a city in Germany in 18th century. There was a river named Pregel that divided the city into four distinct regions.

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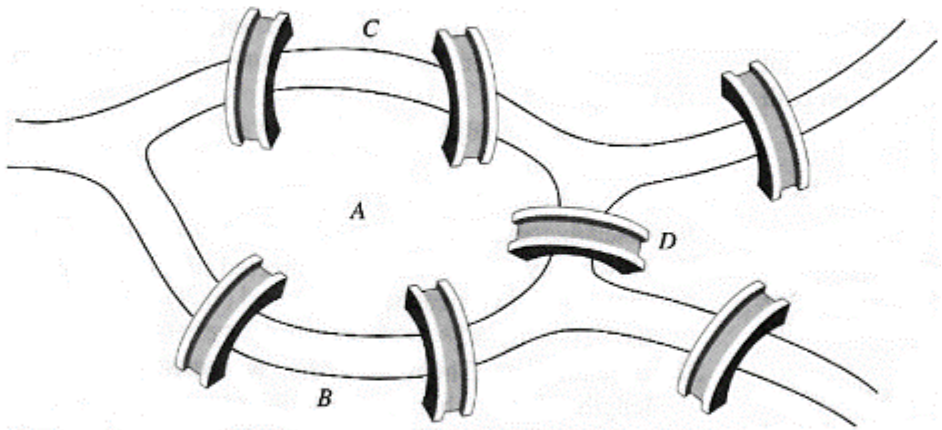
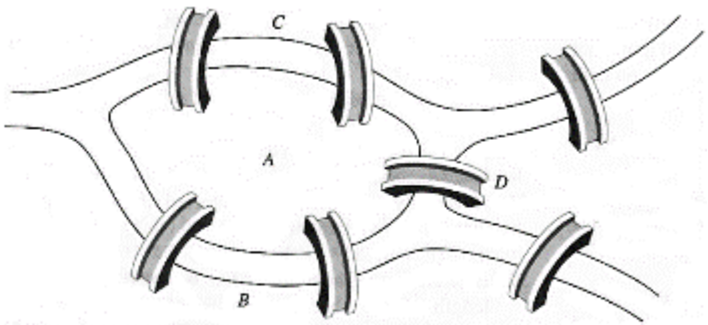


- Königsberg was a city in Germany in 18th century. There was a river named Pregel that divided the city into four distinct regions.
- There was a natural question for the people of Königsberg :

'Is it possible to take a walk around the city that crosses each bridge exaactly once?'

Graph Theory



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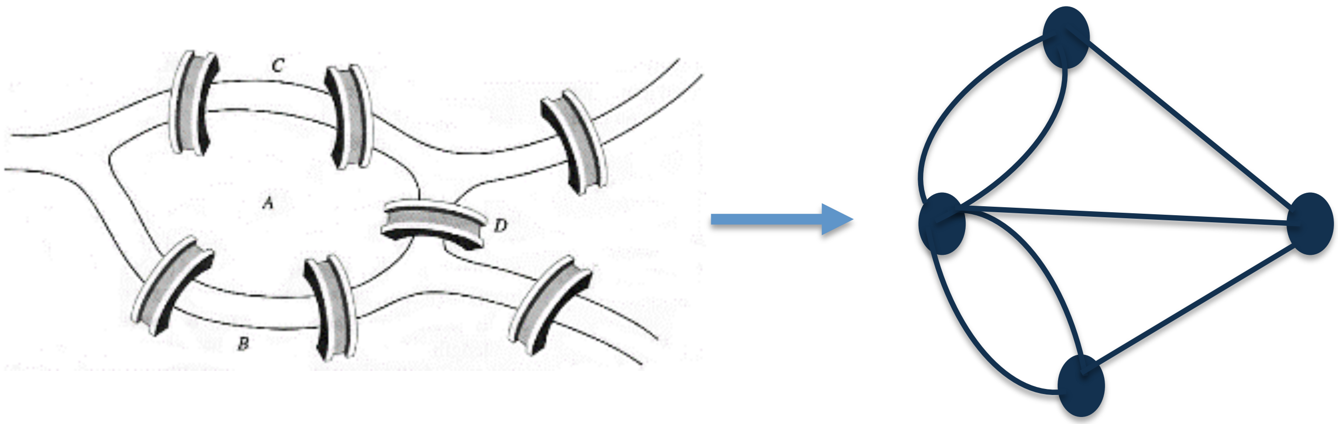

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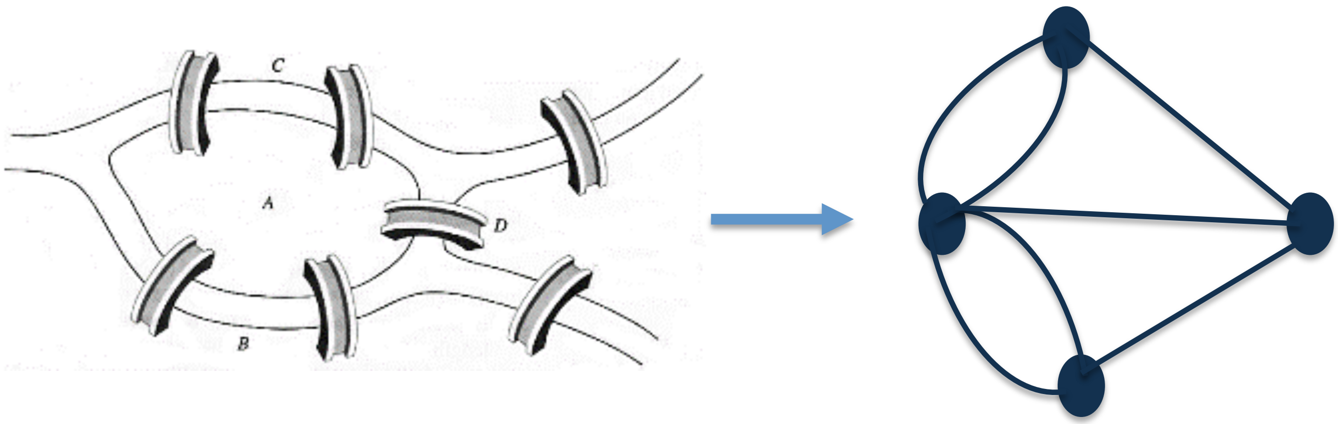
- Euler represented four distinct lands with four points (or nodes), and seven bridges with seven lines connecting those points.

Graph Theory



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- Euler represented four distinct lands with four points (or nodes), and seven bridges with seven lines connecting those points.

'Can you find a path that includes every edge exactly once?'
'Is the given graph traversable?'

Graph Theory

$$G = (V, E)$$

set of nodes (or vertices)

set of edges (or arc)

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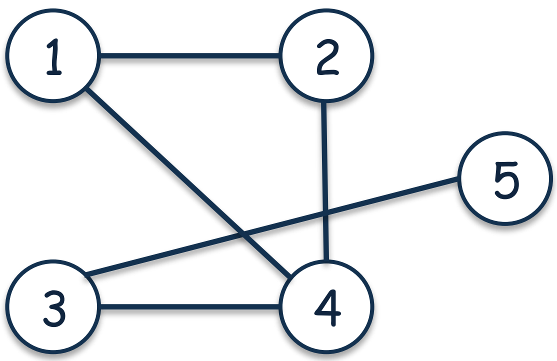

undirected graph

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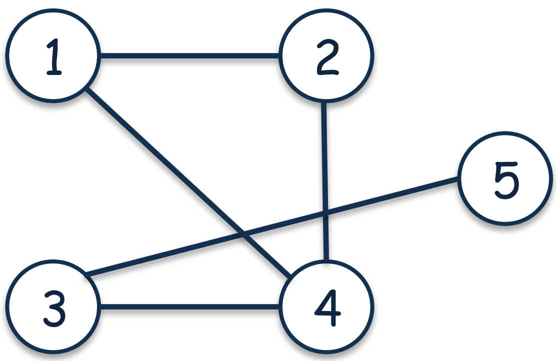

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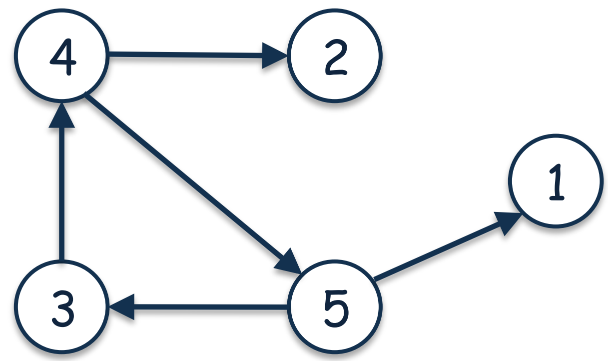

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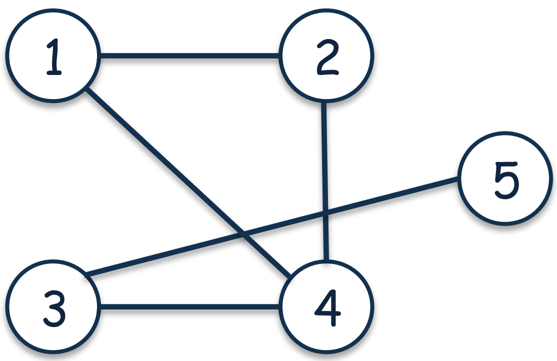

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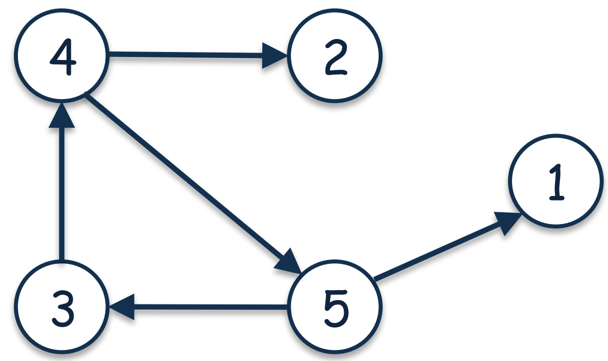

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deg(v)= # of edges at that vertex

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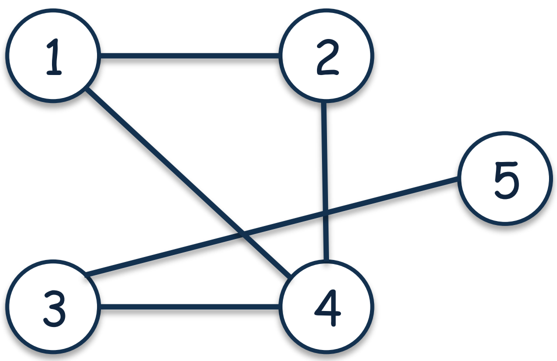

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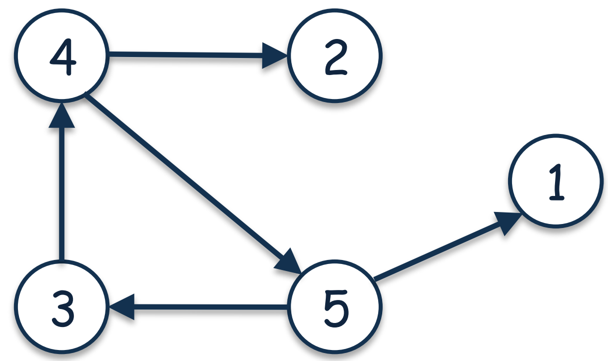

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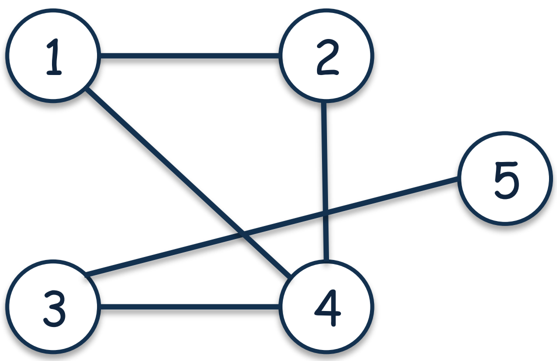

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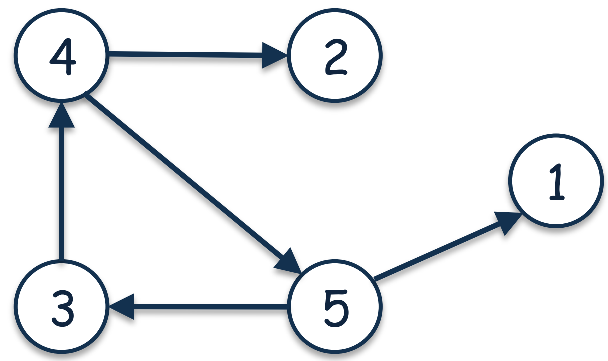

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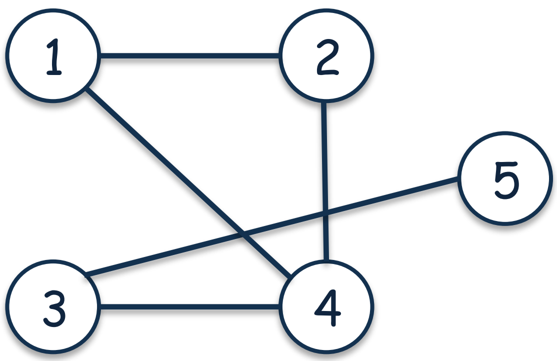

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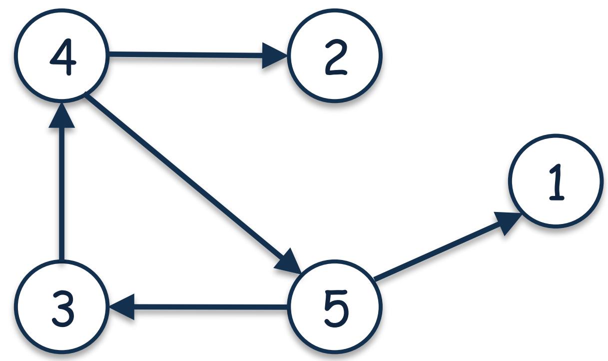

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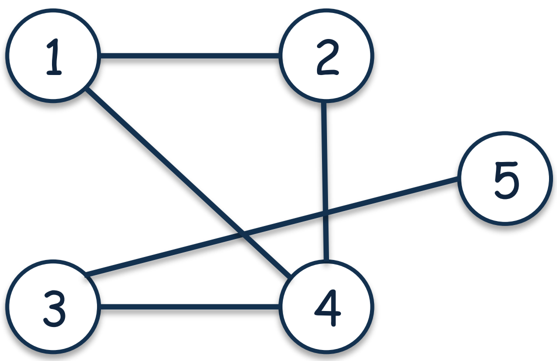

undirected graph

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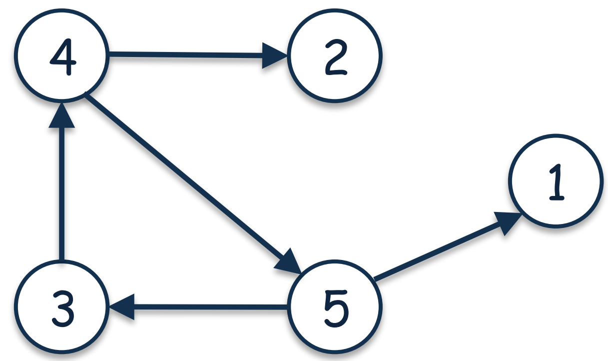
- a vertex v is called odd vertex if deg(v) is odd
- a vertex v is called even vertex if deg(v) is even



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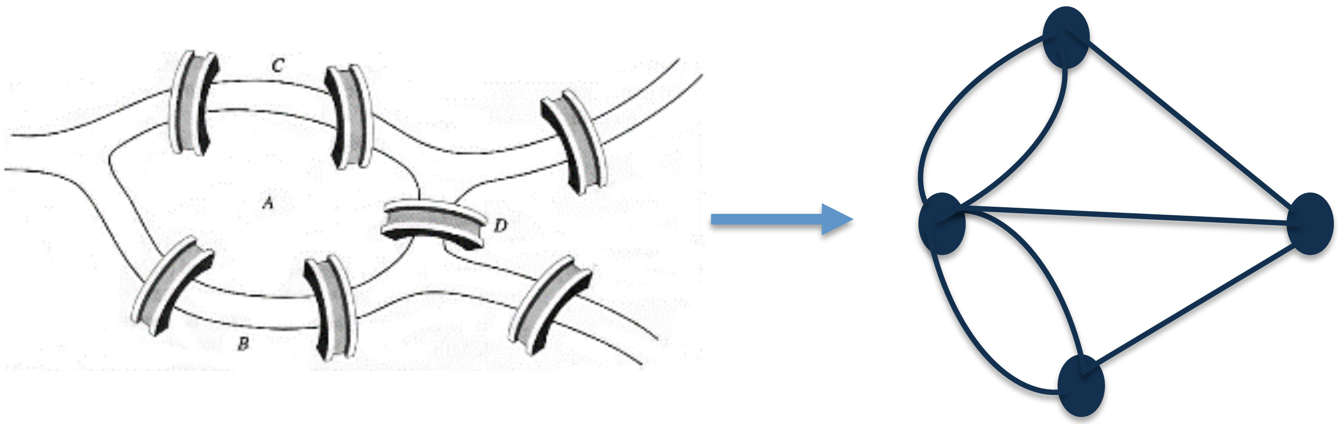

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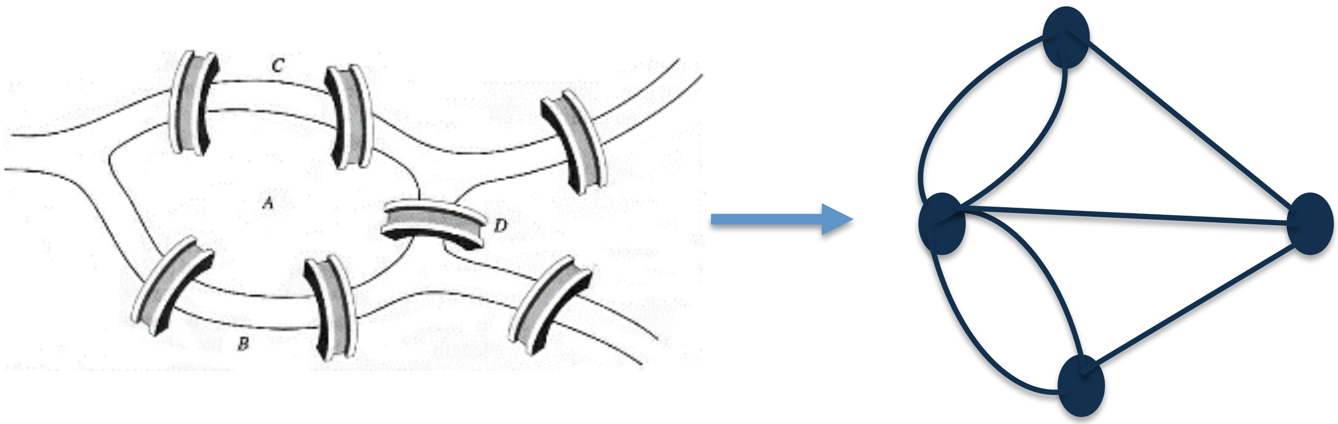

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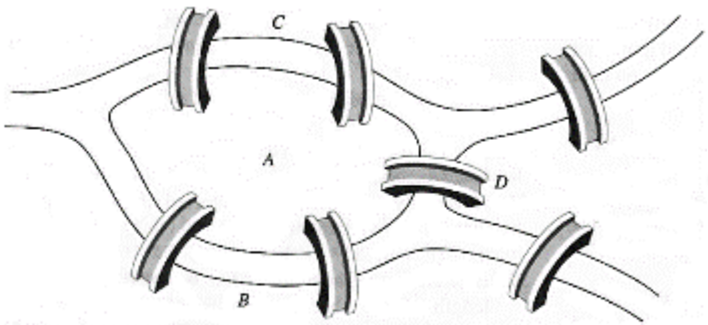

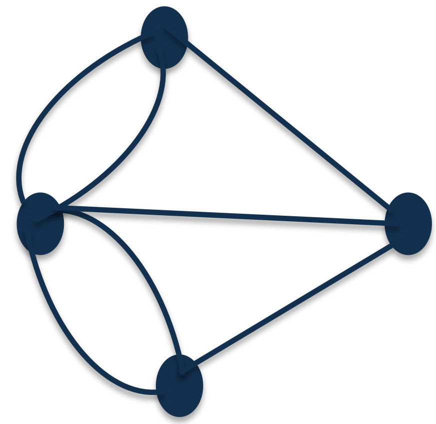

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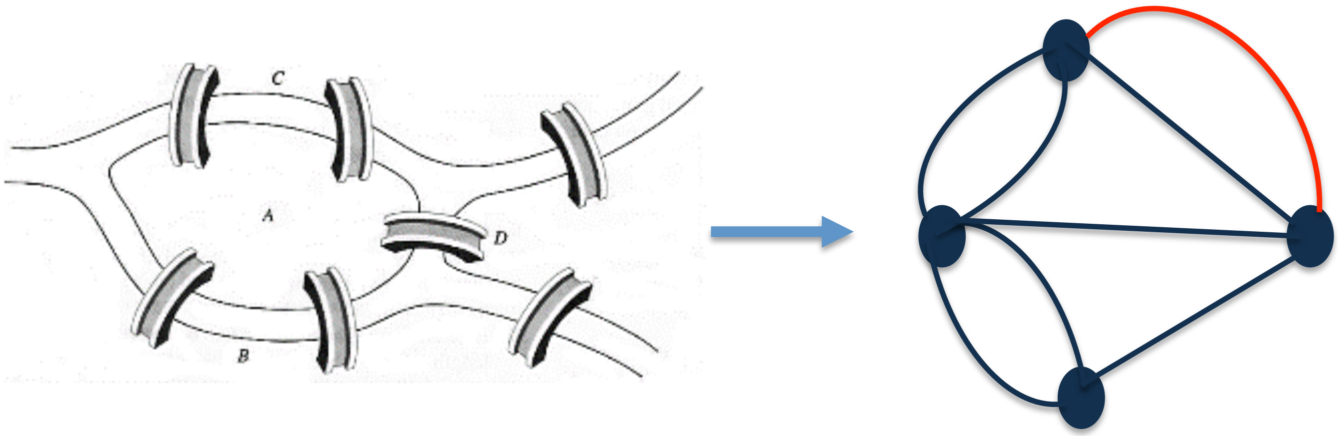

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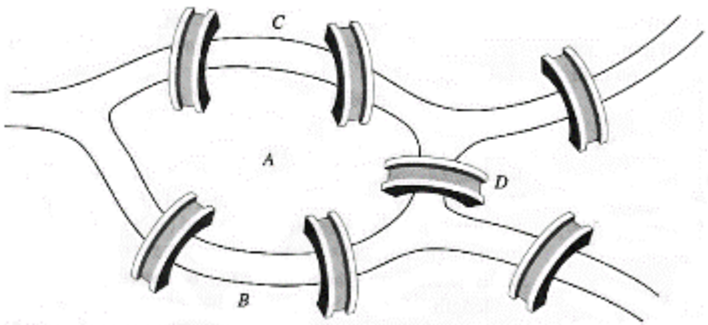

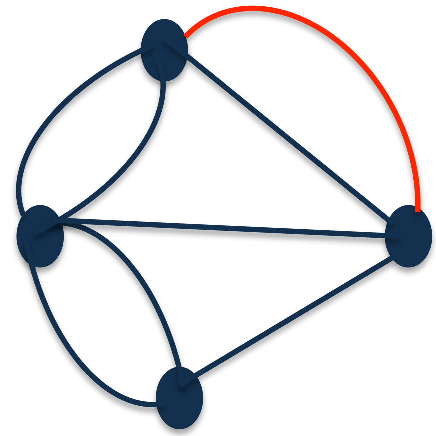

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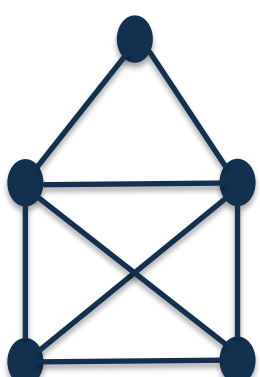
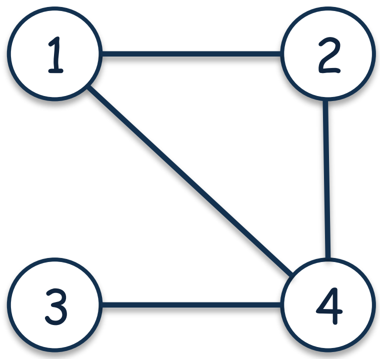
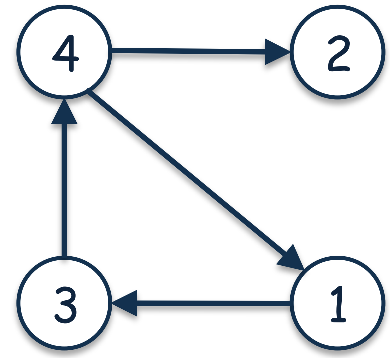


Can you draw an envelope without lifting your pen from the paper?

