**CHAPTER-3**

**Case 2: One-dimensional steady-state conduction with heat generation**

Consider the plane wall in which there is uniform heat generation per unit volume

$\dot{q}\left(\frac{W}{m^{3}}\right)$, the surfaces are maintained at Ts1 and Ts2 . For constant thermal conductivity, k, the appropriate form of the equation is

$\frac{d^{2}T}{dx^{2}}+\frac{\dot{q}}{k}=0 $ 1-D conduction only in x-direction steady-state conduction with uniform heat

generation

$\frac{d^{2}T}{dx^{2}}=-\frac{\dot{q}}{k}$ …………. $\frac{dT}{dx}= -\frac{\dot{q}x}{k}+C\_{1}$

T(x) **=**$-\frac{\dot{q}x^{2}}{2k}+C\_{1}x+C\_{2}$ ………. Temperature distribution

Hot fluid

$T\_{\infty 1}$, h1 Ts2 Uniform heat generation inside the plane wall (W/m3)

**qge**n

 Ts1

 Cold fluid

 $T\_{\infty 2}$, h2

 **X=-L x=0 x=L**

Boundary conditions

X=- L T=Ts1

X=+L T=Ts2

Substituting BCs will give :

Ts1 **=**$-\frac{\dot{q}L^{2}}{2k}-C\_{1}L+C\_{2}$

Ts2 **=**$-\frac{\dot{q}L^{2}}{2k}+C\_{1}L+C\_{2}$

Solving the equations simultaneously will give.

T(x) = $\frac{\dot{q}L^{2}}{2k}\left(1-\frac{x^{2}}{L^{2}}\right)+\frac{T\_{s2}-T\_{s1}}{2}\frac{x}{L}+\frac{T\_{s2}+T\_{s1}}{2}$

Heat flux = $q\_{x}^{''}=-k\frac{dT}{dx}=-k\left[\frac{\dot{q}L^{2}}{2k}\left(-2x\right)+\frac{T\_{s2}-T\_{s1}}{2L}\right]=\dot{q}L^{2}x-(\frac{T\_{s2}-T\_{s1}}{2L}) k $

As you see heat flux is dependent on x under conditions with heat generation

* If you have symmetrical BCs, according to the scheme below:

 T0 (max. temperature)

**T0**

 Ts Ts

 x=-L x=0 x=+L

x=0 T=T0

x=$\pm $L T=Ts

Temperature distribution will take the following form:

T(x) = $\frac{\dot{q}L^{2}}{2k}\left(1-\frac{x^{2}}{L^{2}}\right)+T\_{s}$ (1)

Maximum temperature exists at the plane ……………. $T\_{0}=\frac{\dot{q}L^{2}}{2k}+T\_{s}$ (2)

Taking the ratio of eqns (1) and (2) will give:

$\frac{T\left(x\right)-T\_{s}}{T\_{0}-T\_{s}}=1-\frac{x^{2}}{L^{2}}$ ……………Temperature distribution for a system at:

* Steady-state
* Cartesian coordinates
* 1-D conduction (only x-direction)
* Uniform heat generation, $\dot{q}$
* Symmetrical BCs