Graph Theory

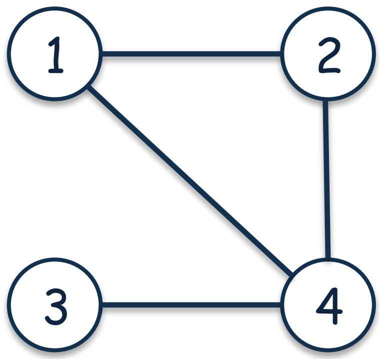


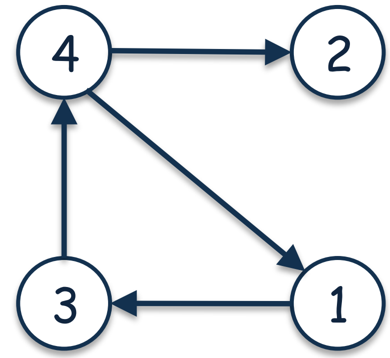
Adjacency List



Adjacency List

Graph Theory





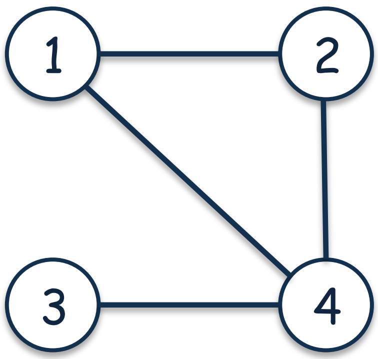
Adjacency List

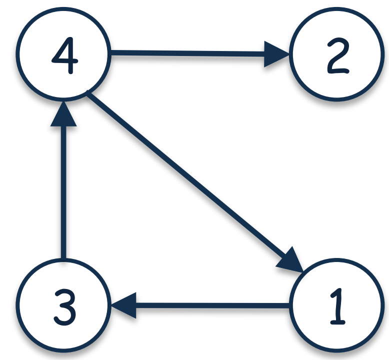
1 - 2,4
2 - 1,4
3 - 4
4 - 1,2,3

Adjacency List

1 - 3
2 -
3 - 4
4 - 1,2

Graph Theory





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Adjacency Matrix

	1	2	3	4
1	0	1	0	1
2	1	0	0	1
3	0	0	0	1
4	1	1	1	0

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Graph Theory

Adjacency List | **Adjacency Matrix**

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• space	O(|E|+|V|)	$O(\|V\|^2)$

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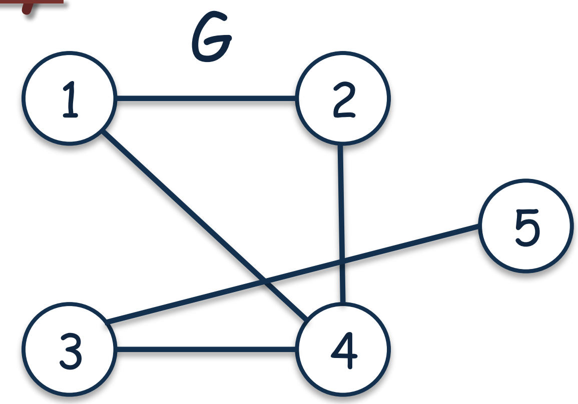
If graph is sparse, use adjacency list;
if graph is dense, use adjacency matrix

Graph Theory



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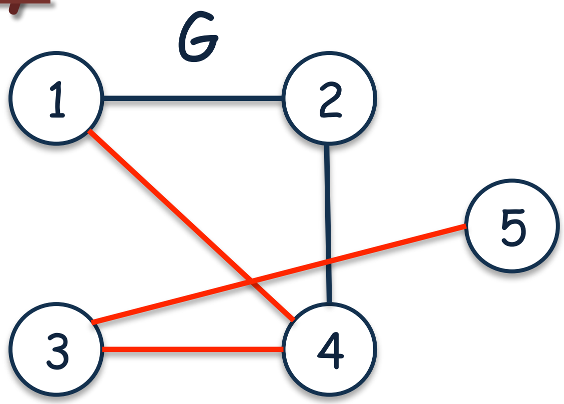

5, 3, 4, 1 is a simple path in G

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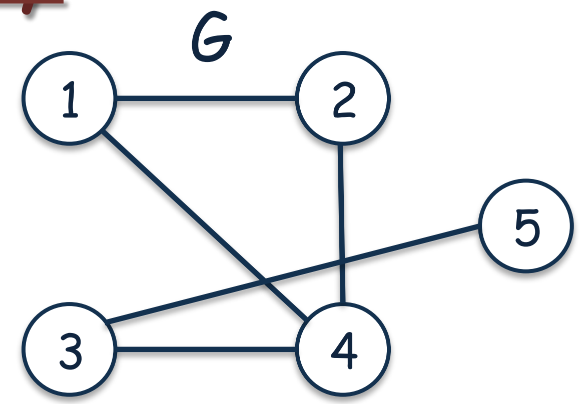

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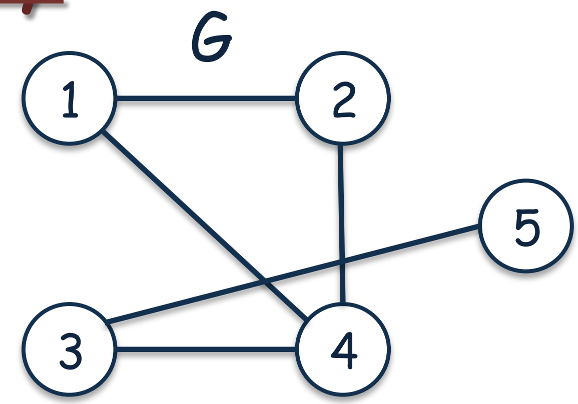

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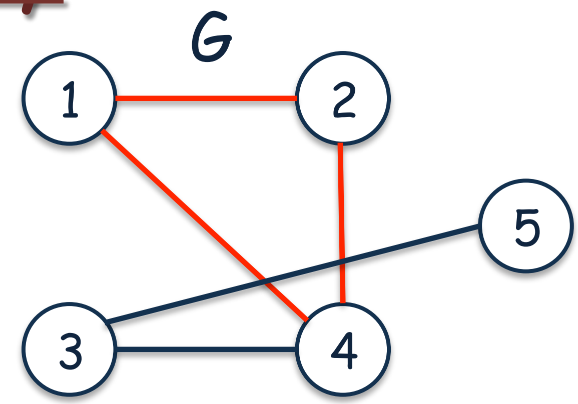
Graph Theory

4, 1, 2, 4 is a simple cycle in G



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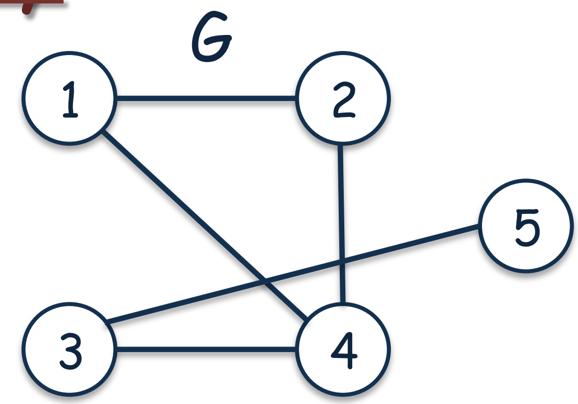

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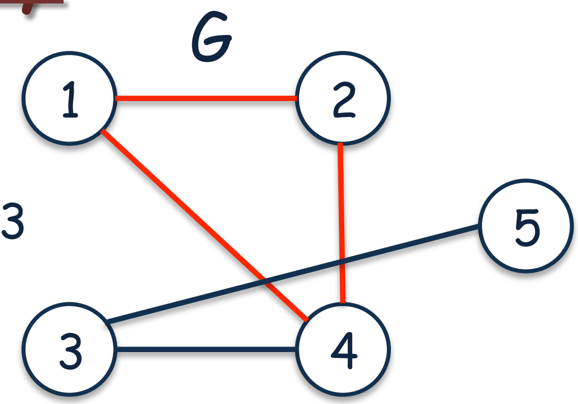

4, 1, 2, 4 is a simple cycle with length 3

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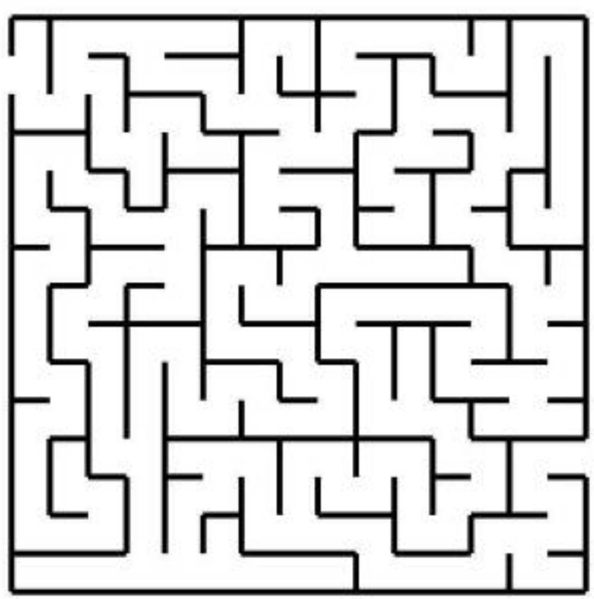

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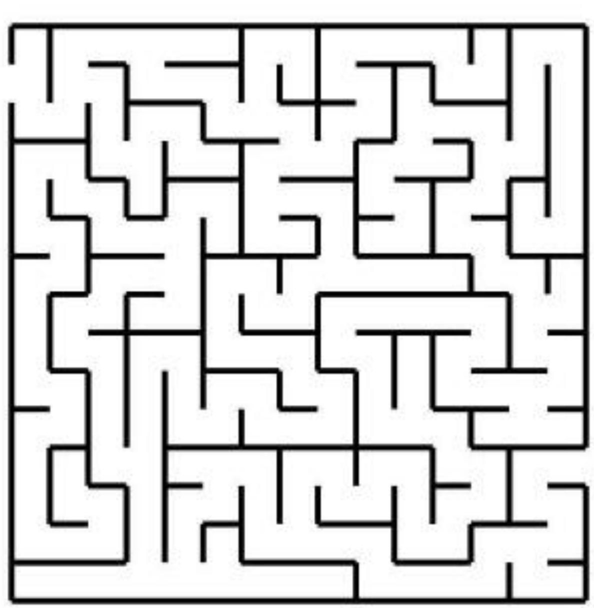

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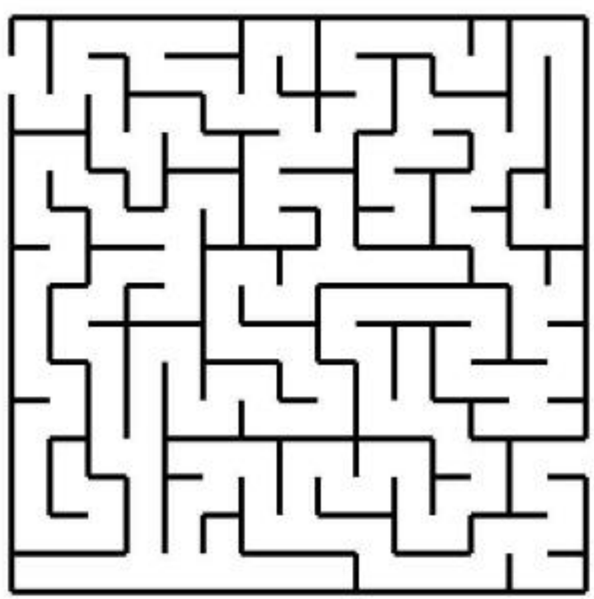

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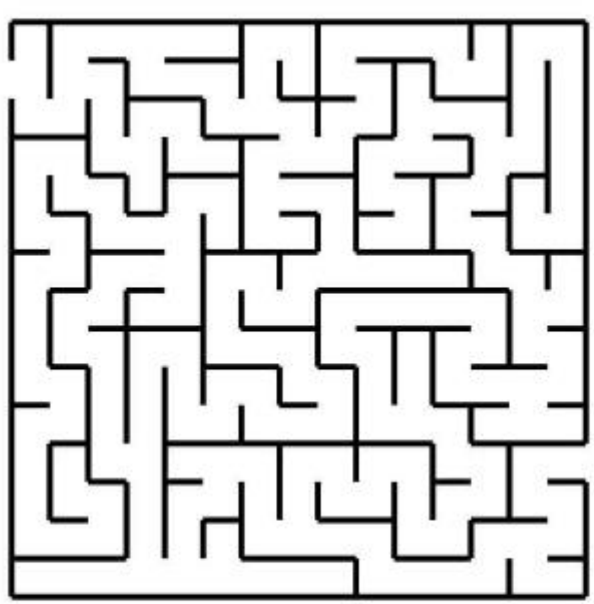

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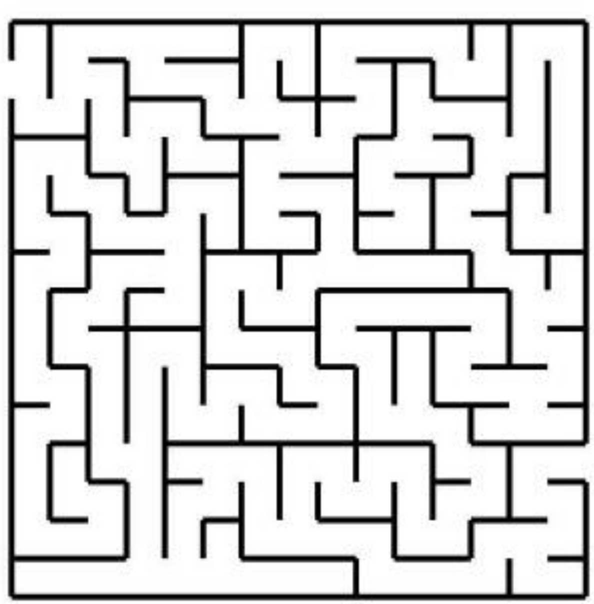

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- For correctness, we do the traversal in a way that we get out of the maze

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state of each vertex changes from left to right

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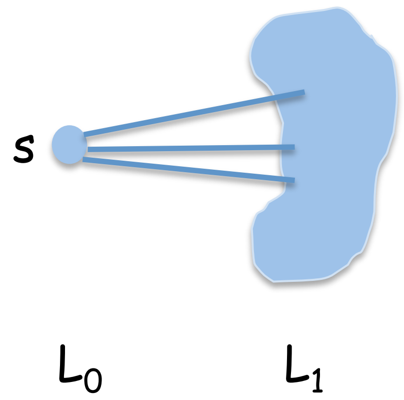
- instead of going deep in a graph, just go in cross-wise fashion

- explore the graph outward from a starting point (a node s) in all possible directions – add one layer of nodes at a time



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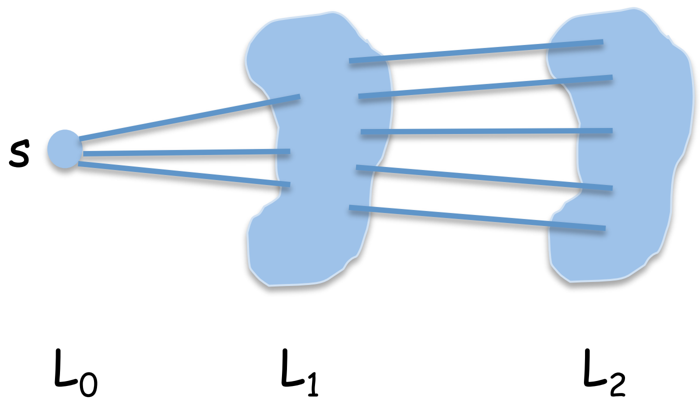

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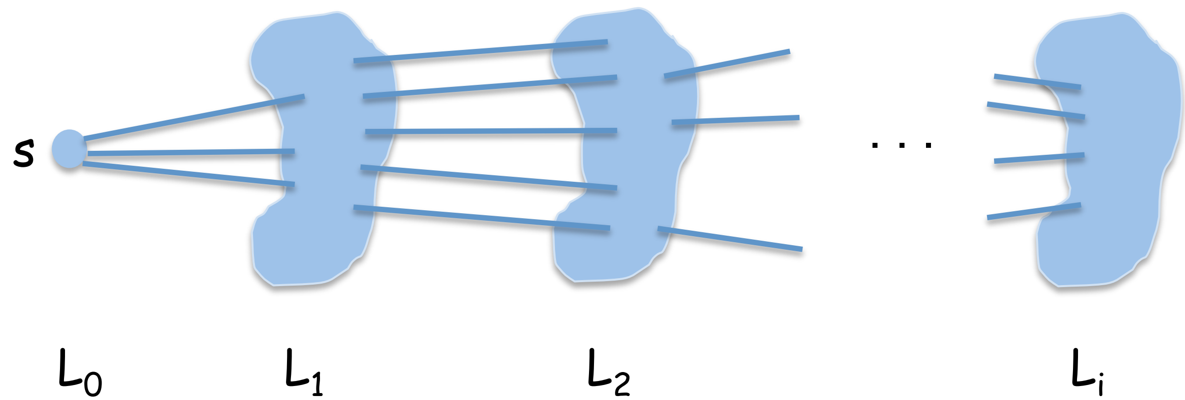

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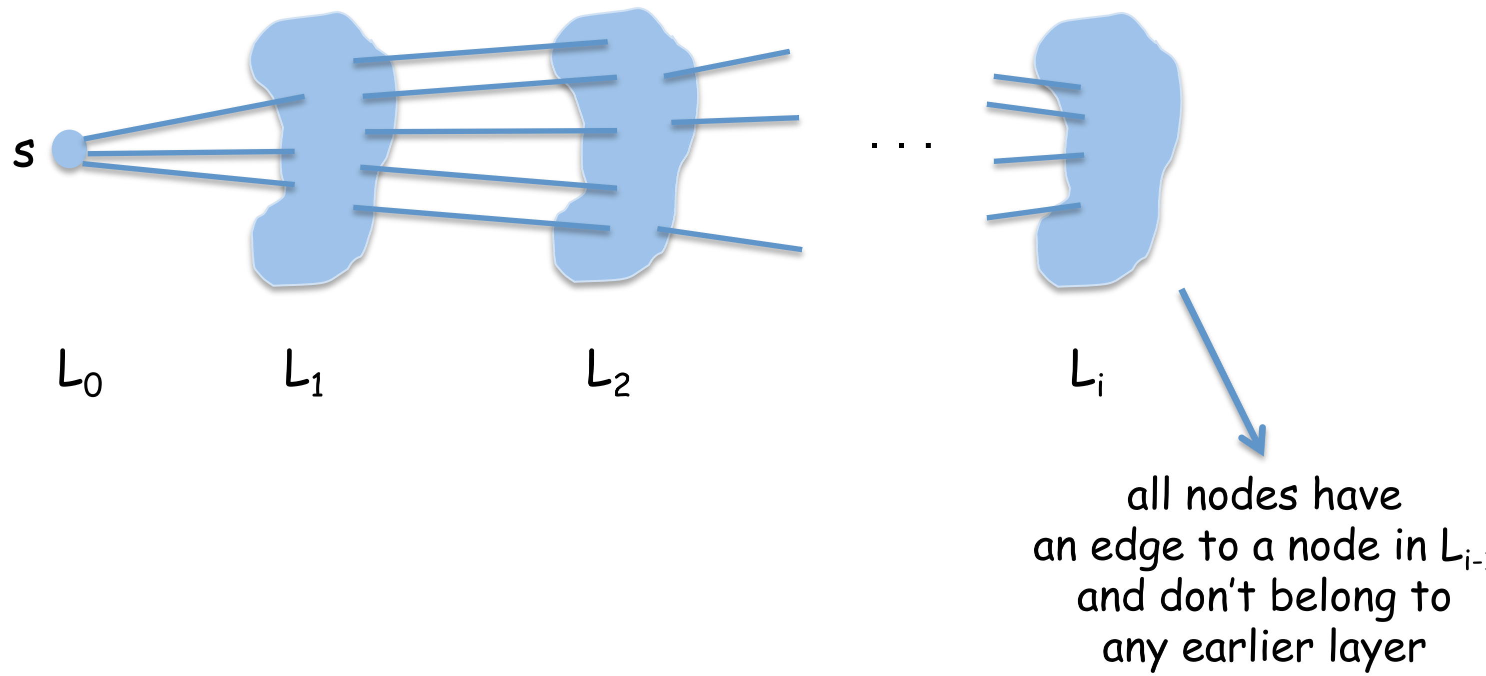

Breadth First Search

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The length of the shortest path from s to u

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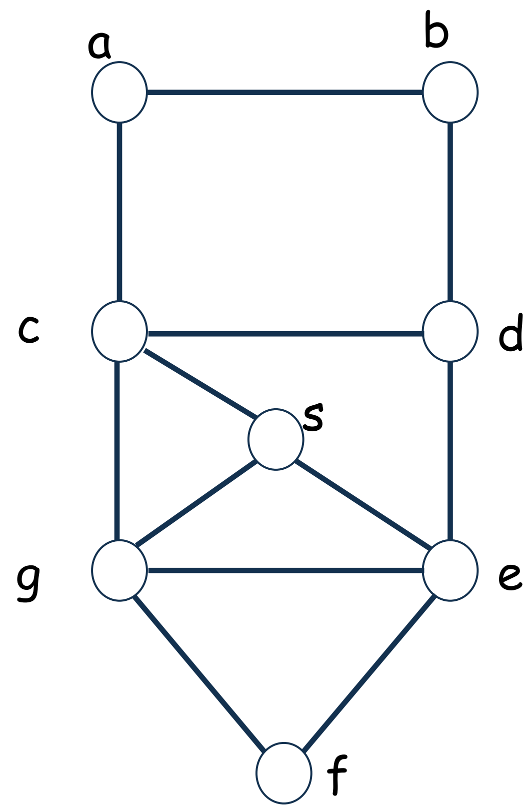

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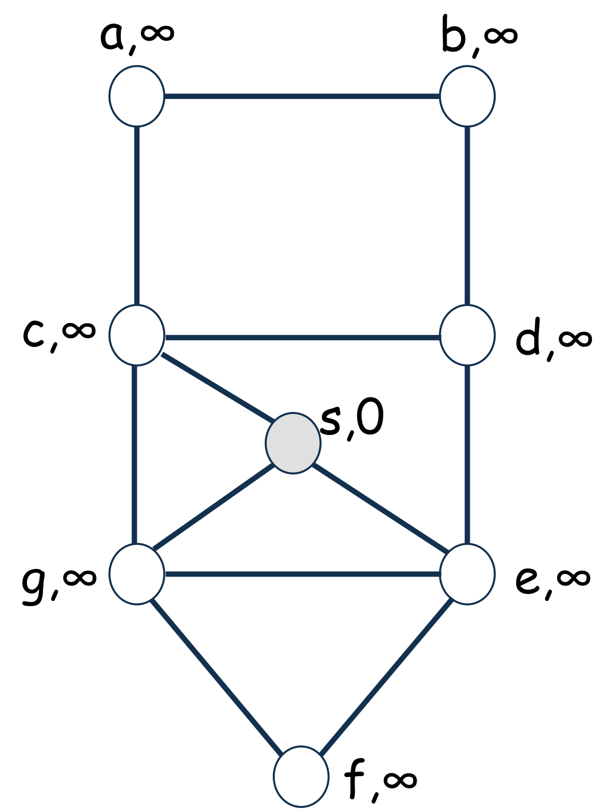

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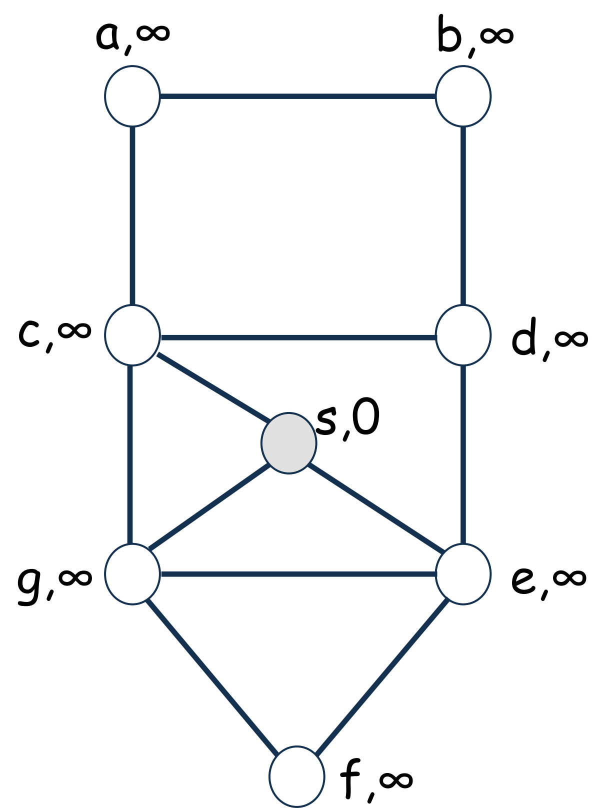

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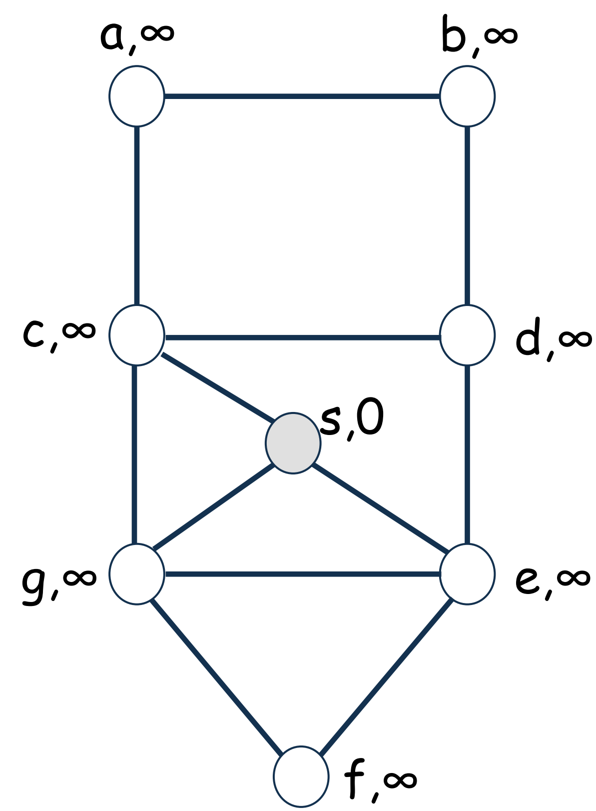

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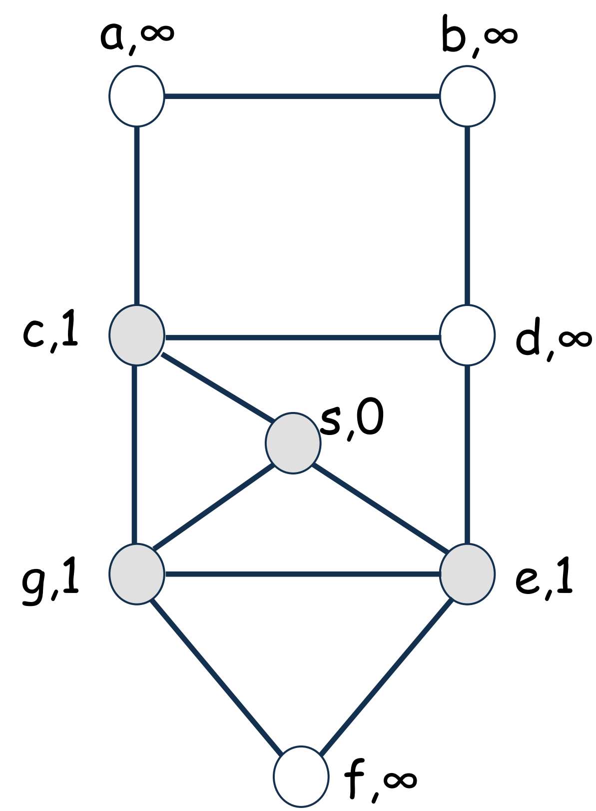

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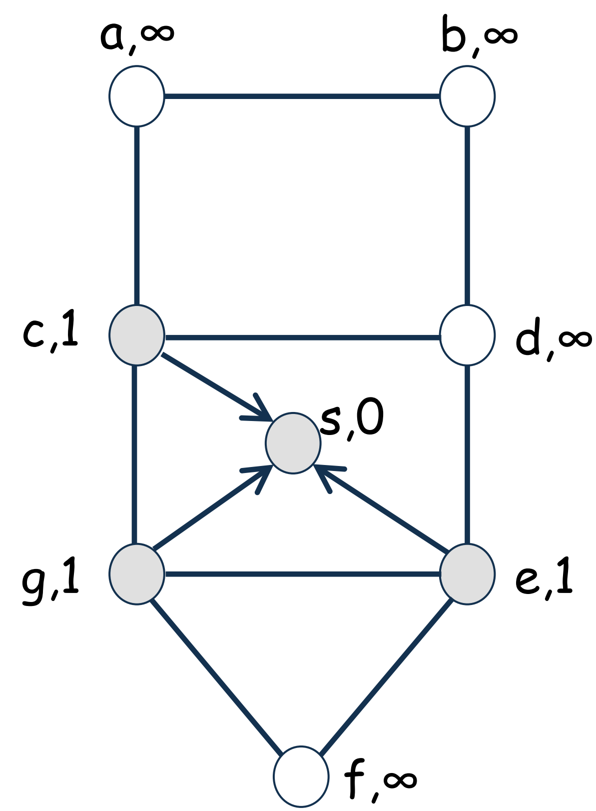

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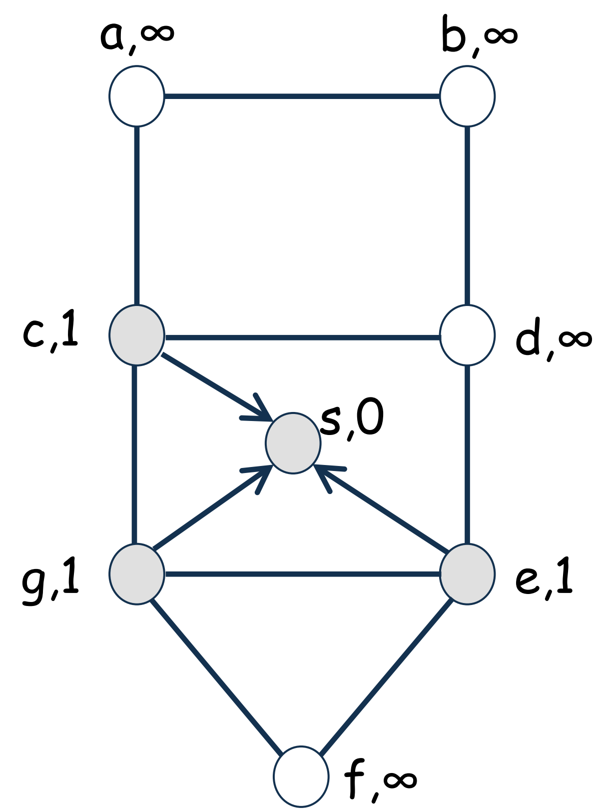

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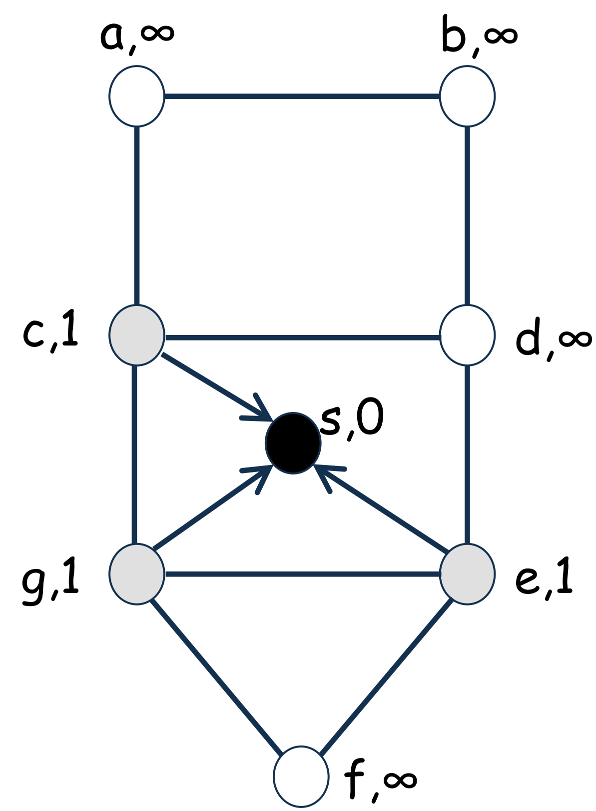

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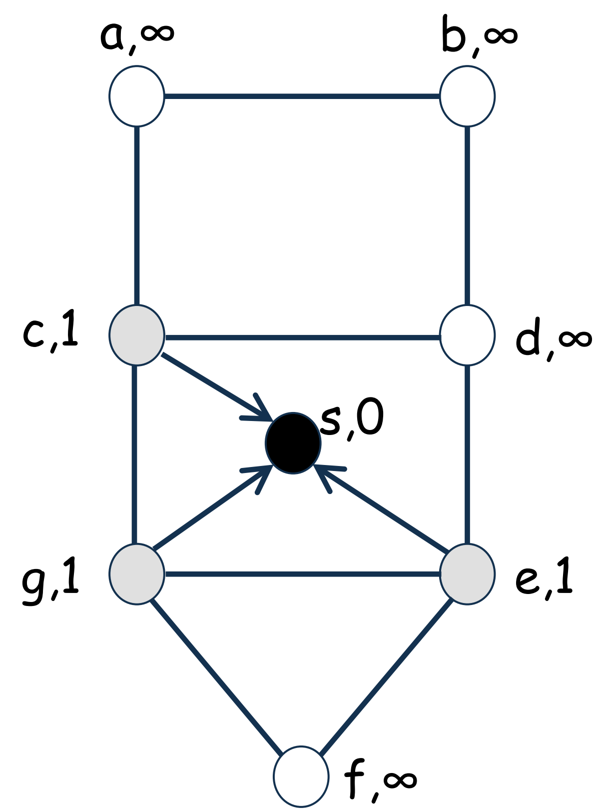

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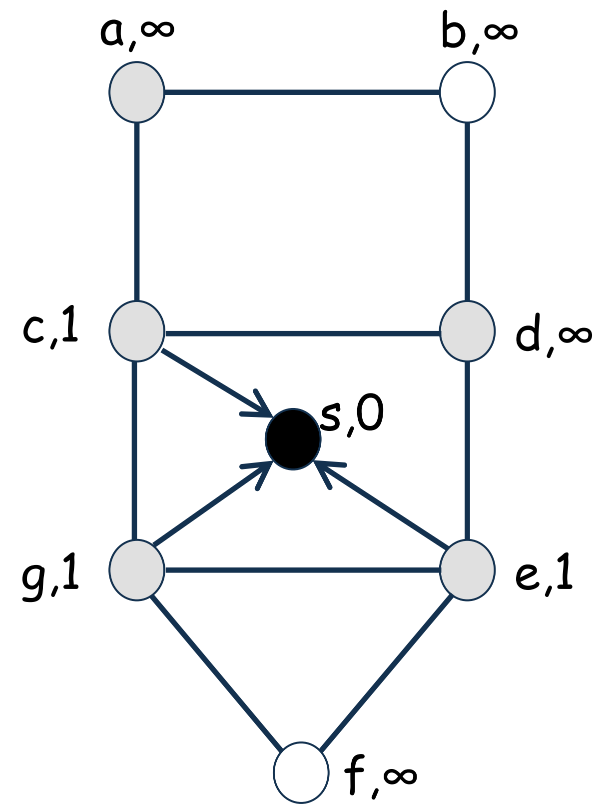

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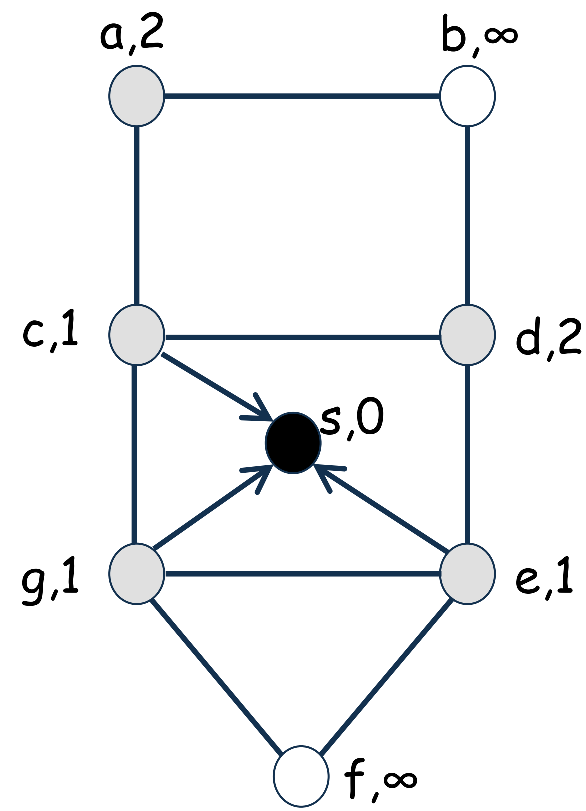

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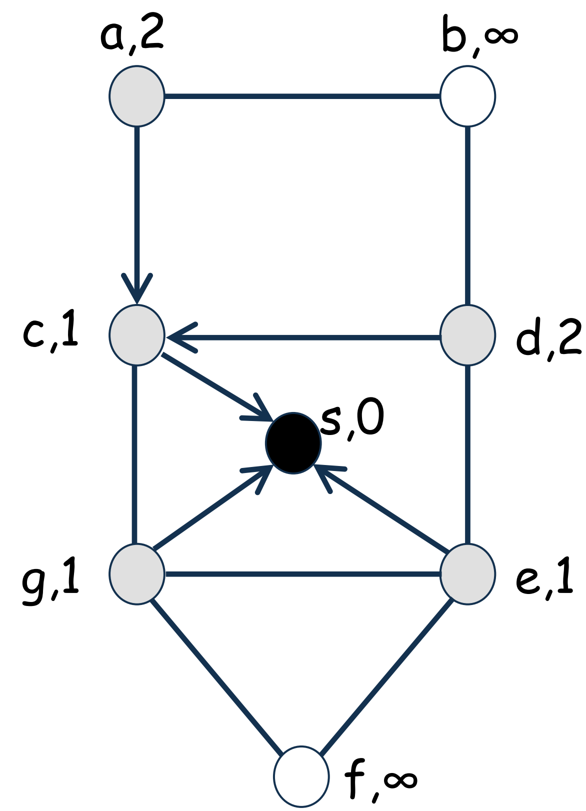

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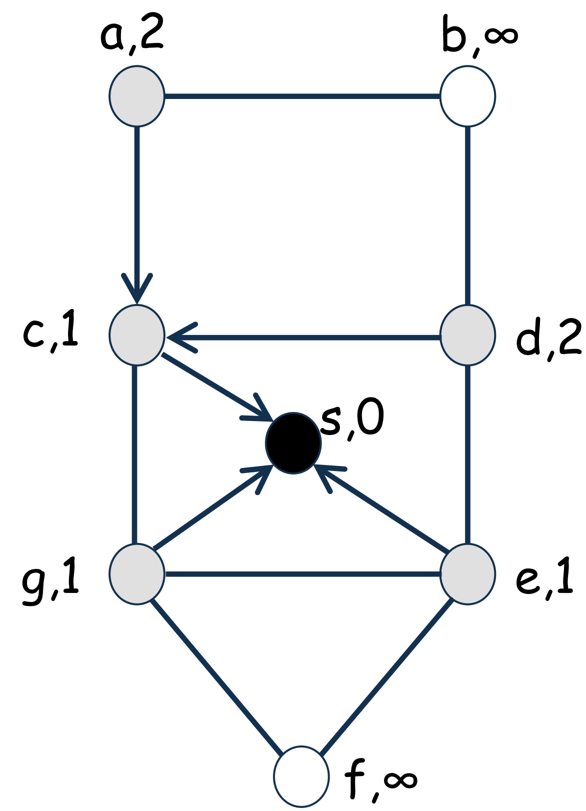

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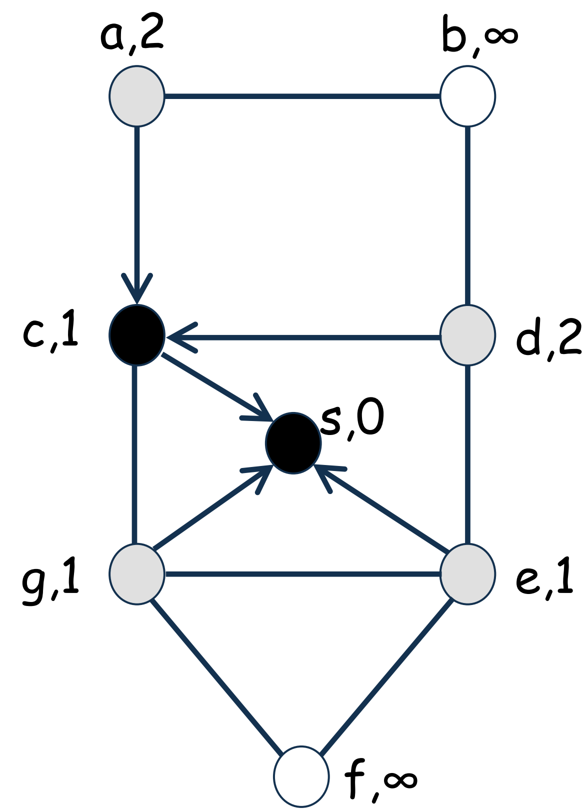

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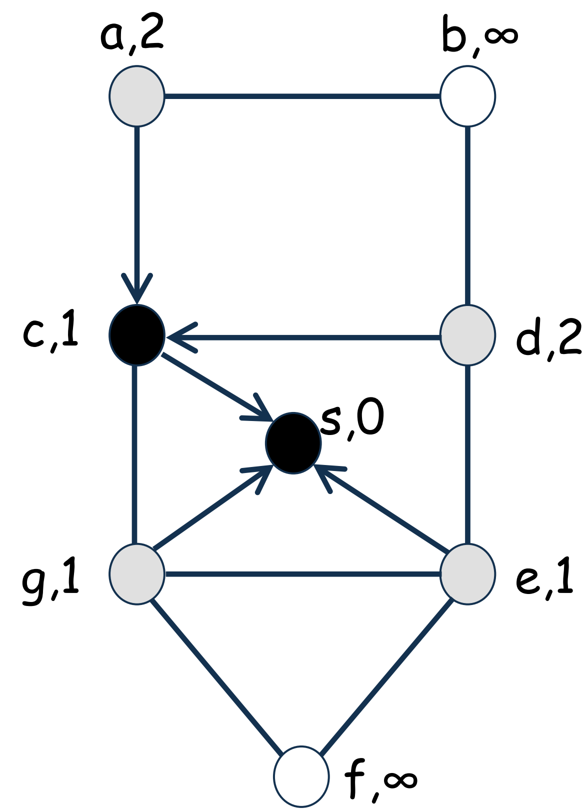

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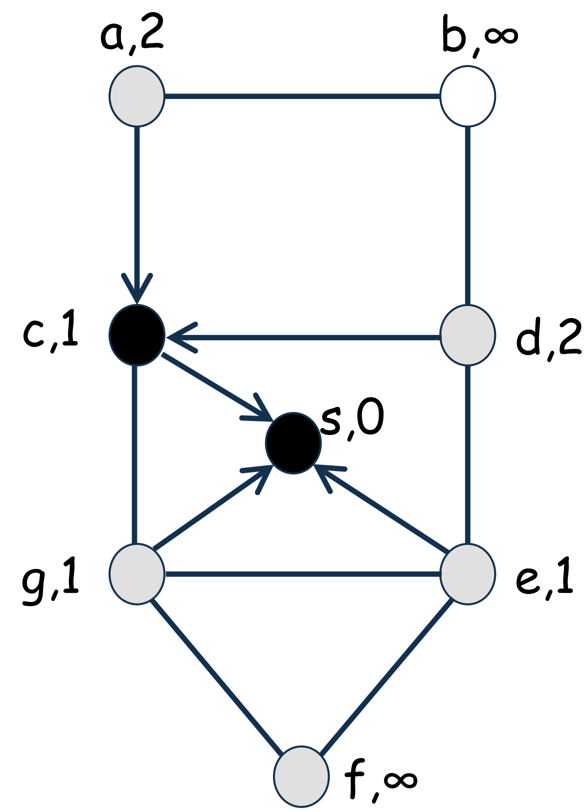

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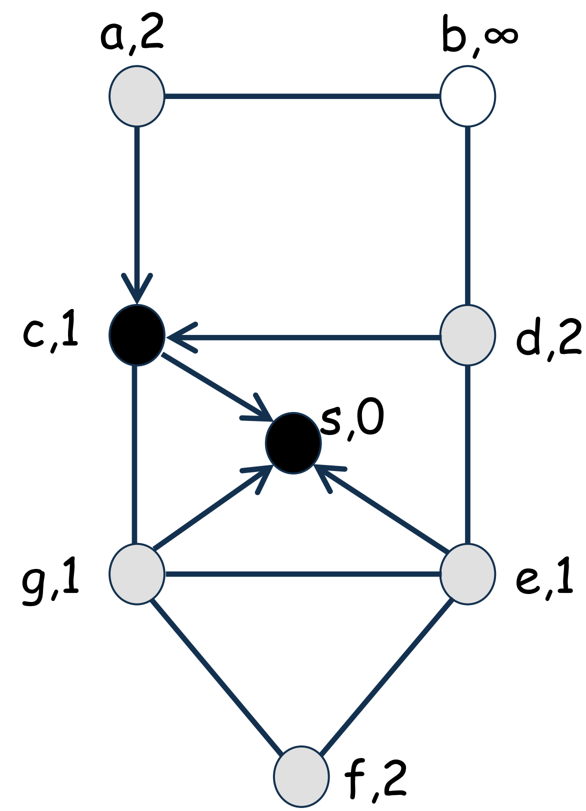

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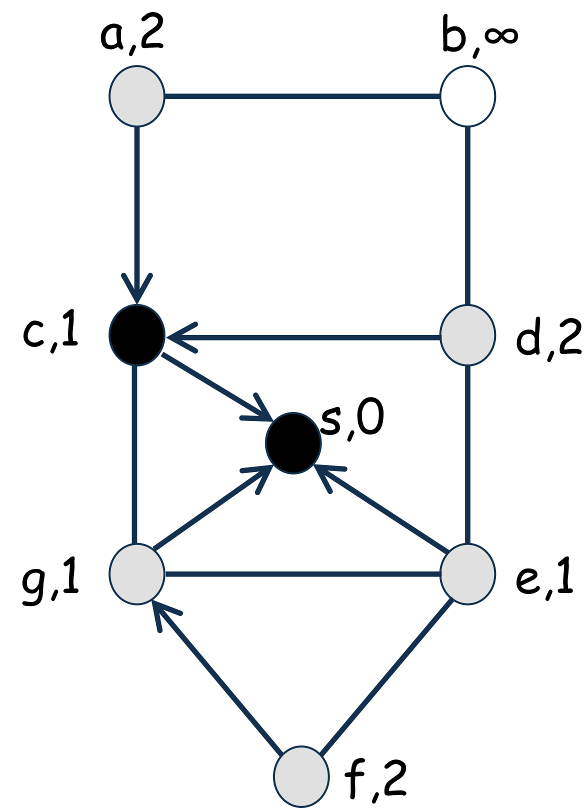

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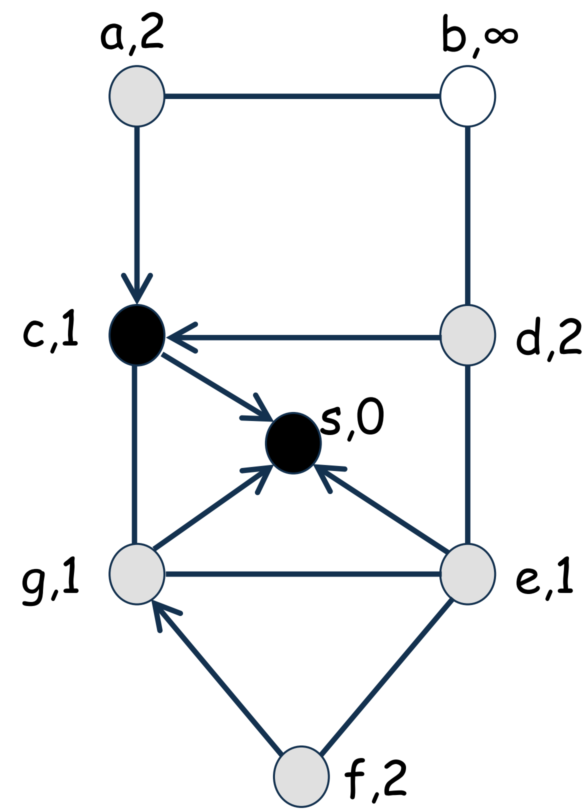

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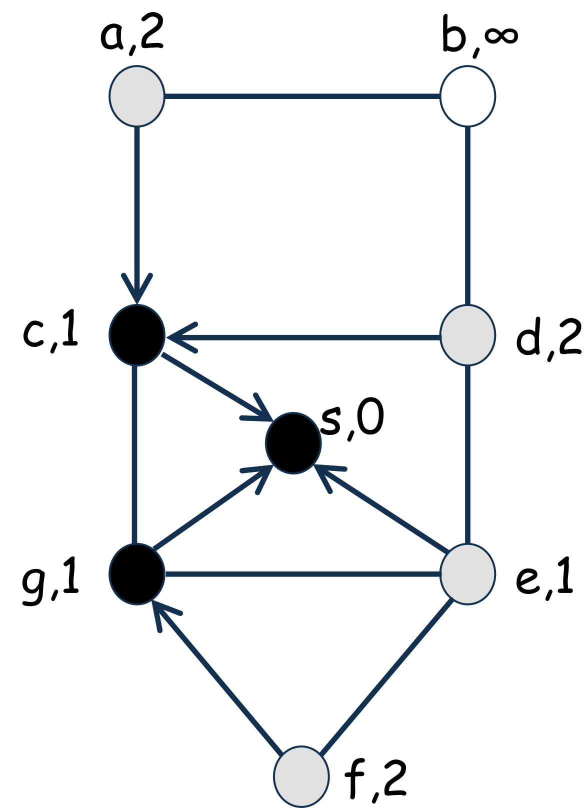

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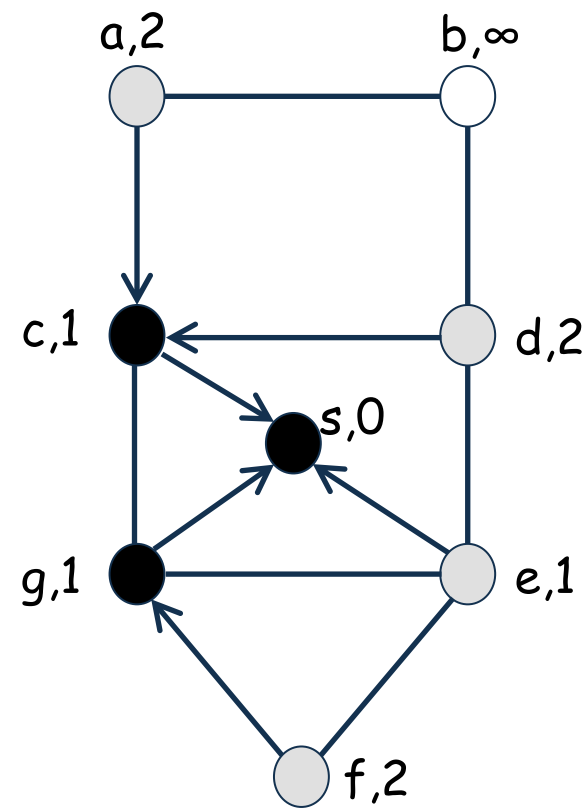

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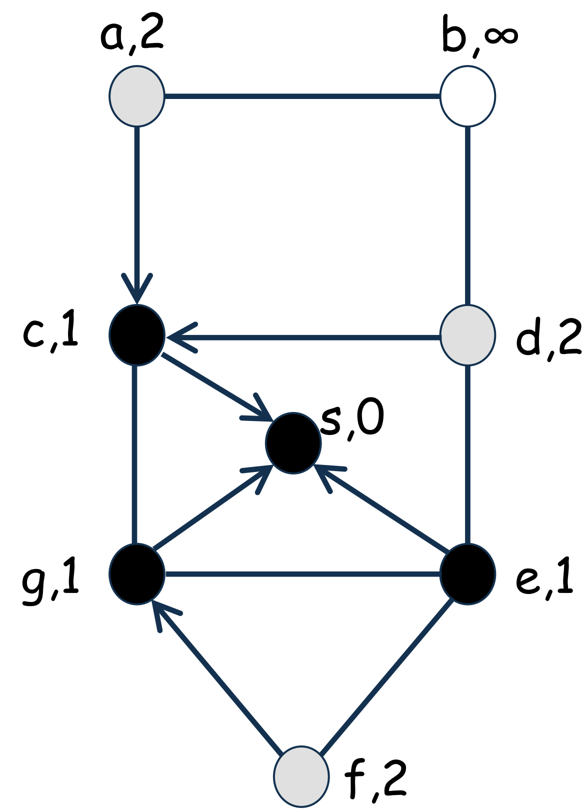

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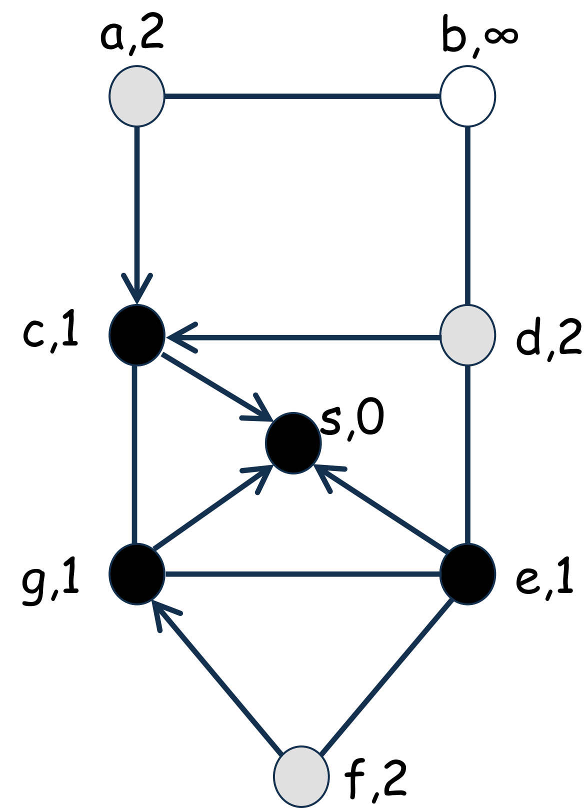

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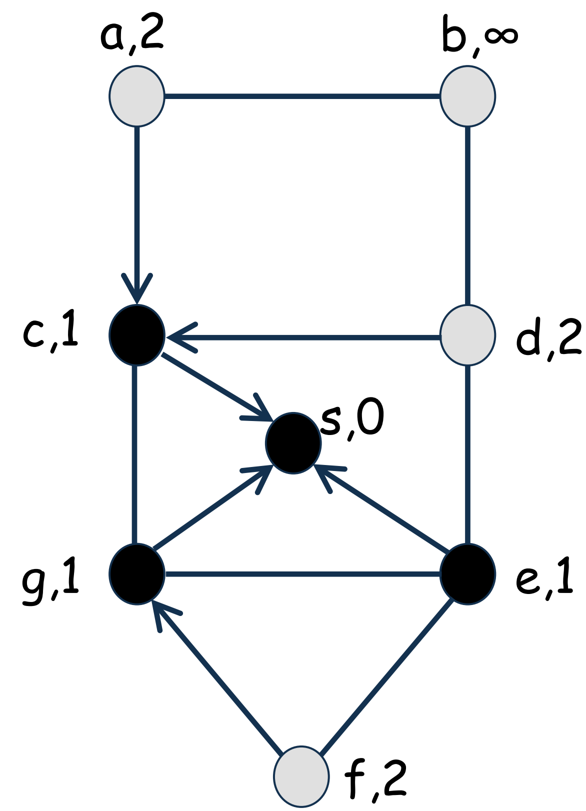

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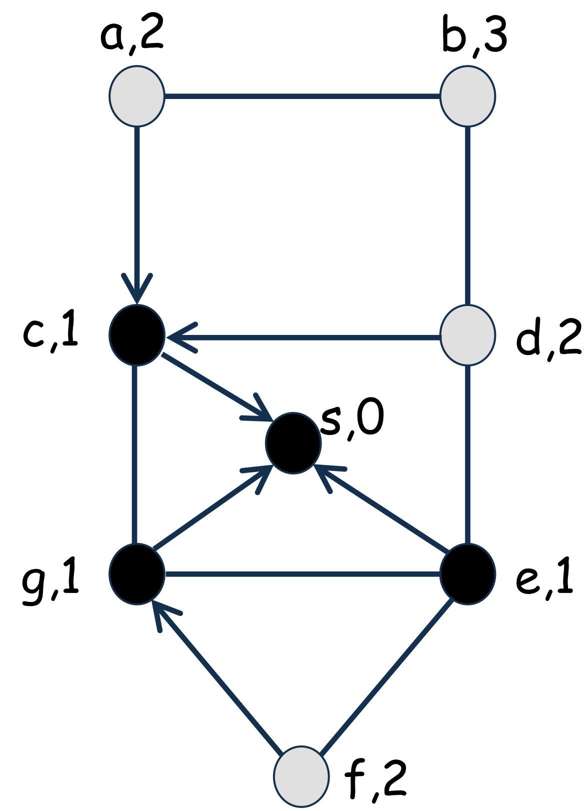

a Q = {d f}

Breadth First Search

BFS(G,s)

```
for each vertex u of V
        u.color = white
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s.color = gray
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while Q ≠ ∅
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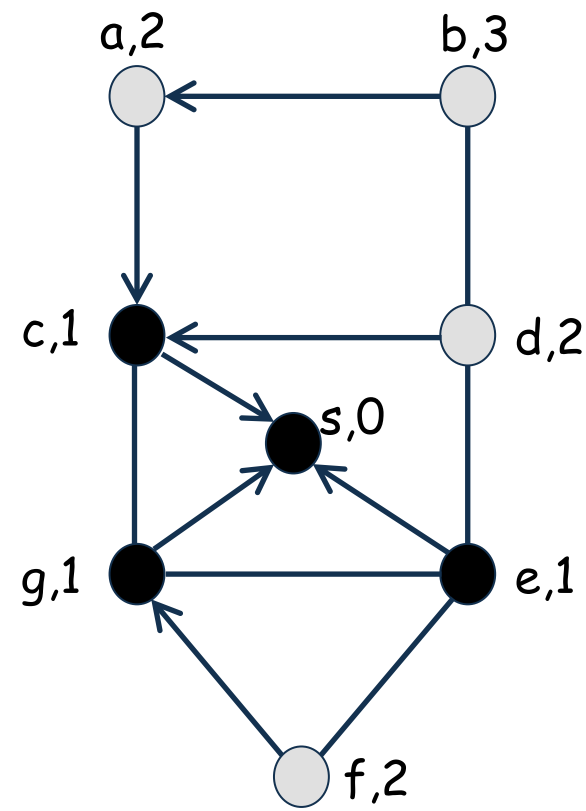
      for each v∈Adj(u)
            if v.color = white
                  v.color = gray
                  v.dis = u.dis + 1
                  v.par = u
                  Enqueue(Q,v)
      u.color = black
```



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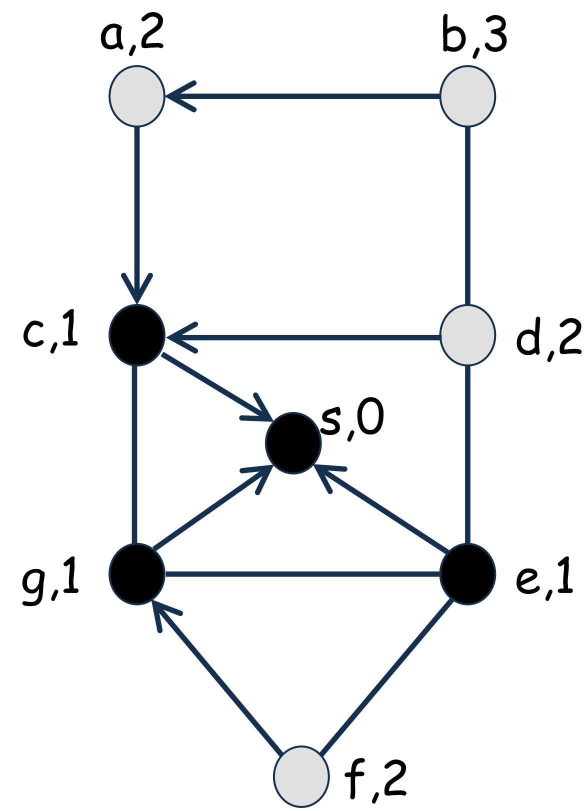

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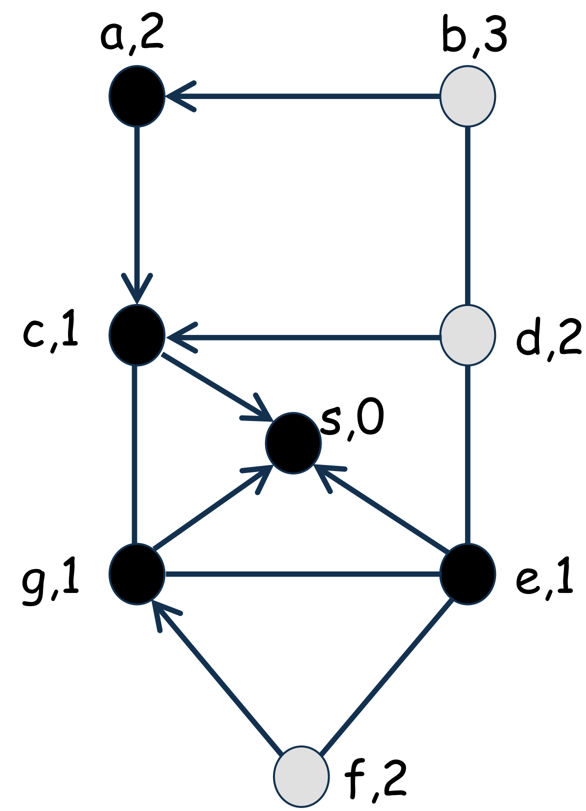

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Breadth First Search

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                        v.color = gray
                        v.dis = u.dis + 1
                        v.par = u
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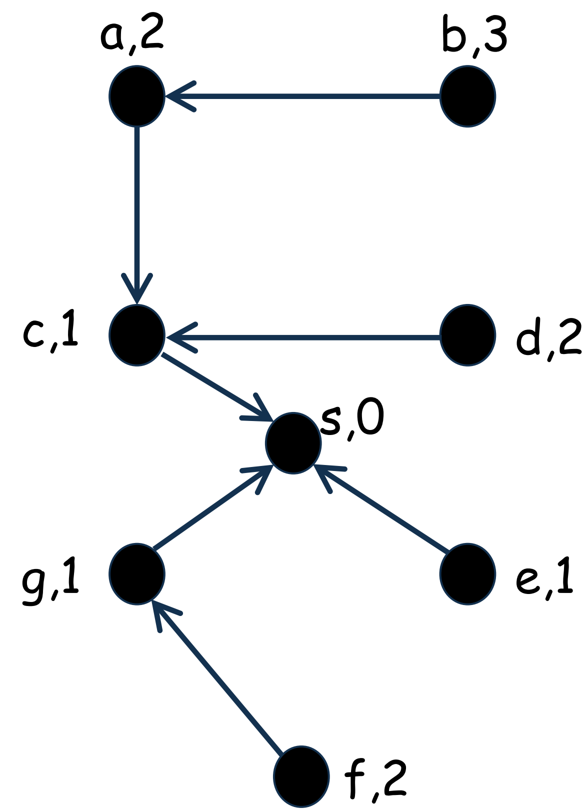
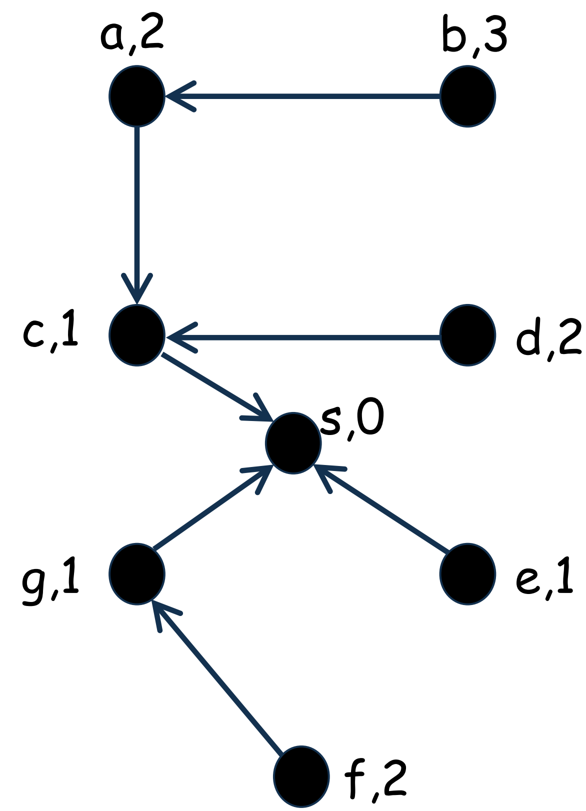


b Q = { }

Breadth First Search

BFS(G,s)

```
for each vertex u of V          ⎫
        u.color = white         ⎬ O(|V|)
        u.dis = ∞               ⎪
        u.par = nil             ⎭
s.color = gray                  ⎫
s.dis = 0                       ⎬ O(1)
initialize an empty queue Q     ⎪
Enqueu(Q,s)                     ⎭
while Q ≠ ∅
      u = Dequeu(Q)
      for each v∈Adj(u)
            if v.color = white
                  v.color = gray
                  v.dis = u.dis + 1
                  v.par = u
                  Enqueue(Q,v)
      u.color = black
```



b Q = { }

Breadth First Search

BFS(G,s)

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for each vertex u of V
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            if v.color = white
                  v.color = gray
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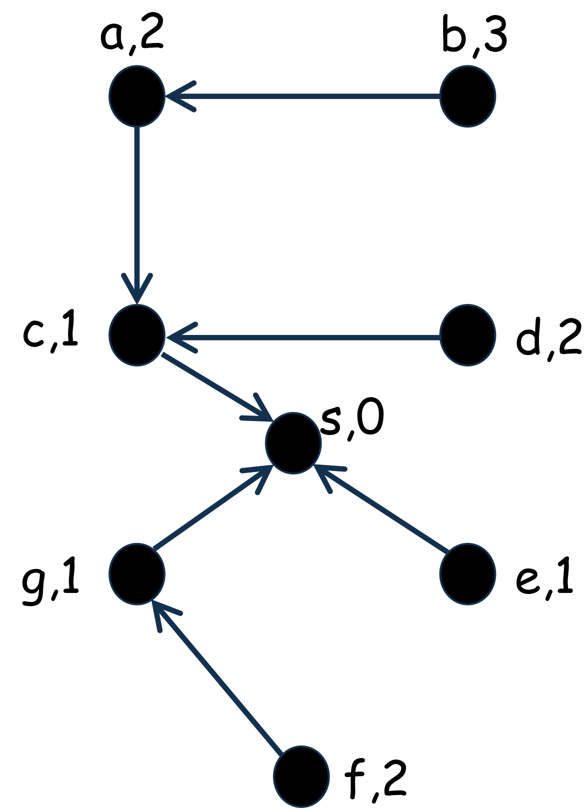
                  v.par = u
                  Enqueue(Q,v)
      u.color = black
```

O(|V|)

O(1)

O(|E|)



b Q = { }

Breadth First Search

BFS(G,s)

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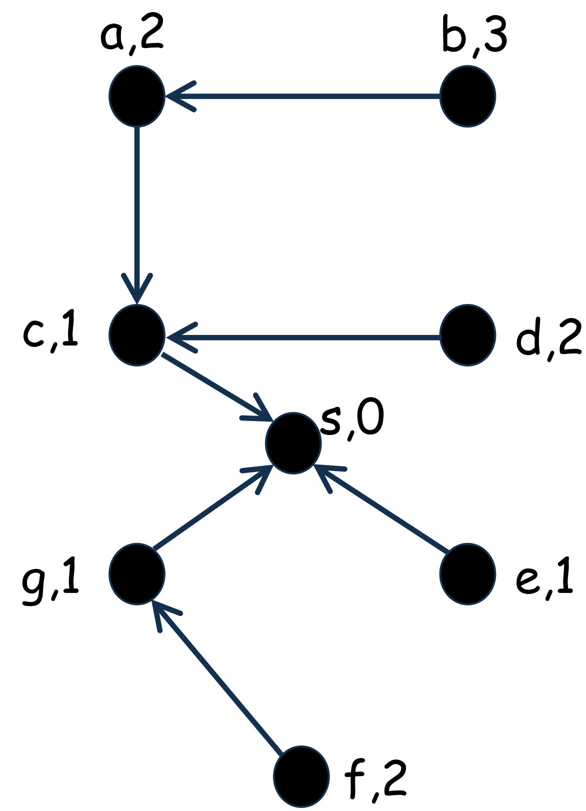
                  v.color = gray
                  v.dis = u.dis + 1
                  v.par = u
                  Enqueue(Q,v)
      u.color = black
```

$\left.\right\}$ O(|V|) (first four lines)

$\left.\right\}$ O(1) (s.color through Enqueu(Q,s))

$\left.\right\}$ O(|E|) (for each loop)

total O(|V| + |E|)



b Q = { }

Breadth First Search

BFS(G,s)

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                        v.par = u
                        Enqueue(Q,v)
        u.color = black
```

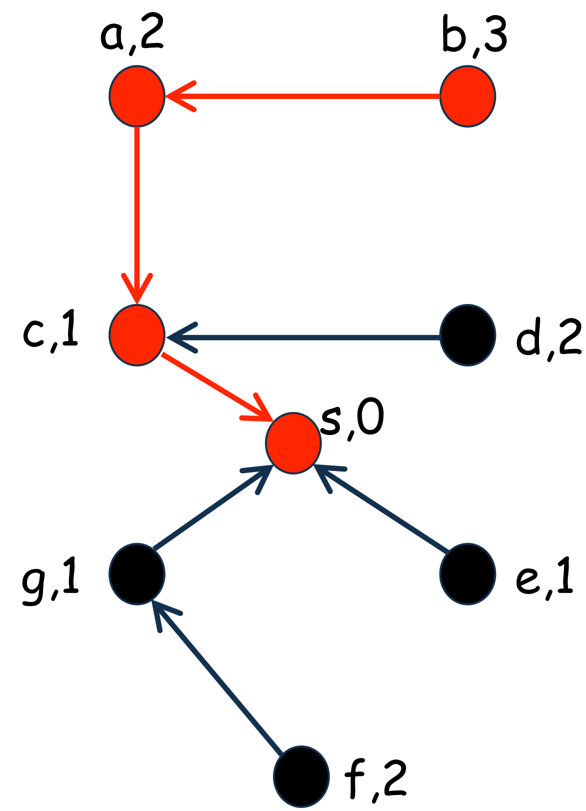
O(|V|)

O(1)

O(|E|)

total O(|V| + |E|)

parent pointer used to find the shortest path



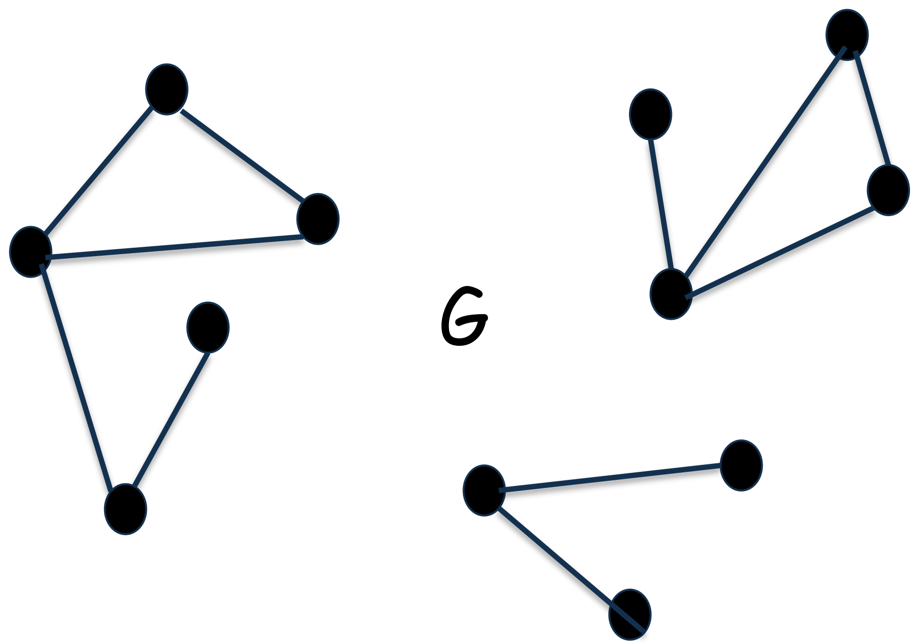
b Q = { }

Connected Components

- a connected component is a maximal subgraph where there is a path between any two nodes of it

Connected Components

- a connected component is a maximal subgraph where there is a path between any two nodes of it
- a graph can be made up of seperate connected components



Connected Components

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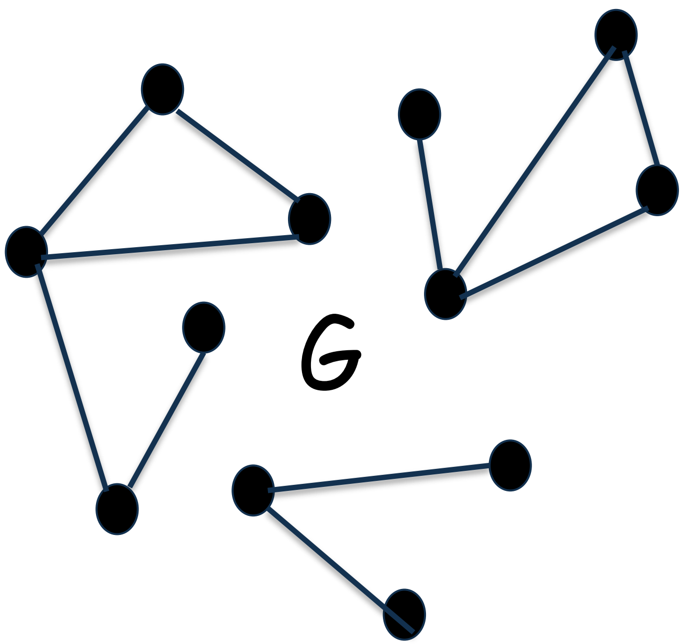

- find the number of connected components of a given graph (use BFS)

Connected Components

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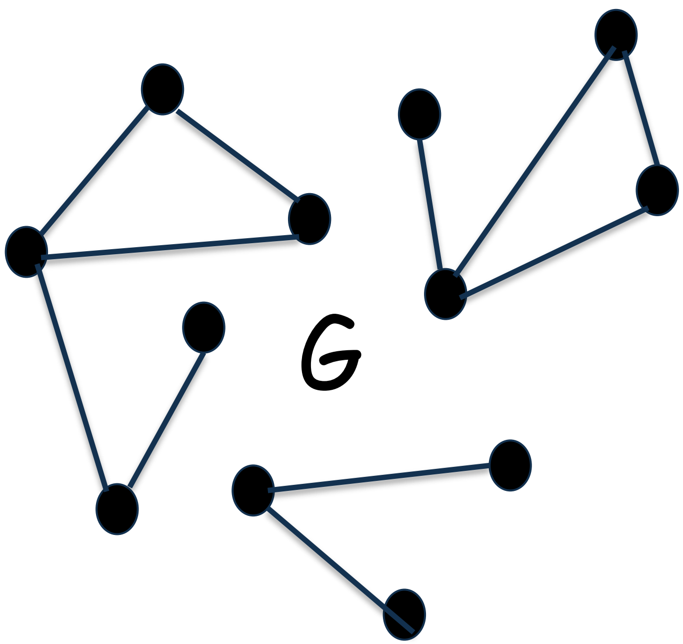

- find the number of connected components of a given graph (use BFS)
- start from the first vertex; any vertex we have discovered during this search should be part of same component.

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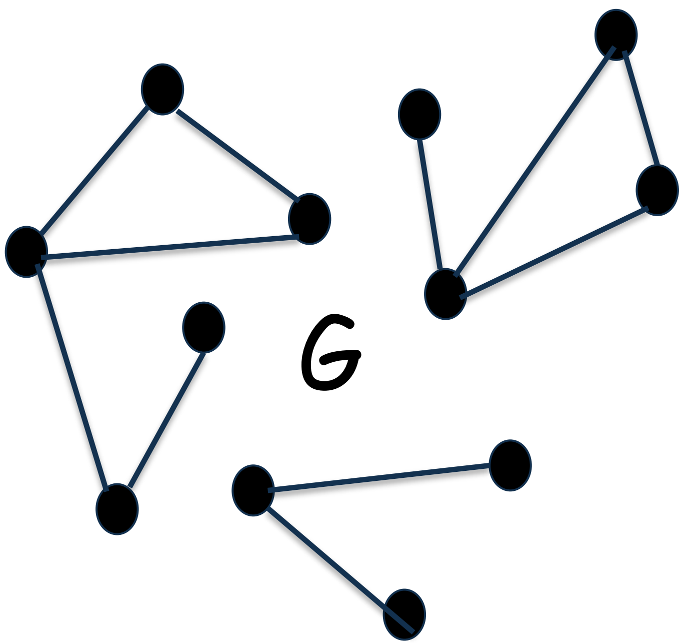

- find the number of connected components of a given graph (use BFS)
- start from the first vertex; any vertex we have discovered during this search should be part of same component.
- so, repeat the process with an undiscovered vertex.

Connected Components

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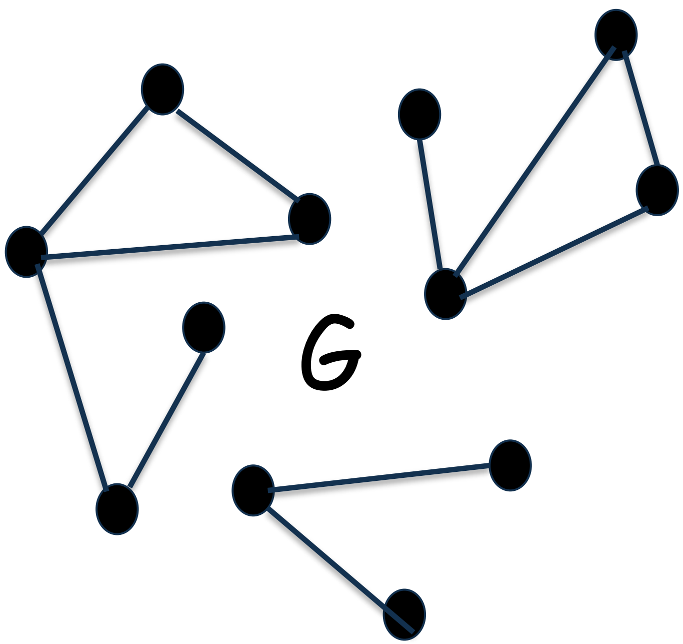
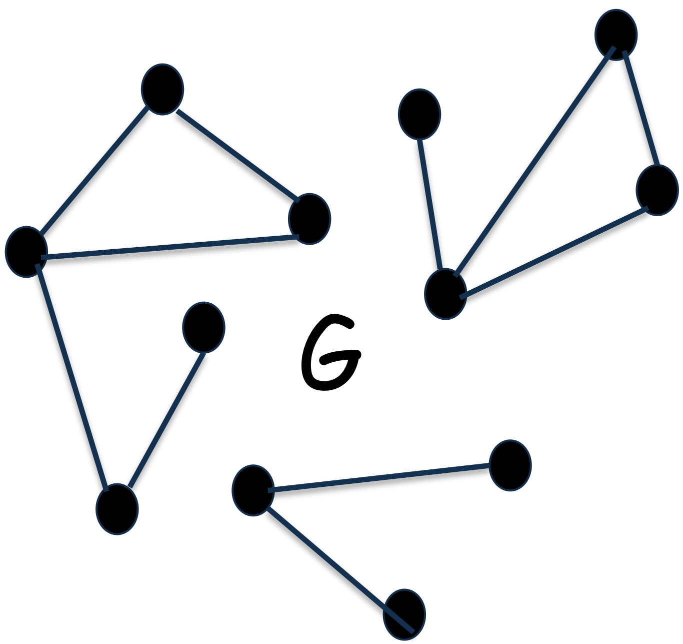


- find the number of connected components of a given graph (use BFS)
- start from the first vertex; any vertex we have discovered during this search should be part of same component.
- so, repeat the process with an undiscovered vertex.

```
int num = 0
for (i=1 to n)
      if (v_i has not been discovered)
            num = num + 1
            BFS(G, v_i)
return num
```

Connected Components

- a connected component is a maximal subgraph where there is a path between any two nodes of it
- a graph can be made up of seperate connected components



- It would be useful to clasify each vertex by which connected component it belongs
- When we run BFS on G from v, we mark each vertex as being owned by v.
- If we iterate through all vertices, each vertex will be marked by its owner that represents a different connected component

Depth First Search

- instead of going cross-wise, just go deep in the graph

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- when we process a node u, we pick each neighbor v of u in order

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- when we process a node u, we pick each neighbor v of u in order

  at the time of checking each v, we check again each neighbor of v in order

  only after processing all descendants of v, we pass to the next neighbor of u

Depth First Search

- instead of going cross-wise, just go deep in the graph
- when we process a node u, we pick each neighbor v of u in order

  at the time of checking each v, we check again each neighbor of v in order

  only after processing all descendants of v, we pass to the next neighbor of u

- the process continues until all verices reachable from the source have been discovered
- if any undiscovered vertices remain, choose one of them as new source and repeat the process

Depth First Search

- every node u is associated with four parameters :

Depth First Search

- every node u is associated with four parameters :

discovery finish parent color

Depth First Search

- every node u is associated with four parameters :

discovery

the time we have
discovered the node u

finish

the time we have
processed the node u

parent

u's predecessor on
the shortest path
from s to u

color

shows the state of u
white : undiscovered
gray : discovered
Black : processed

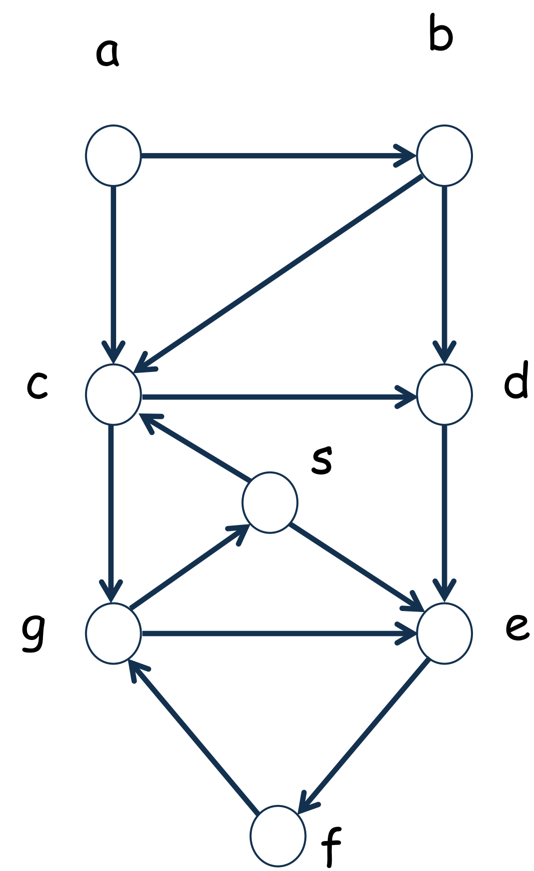
Depth First Search

<u>DFS(G)</u>

```
for each vertex u of V
        u.color = white
        u.par = nil
time = 0
for each vertex u of V
        if u.color = white
              DFS_Visit(u)
```

<u>DFS_Visit(u)</u>

```
u.color = gray
time = time + 1
u.dis = time
for each v in Adj(u)
     if (v.color = white)
          v.par = u
          DFS_Visit(v)
u.color = black
time = time + 1
u.fin = time
```



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for each vertex u of V
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time = 0
for each vertex u of V
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<u>DFS_Visit(u)</u>

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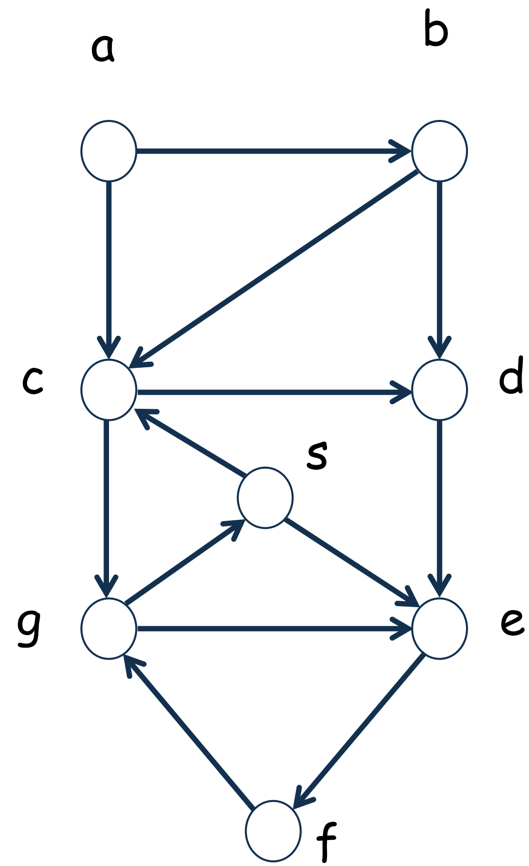
u.color = gray
time = time + 1
u.dis = time
for each v in Adj(u)
      if (v.color = white)
            v.par = u
            DFS_Visit(v)
u.color = black
time = time + 1
u.fin = time
```

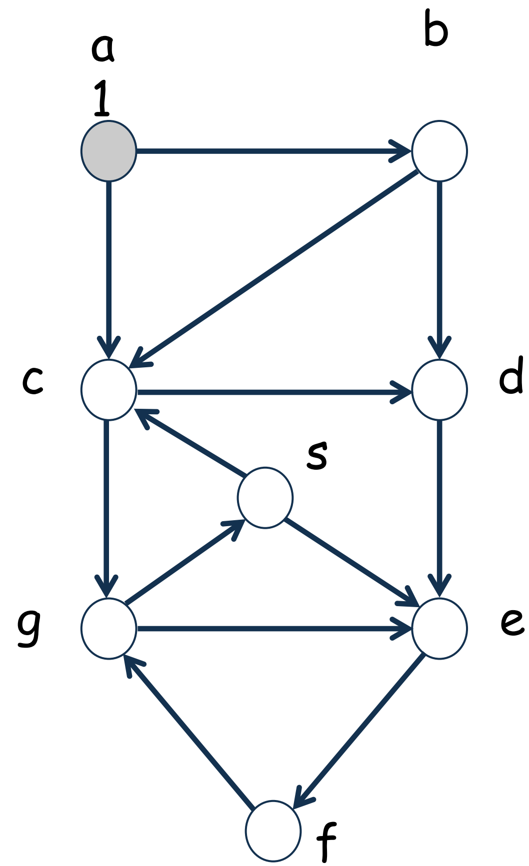


time = 0

Depth First Search

```
DFS(G)
for each vertex u of V
        u.color = white
        u.par = nil
time = 0
for each vertex u of V
        if u.color = white
            DFS_Visit(u)
```

```
DFS_Visit(u)
u.color = gray
time = time + 1
u.dis = time
for each v in Adj(u)
    if (v.color = white)
        v.par = u
        DFS_Visit(v)
u.color = black
time = time + 1
u.fin = time
```



time = 1

Depth First Search

DFS(G)

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for each vertex u of V
        u.color = white
        u.par = nil
time = 0
for each vertex u of V
        if u.color = white
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```

DFS_Visit(u)

```
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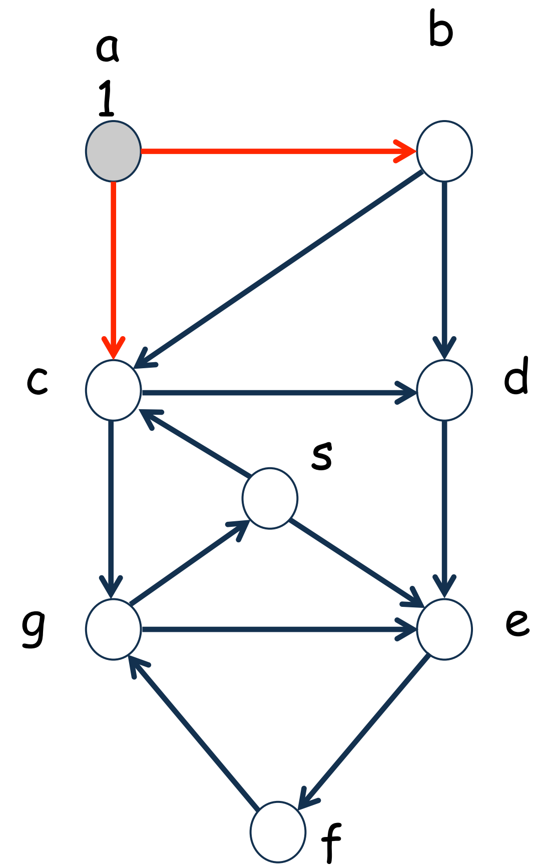
u.dis = time
for each v in Adj(u)
      if (v.color = white)
            v.par = u
            DFS_Visit(v)
u.color = black
time = time + 1
u.fin = time
```



time = 1

Depth First Search

DFS(G)

```
for each vertex u of V
        u.color = white
        u.par = nil
time = 0
for each vertex u of V
        if u.color = white
              DFS_Visit(u)
```

DFS_Visit(u)

```
u.color = gray
time = time + 1
u.dis = time
for each v in Adj(u)
      if (v.color = white)
            v.par = u
            DFS_Visit(v)
u.color = black
time = time + 1
u.fin = time
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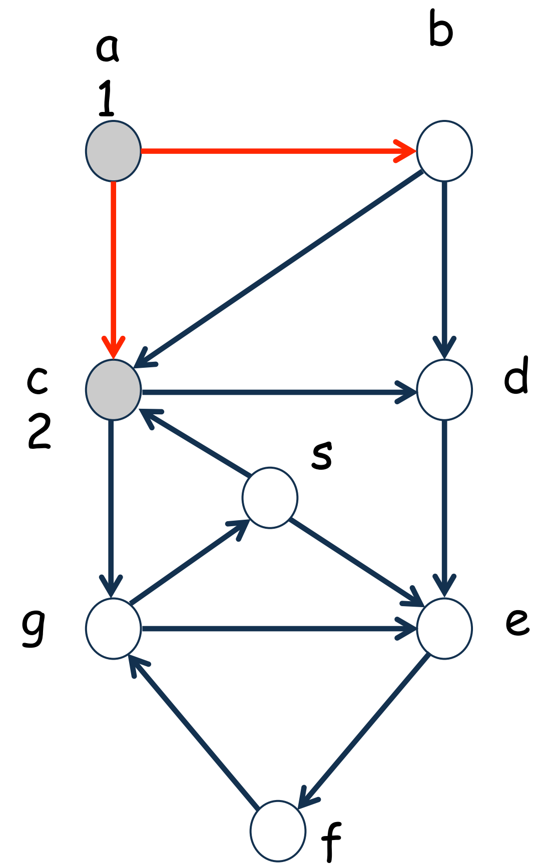


time = 2

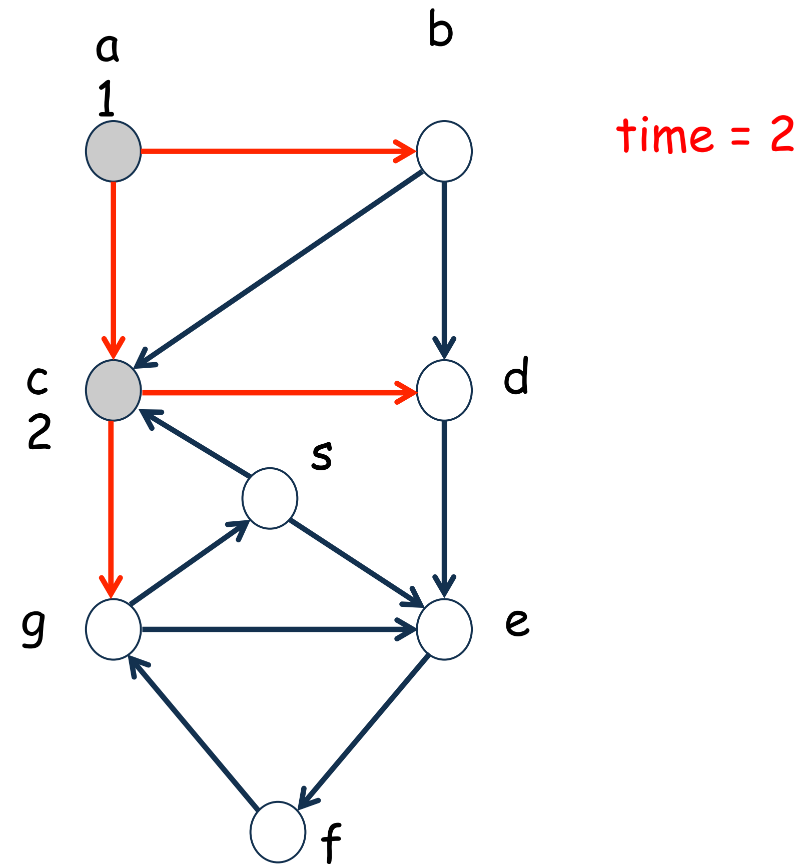
Depth First Search

<u>DFS(G)</u>

```
for each vertex u of V
        u.color = white
        u.par = nil
time = 0
for each vertex u of V
        if u.color = white
              DFS_Visit(u)
```

<u>DFS_Visit(u)</u>

```
u.color = gray
time = time + 1
u.dis = time
for each v in Adj(u)
     if (v.color = white)
          v.par = u
          DFS_Visit(v)
u.color = black
time = time + 1
u.fin = time
```



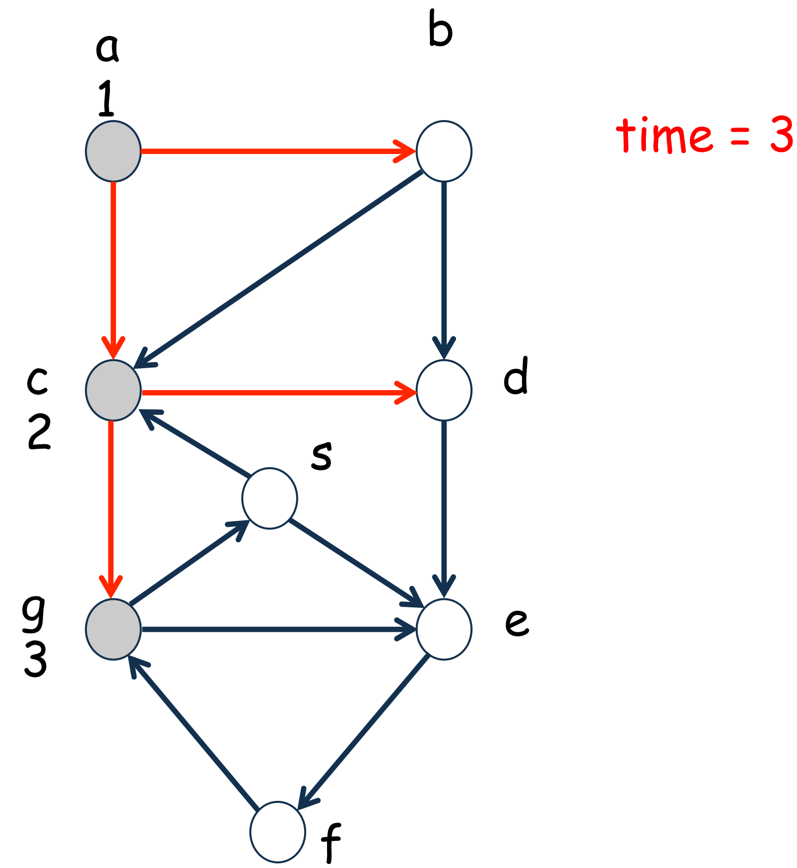
Depth First Search

DFS(G)

```
for each vertex u of V
        u.color = white
        u.par = nil
time = 0
for each vertex u of V
        if u.color = white
              DFS_Visit(u)
```

DFS_Visit(u)

```
u.color = gray
time = time + 1
u.dis = time
for each v in Adj(u)
      if (v.color = white)
            v.par = u
            DFS_Visit(v)
u.color = black
time = time + 1
u.fin = time
```



Depth First Search

<u>DFS(G)</u>
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for each vertex u of V
        u.color = white
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time = 0
for each vertex u of V
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              DFS_Visit(u)
```

<u>DFS_Visit(u)</u>
```
u.color = gray
time = time + 1
u.dis = time
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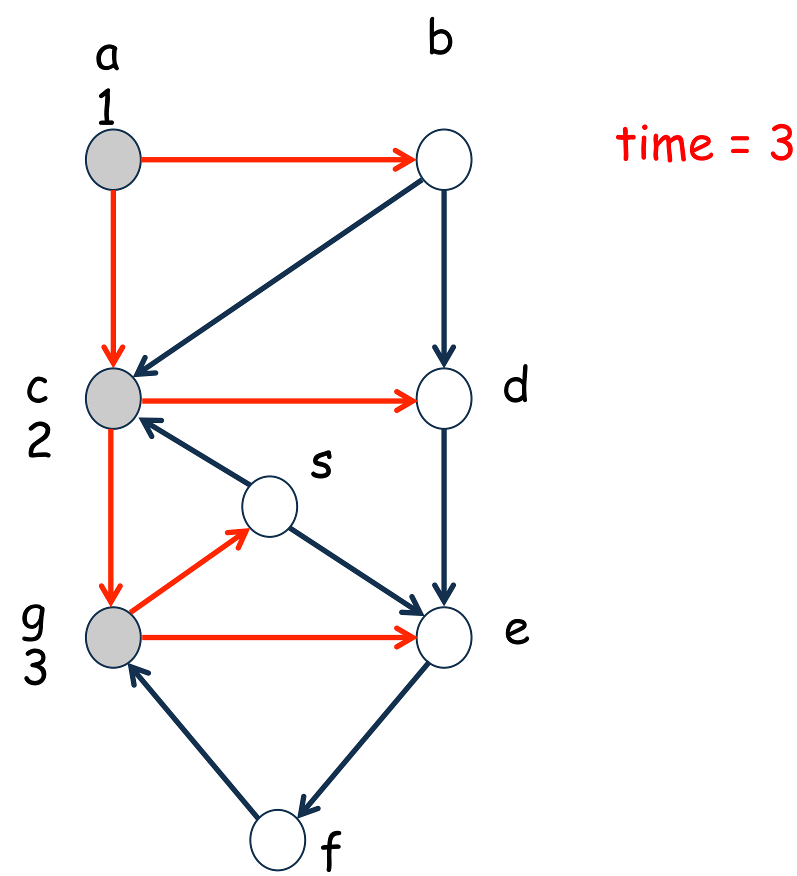
      if (v.color = white)
            v.par = u
            DFS_Visit(v)
u.color = black
time = time + 1
u.fin = time
```



Depth First Search

DFS(G)

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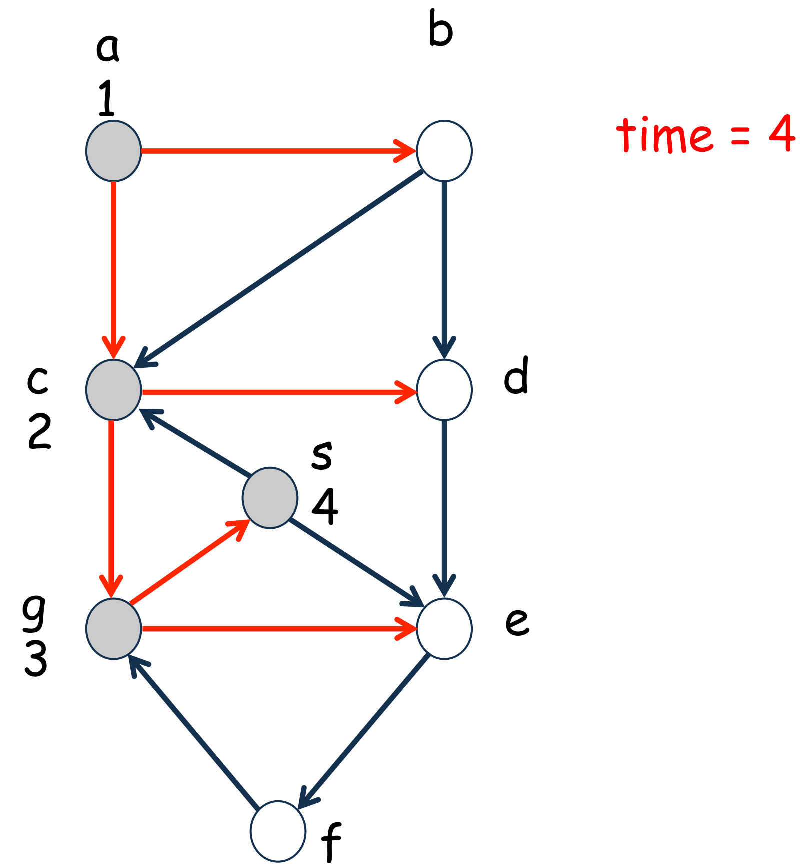
for each vertex u of V
        u.color = white
        u.par = nil
time = 0
for each vertex u of V
        if u.color = white
                DFS_Visit(u)
```

DFS_Visit(u)

```
u.color = gray
time = time + 1
u.dis = time
for each v in Adj(u)
      if (v.color = white)
            v.par = u
            DFS_Visit(v)
u.color = black
time = time + 1
u.fin = time
```



Depth First Search

DFS(G)

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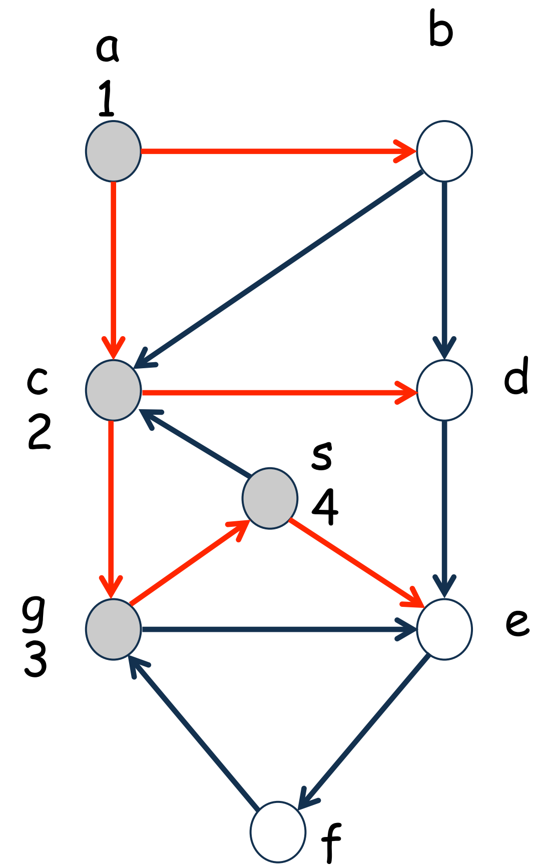
for each vertex u of V
        u.color = white
        u.par = nil
time = 0
for each vertex u of V
        if u.color = white
              DFS_Visit(u)
```

DFS_Visit(u)

```
u.color = gray
time = time + 1
u.dis = time
for each v in Adj(u)
      if (v.color = white)
            v.par = u
            DFS_Visit(v)
u.color = black
time = time + 1
u.fin = time
```



time = 4

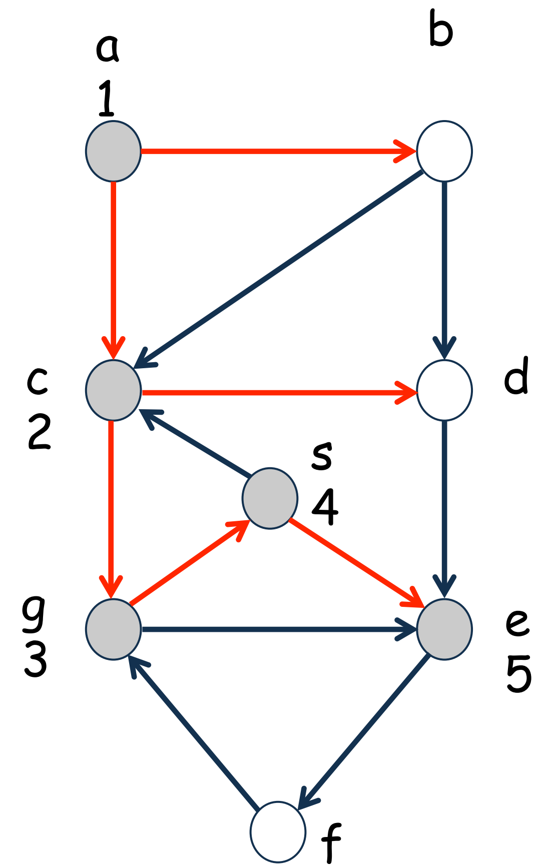
Depth First Search

DFS(G)

```
for each vertex u of V
        u.color = white
        u.par = nil
time = 0
for each vertex u of V
        if u.color = white
            DFS_Visit(u)
```

DFS_Visit(u)

```
u.color = gray
time = time + 1
u.dis = time
for each v in Adj(u)
      if (v.color = white)
            v.par = u
            DFS_Visit(v)
u.color = black
time = time + 1
u.fin = time
```

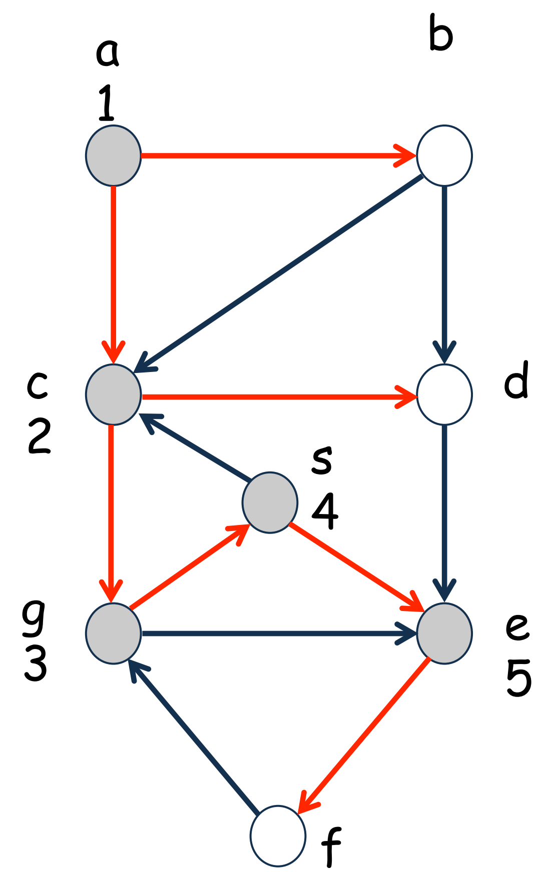


time = 5

Depth First Search

<u>DFS(G)</u>
```
for each vertex u of V
        u.color = white
        u.par = nil
time = 0
for each vertex u of V
        if u.color = white
              DFS_Visit(u)
```

<u>DFS_Visit(u)</u>
```
u.color = gray
time = time + 1
u.dis = time
for each v in Adj(u)
      if (v.color = white)
            v.par = u
            DFS_Visit(v)
u.color = black
time = time + 1
u.fin = time
```

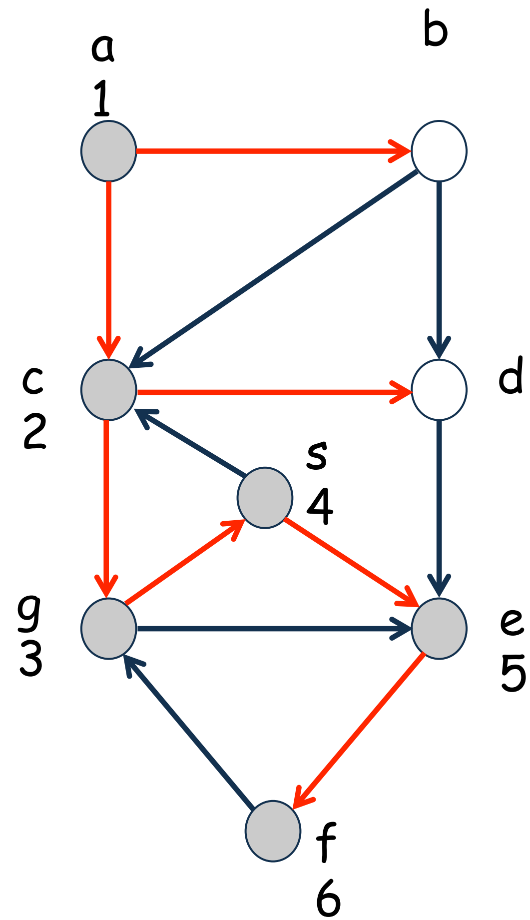


time = 5

Depth First Search

DFS(G)
for each vertex u of V
    u.color = white
    u.par = nil
time = 0
for each vertex u of V
    if u.color = white
        DFS_Visit(u)

DFS_Visit(u)
u.color = gray
time = time + 1
u.dis = time
for each v in Adj(u)
    if (v.color = white)
        v.par = u
        DFS_Visit(v)
u.color = black
time = time + 1
u.fin = time



time = 6

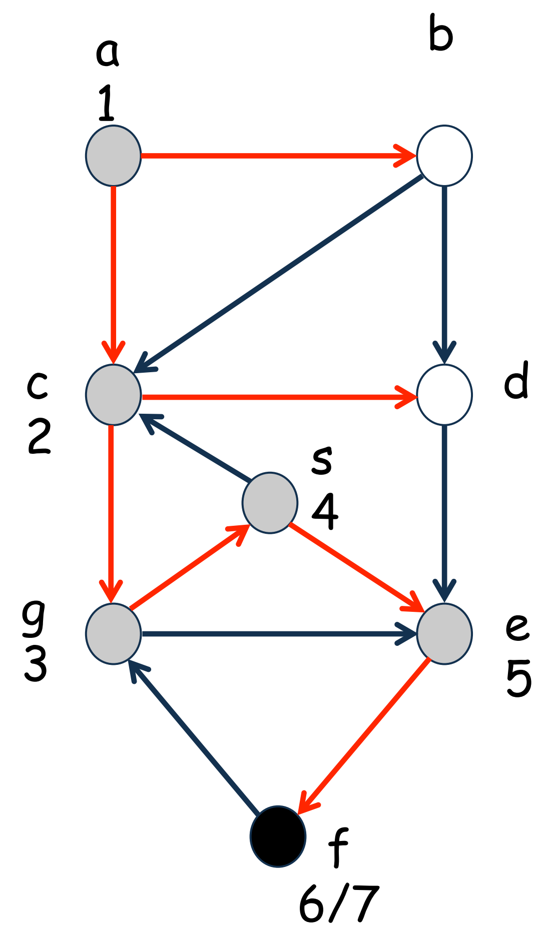
Depth First Search

<u>DFS(G)</u>

```
for each vertex u of V
        u.color = white
        u.par = nil
time = 0
for each vertex u of V
        if u.color = white
              DFS_Visit(u)
```

<u>DFS_Visit(u)</u>

```
u.color = gray
time = time + 1
u.dis = time
for each v in Adj(u)
      if (v.color = white)
            v.par = u
            DFS_Visit(v)
u.color = black
time = time + 1
u.fin = time
```



time = 7

Depth First Search

DFS(G)
for each vertex u of V
    u.color = white
    u.par = nil
time = 0
for each vertex u of V
    if u.color = white
        DFS_Visit(u)

DFS_Visit(u)
u.color = gray
time = time + 1
u.dis = time
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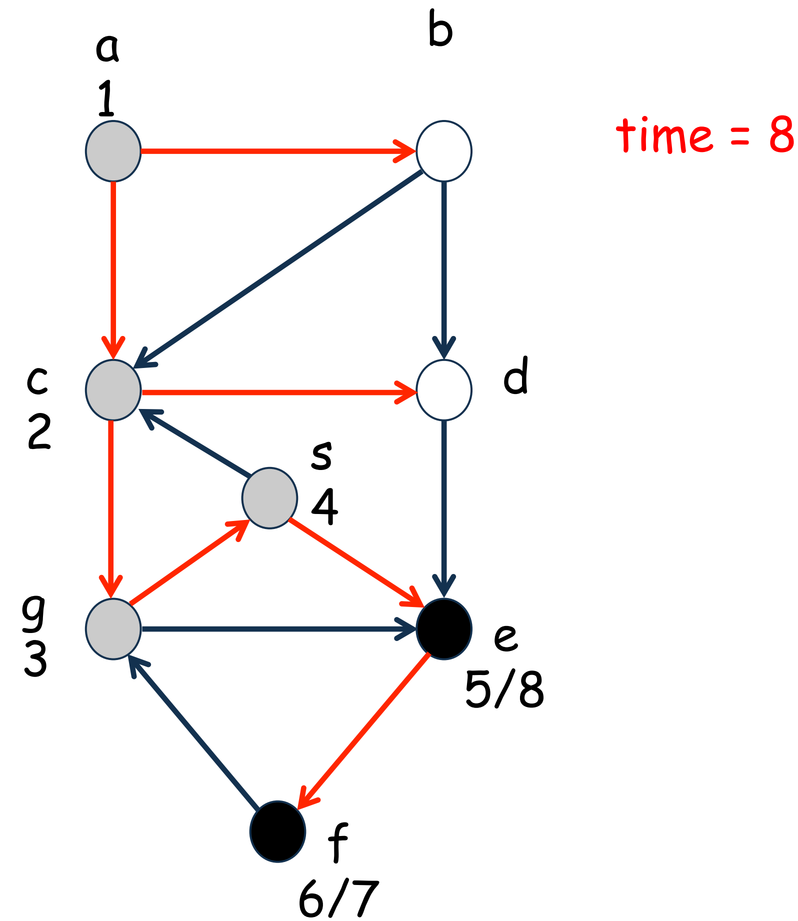

Depth First Search

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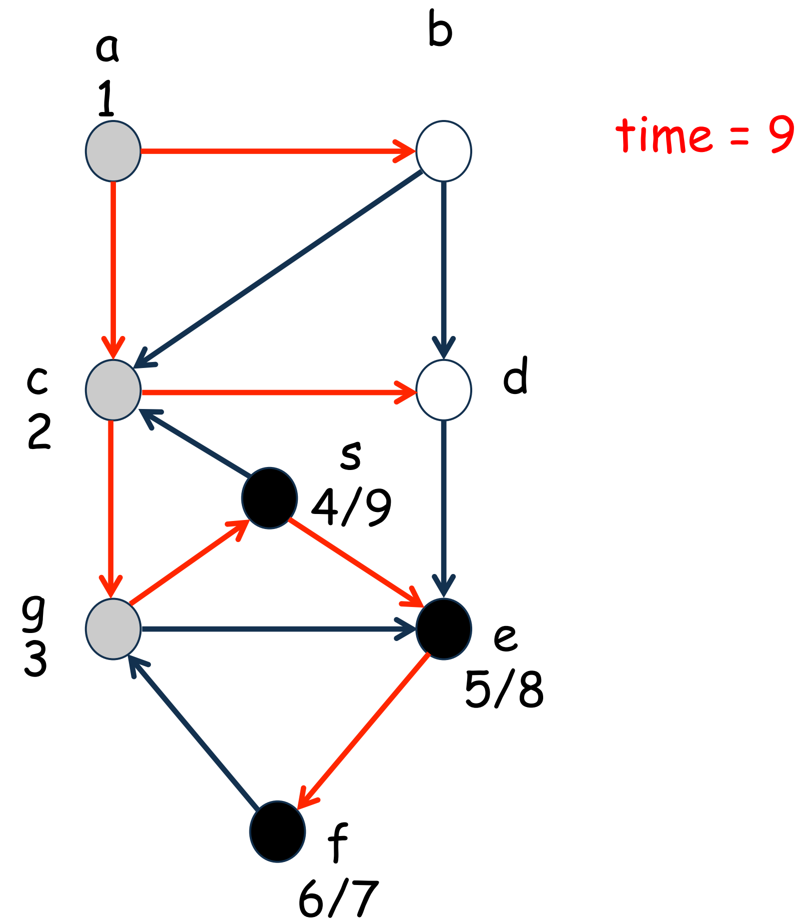

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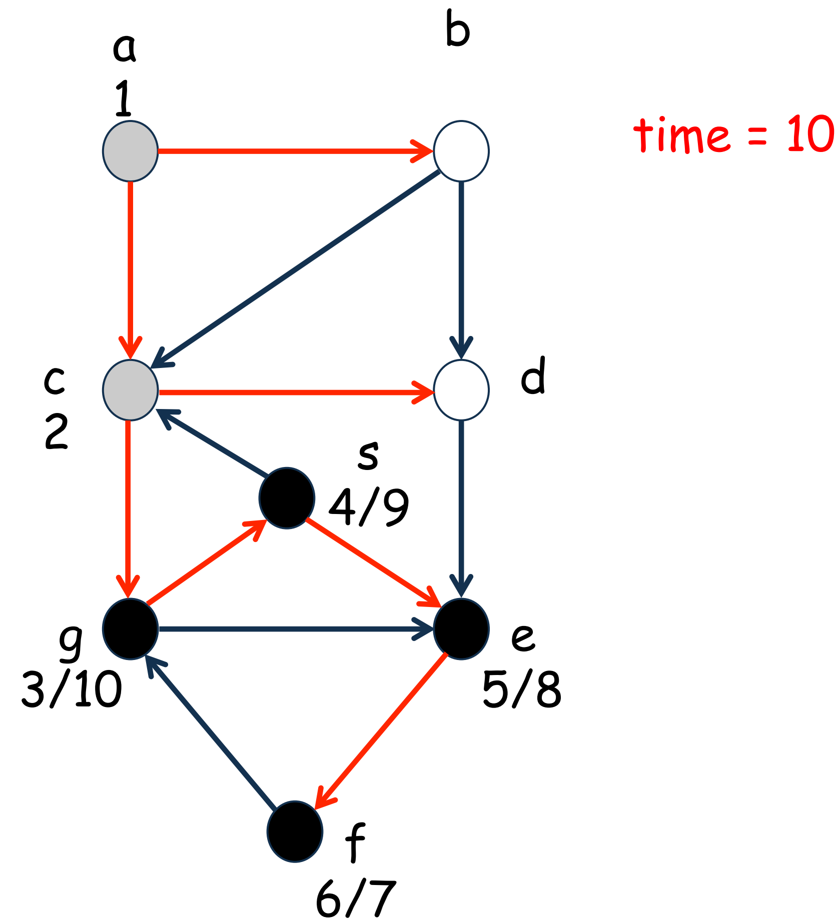

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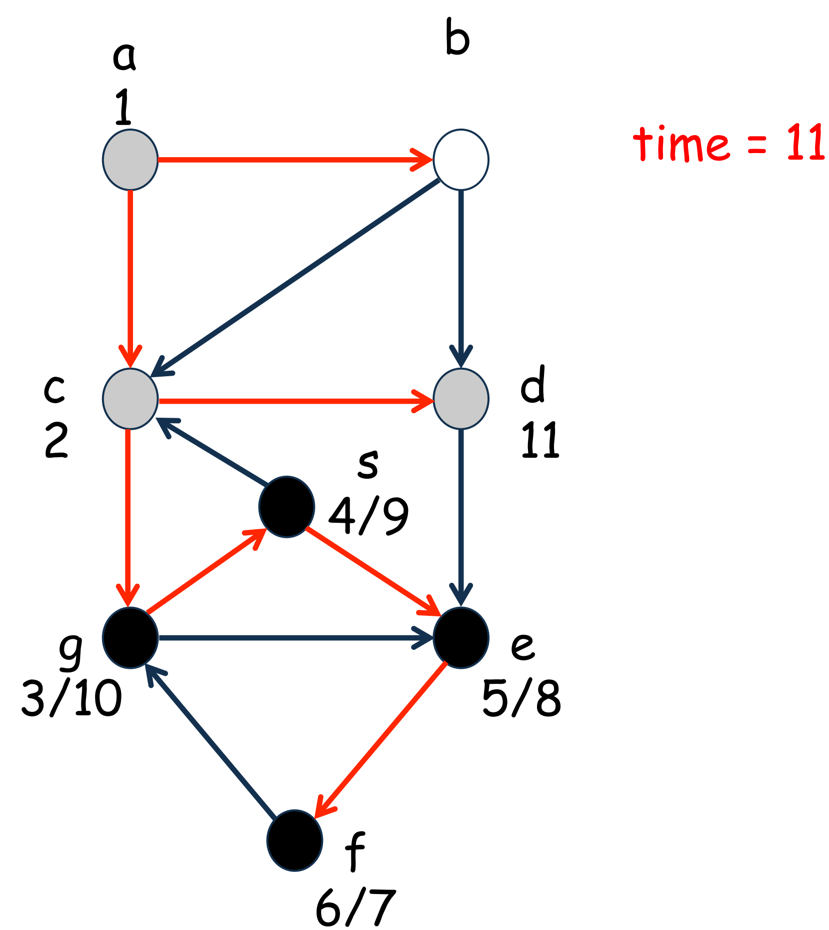

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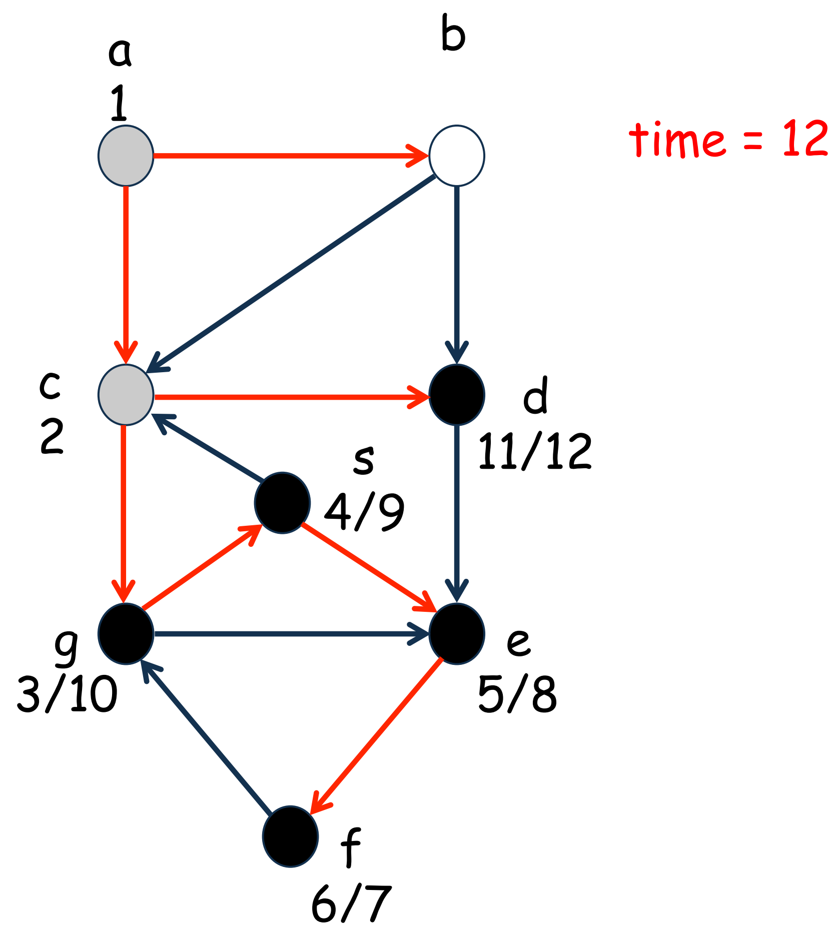

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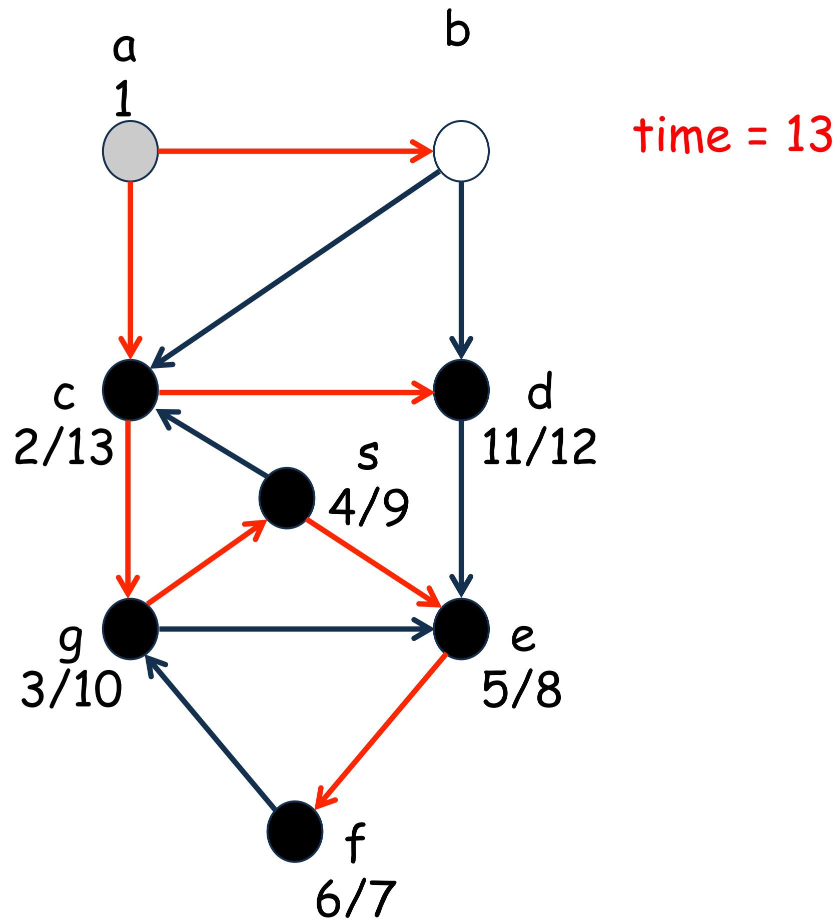

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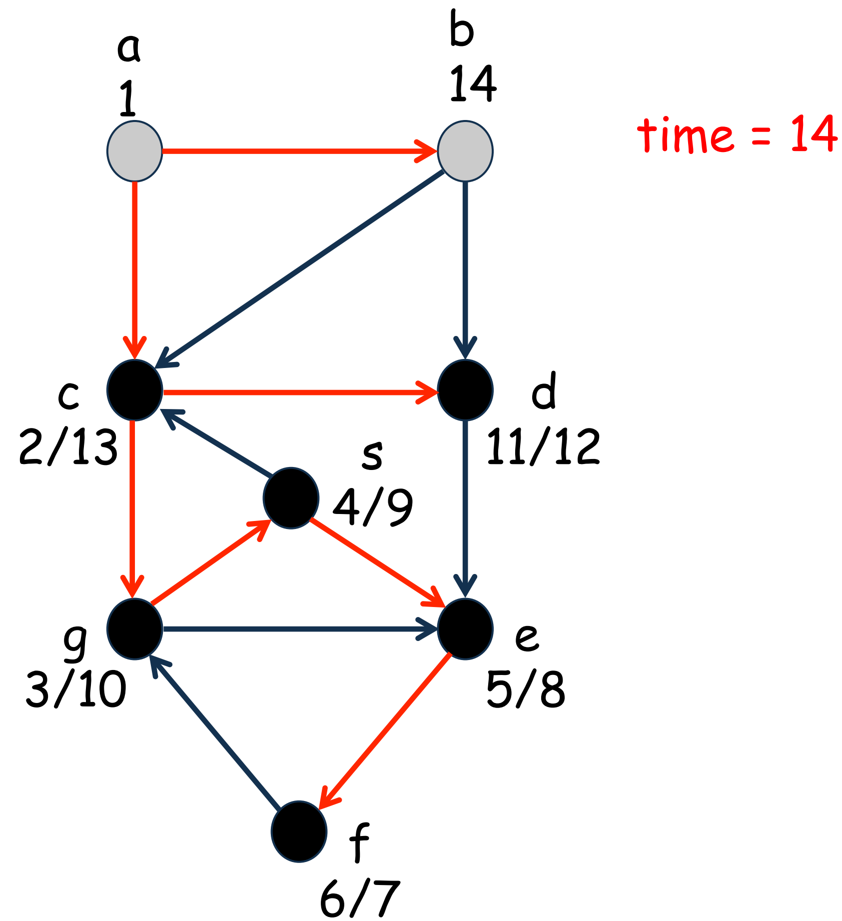

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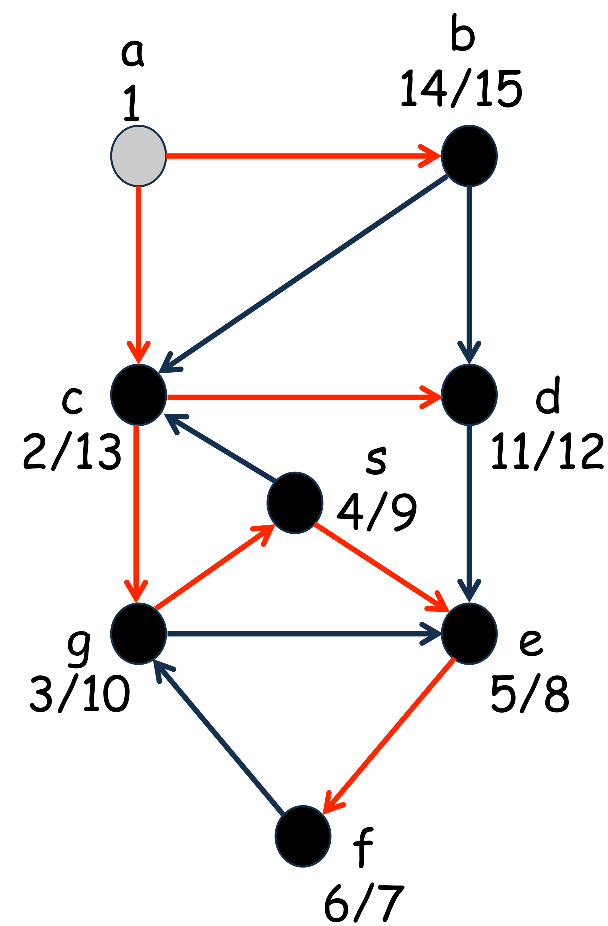

time = 15

Depth First Search

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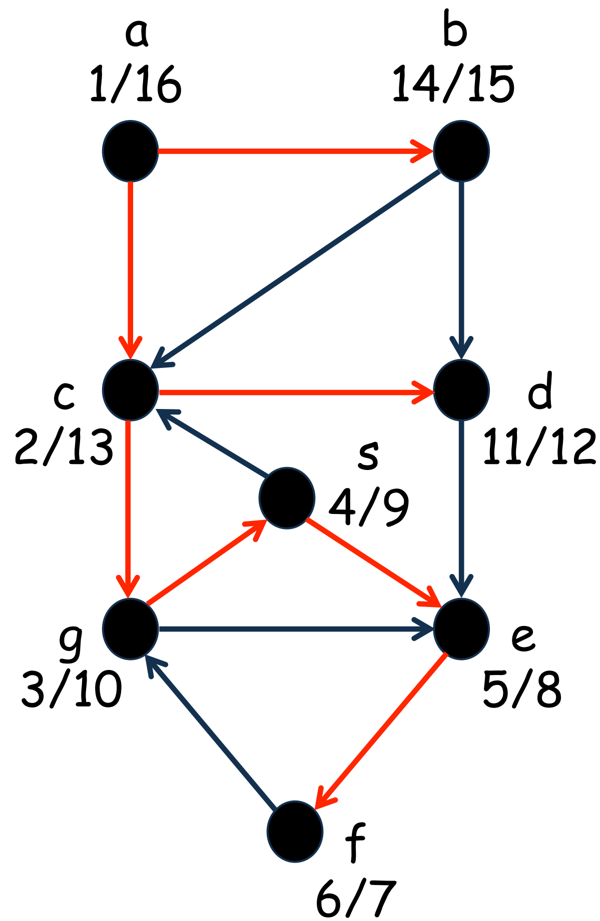
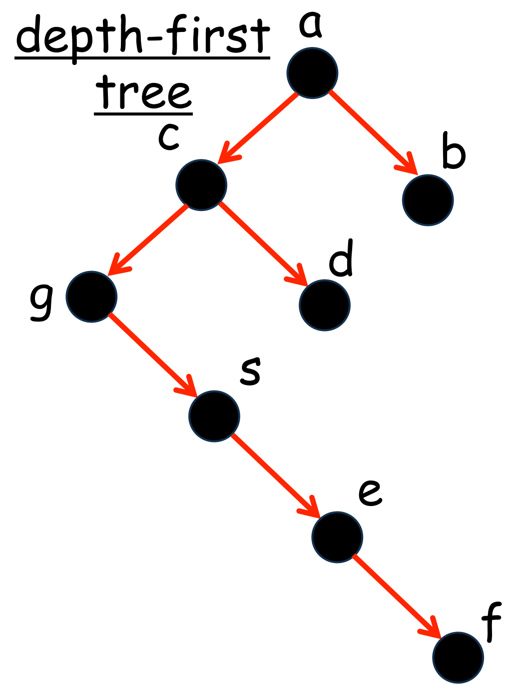
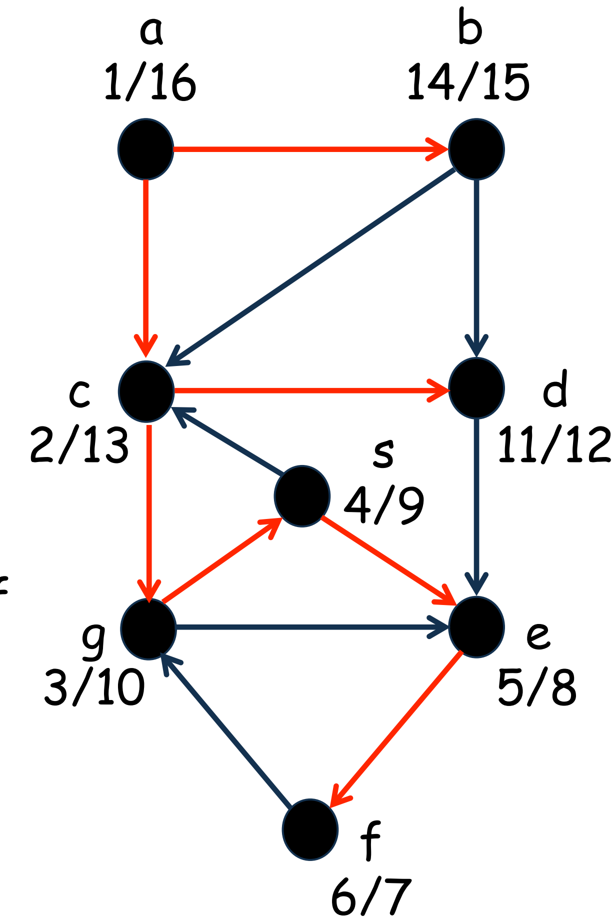


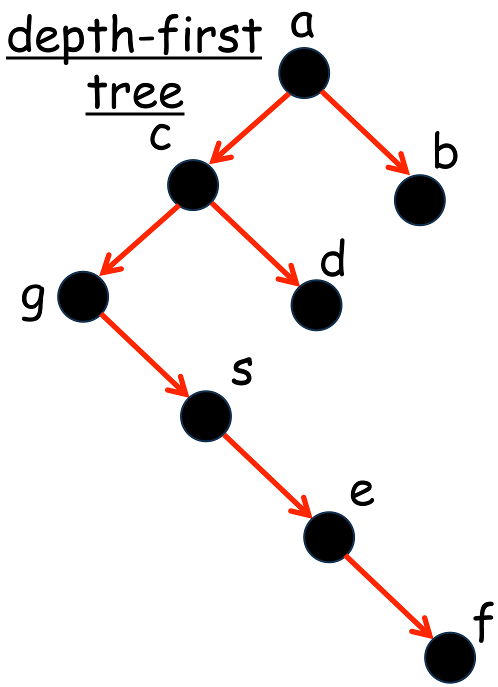
time = 15

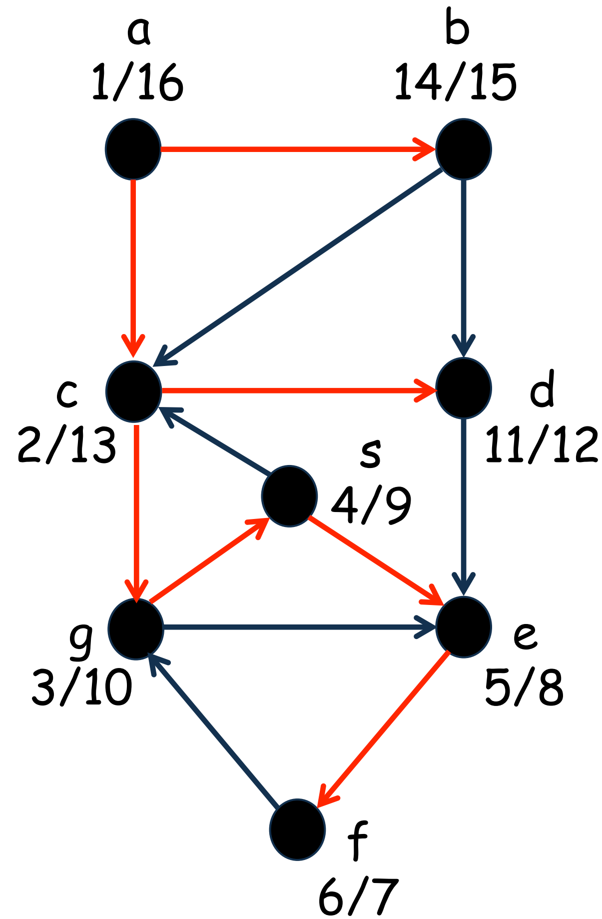
Depth First Search





Depth First Search





tree-edge

(u,v) is called tree-edge if it was first discovered by exploring edge

Depth First Search

tree-edge

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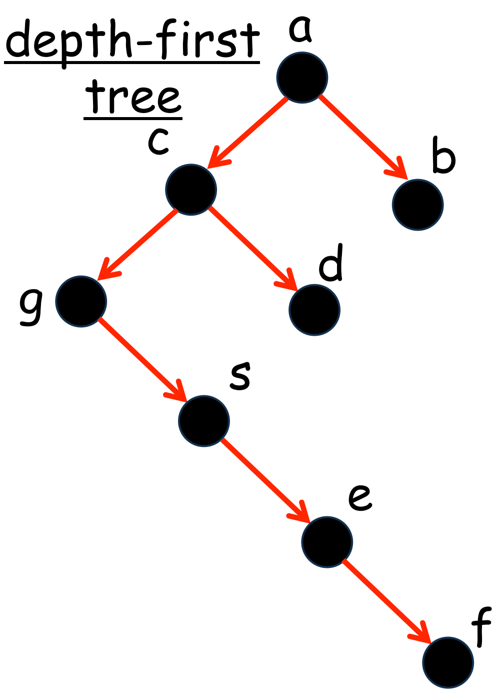
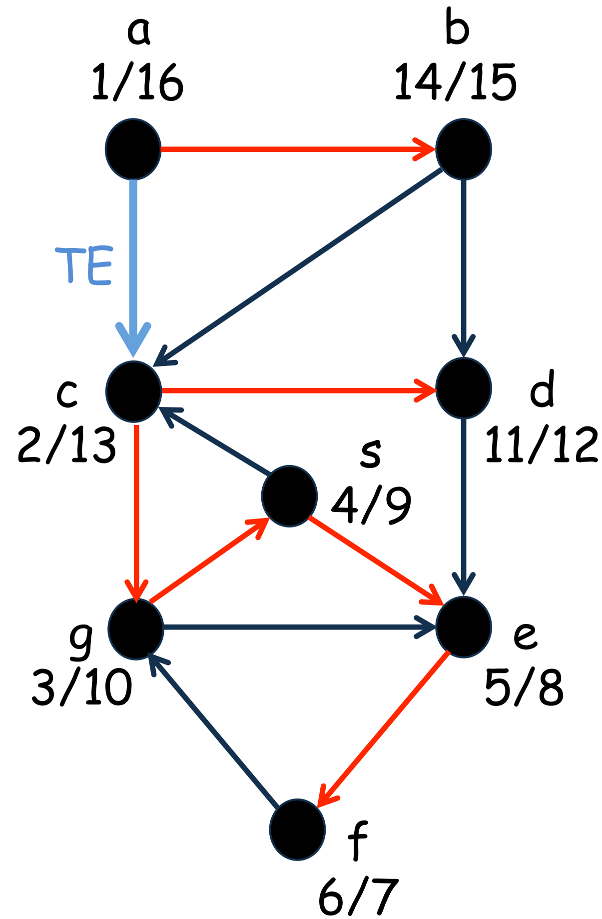
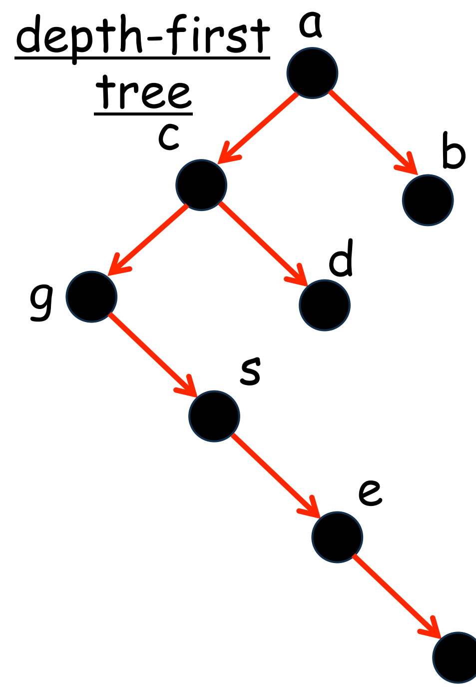
depth-first tree



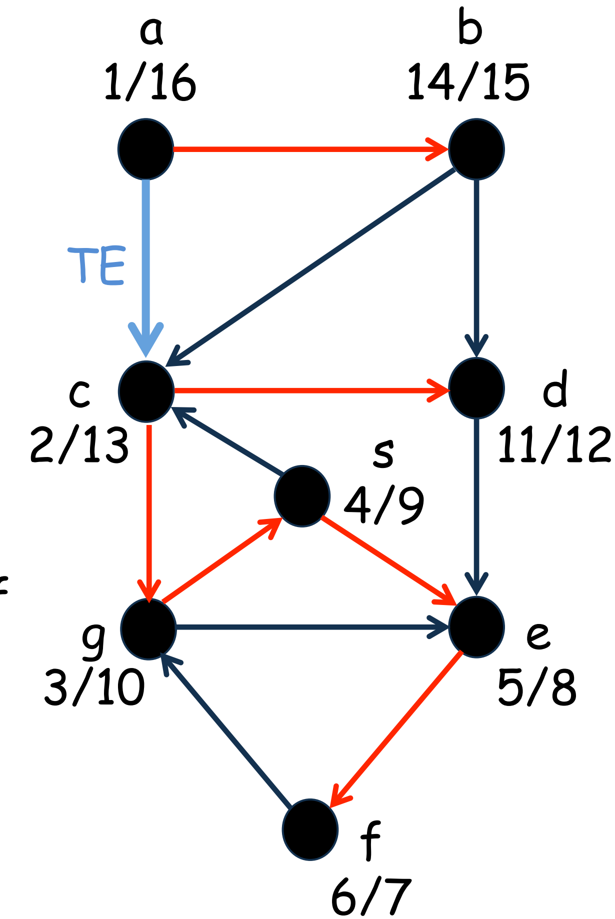


Depth First Search



tree-edge
(u,v) is called tree-edge if it was first discovered by exploring edge

back-edge
(u,v) is called back-edge if it's connecting vertex u to an ancestor v in depth-first tree



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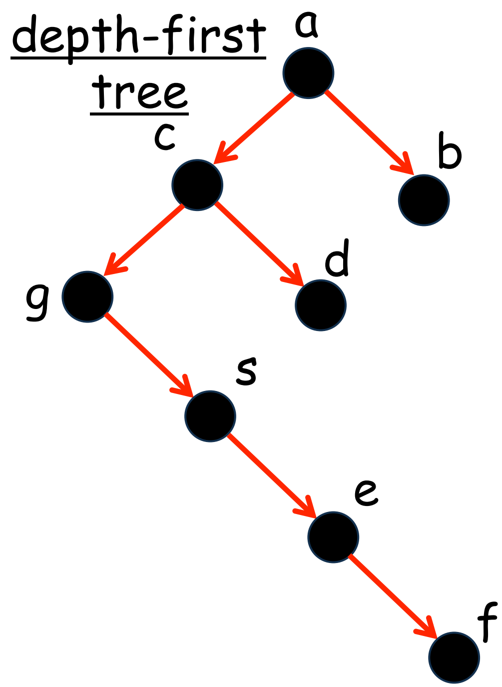

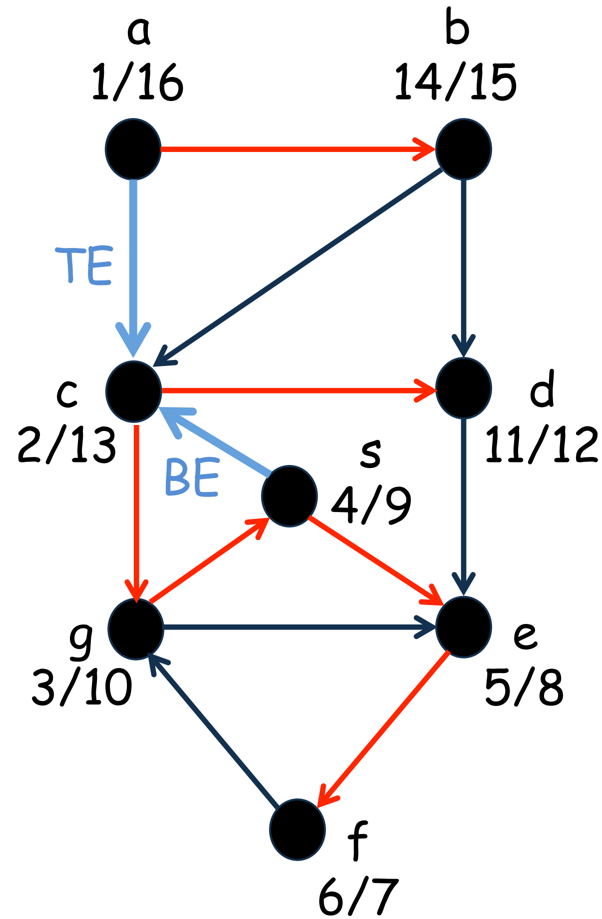

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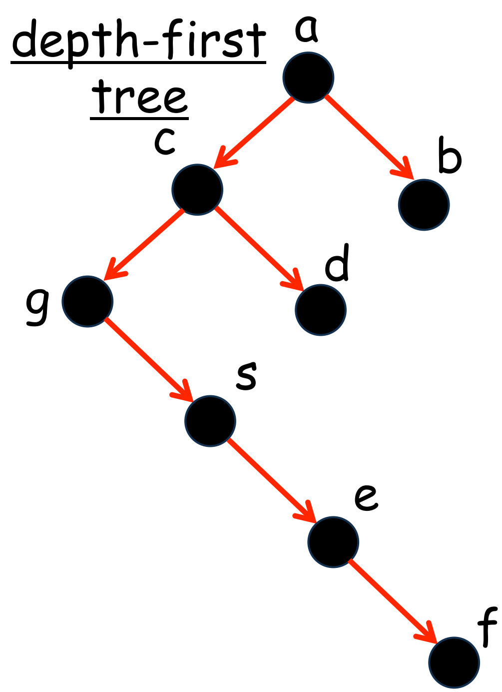

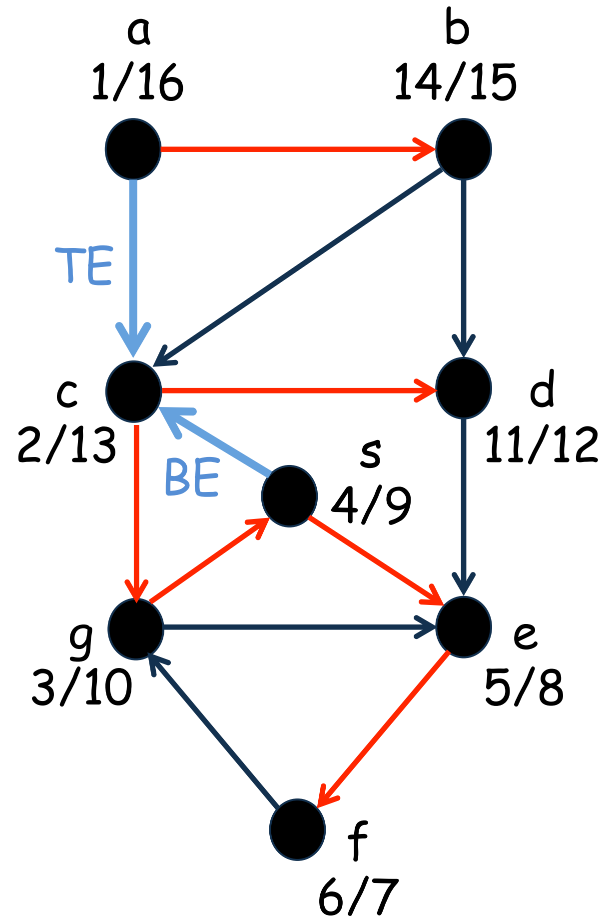

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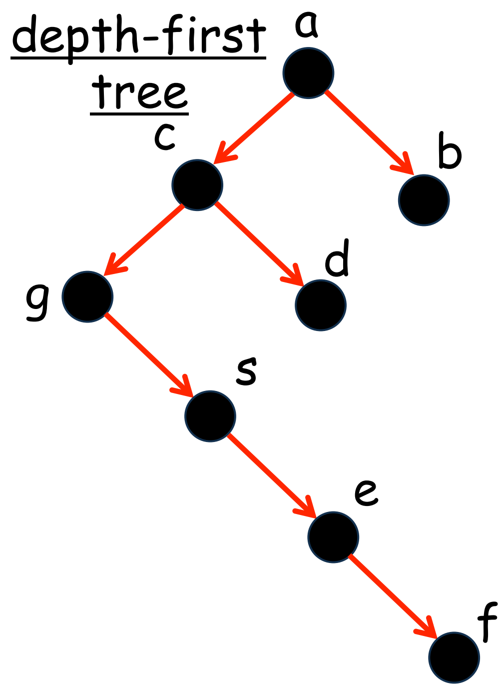
Depth First Search



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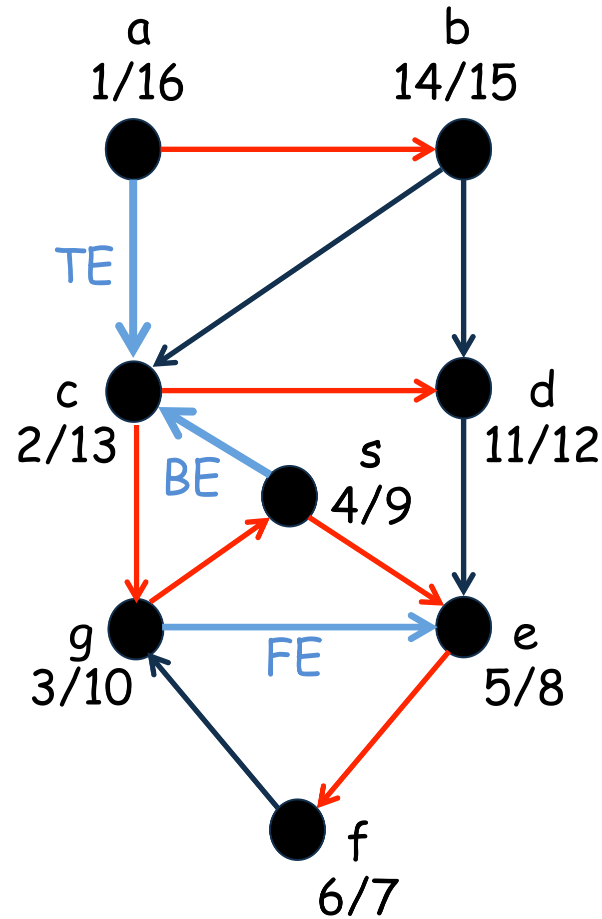

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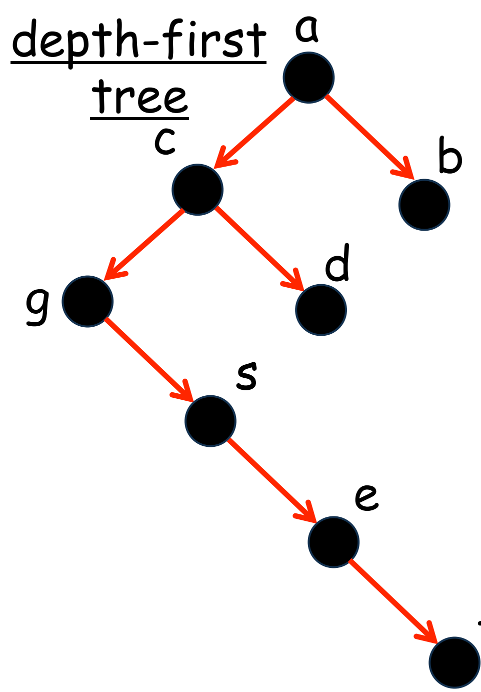

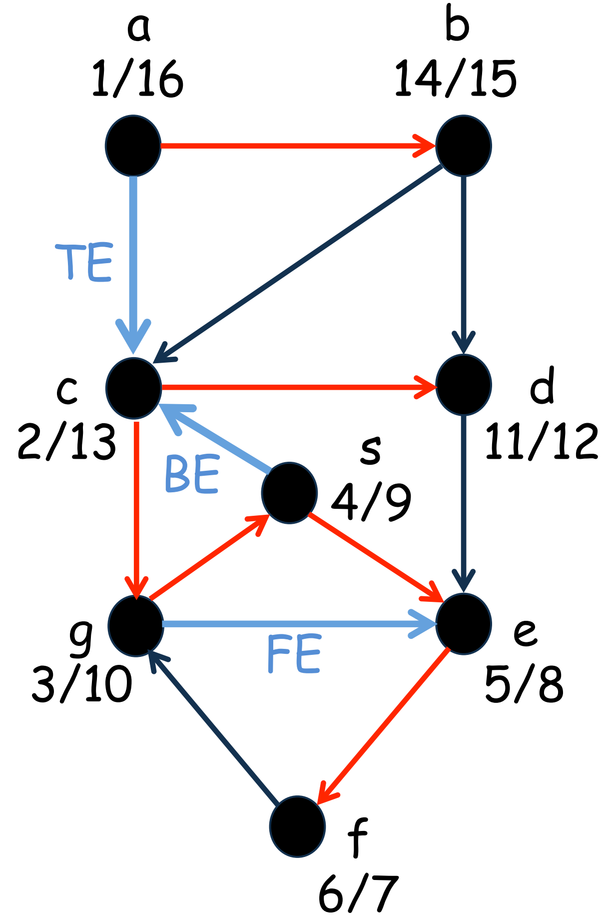

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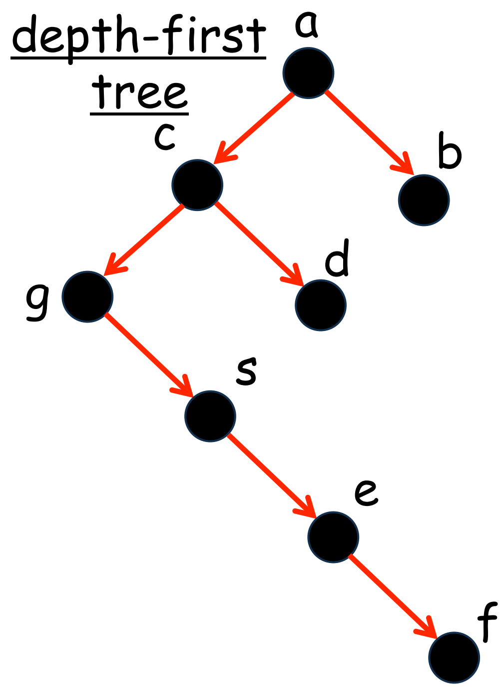
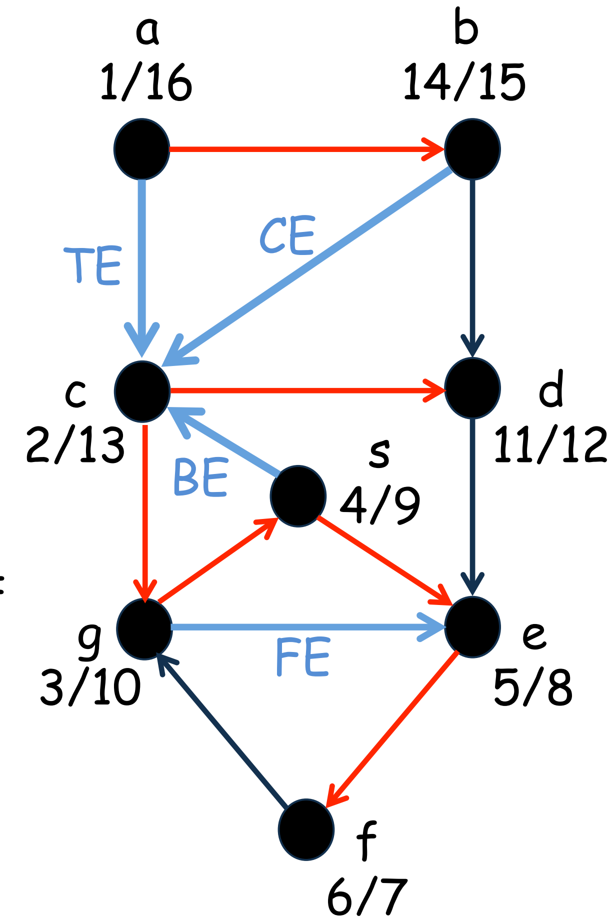
(u,v) is called cross-edge if it's connecting vertex u to vertex v such that there is no ancestor/ descendant relation between them

Depth First Search





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Cycle Detection

- There is a cycle in the graph only if there is back edge in the graph

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u.color = gray
for each v in Adj(u)
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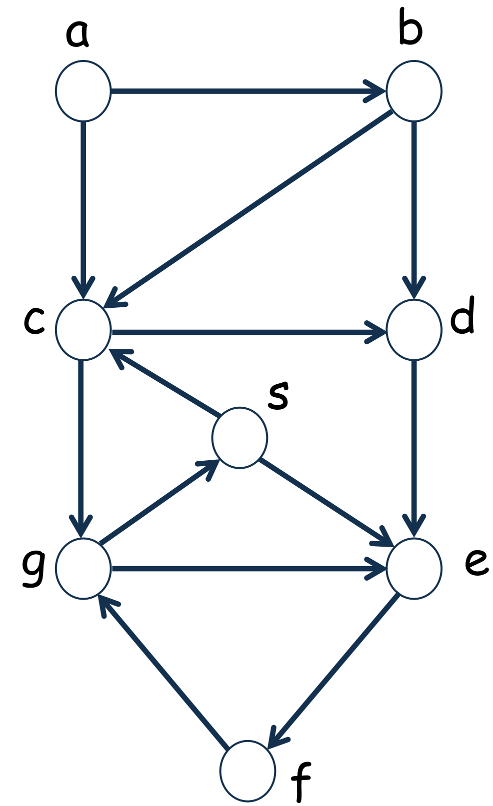

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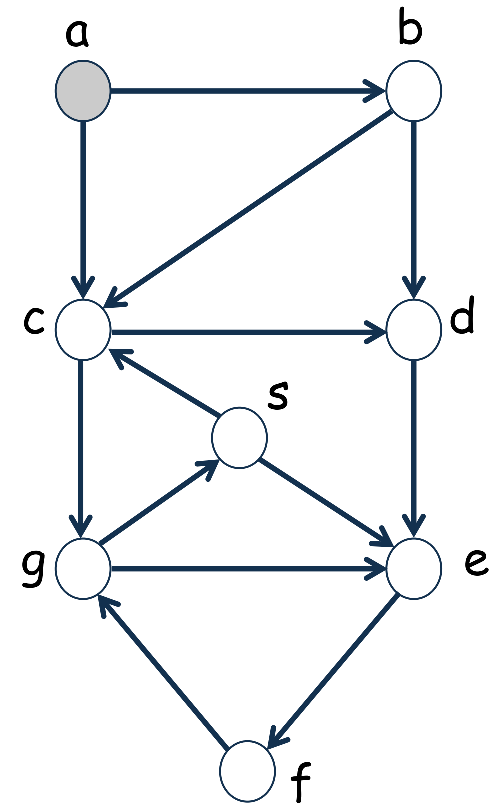


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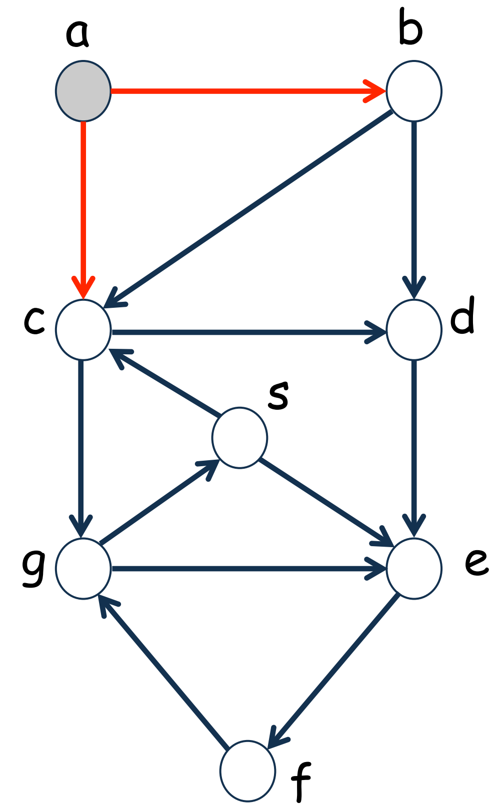

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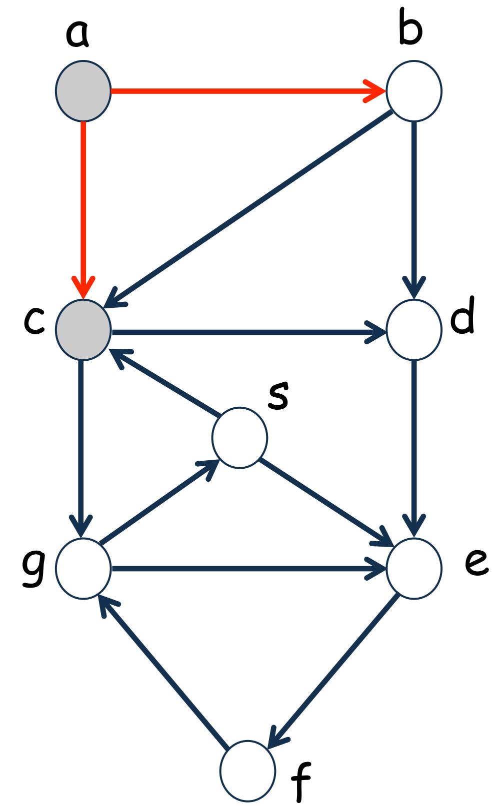

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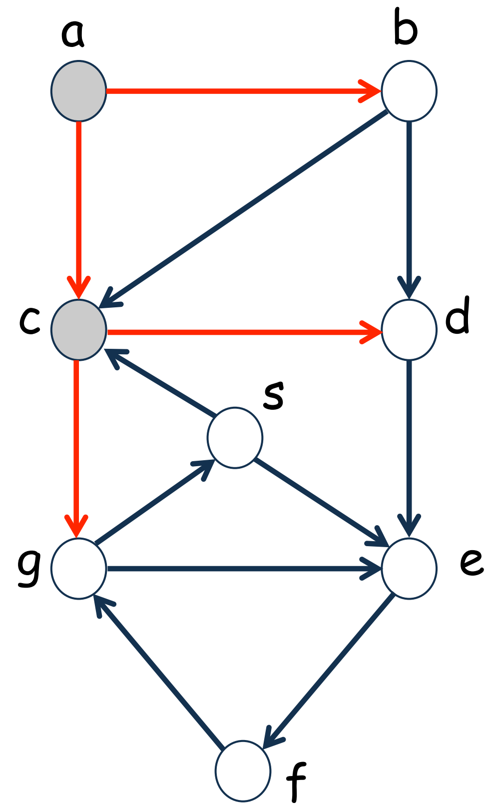


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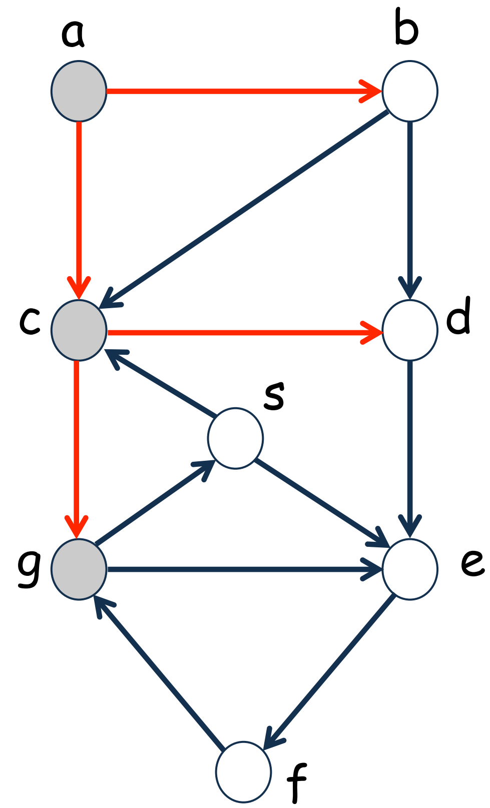

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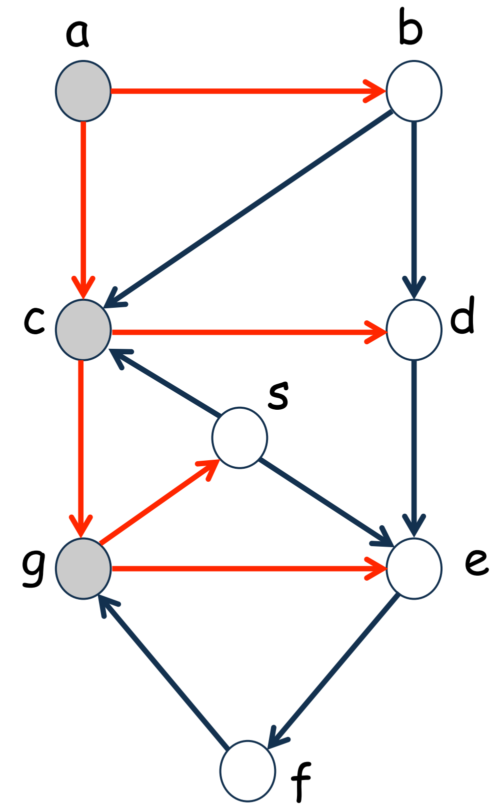

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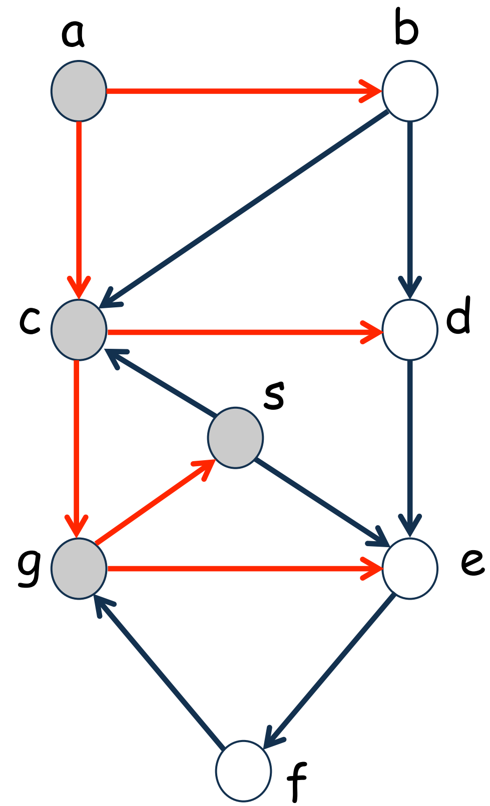

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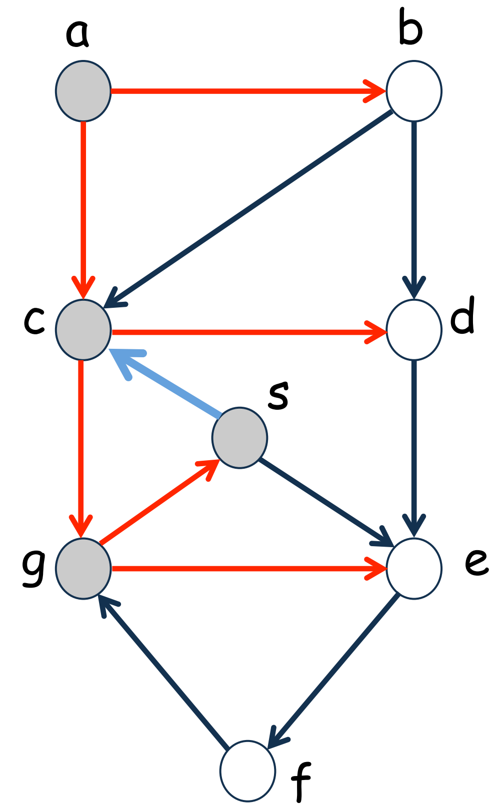


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