Problem - 1

An uninsulated steam pipe passes through a room in which the air and walls are at 25°C. The outside diameter of the pipe is 70 mm, and its surface temperature and emissivity are 200°C and 0.8, respectively. If the coefficient associated with free convection heat transfer from the surface to the air is 15 W/m2 · K.

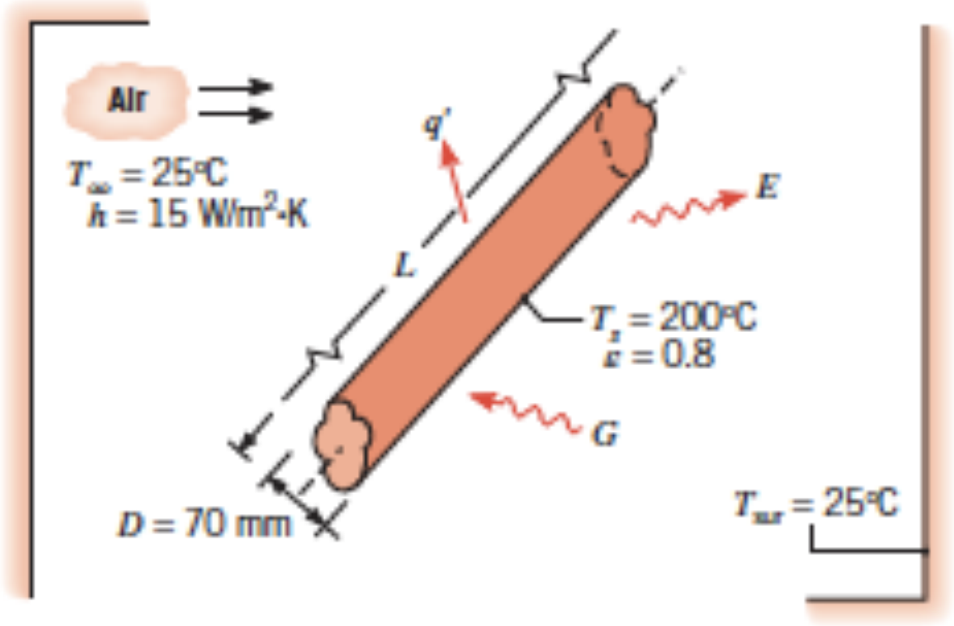
What is the rate of heat loss from the surface per unit length of pipe?

2. Heat loss from the pipe is by convection to the room air and by radiation exchange with the walls. Hence, $q = q_{conv} + q_{rad}$ and from Equation 1.10, with $A = \pi DL$,

$$q = h(\pi DL)(T_s - T_\infty) + \varepsilon(\pi DL)\sigma(T_s^4 - T_{sur}^4)$$

The rate of heat loss per unit length of pipe is then

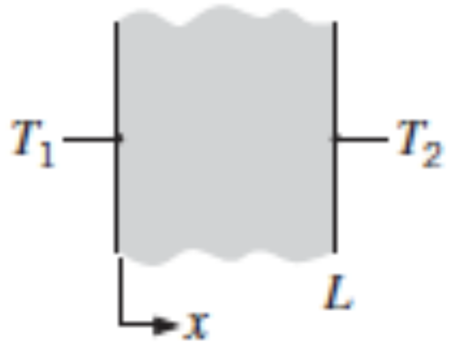
$$q' = \frac{q}{L} = 15\ \mathrm{W/m^2 \cdot K}\,(\pi \times 0.07\,\mathrm{m})(200 - 25)^\circ\mathrm{C}$$
$$+\,0.8(\pi \times 0.07\ \mathrm{m})\ 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}(473^4 - 298^4)\,\mathrm{K^4}$$
$$q' = 577\ \mathrm{W/m} + 421\ \mathrm{W/m} = 998\ \mathrm{W/m}$$



Problem - 2

Consider steady-state conditions for one-dimensional conduction in a plane wall having a thermal conductivity k = 50 W/m · K and a thickness L = 0.25 m, with no internal heat generation.

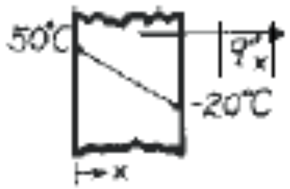
Determine the heat flux and the unknown quantity for each case and sketch the temperature distribution, indicating the direction of the heat flux.



Case	T_1(°C)	T_2(°C)	dT/dx (K/m)
1	50	–20	
2	–30	–10	
3	70		160
4		40	–80
5		30	200

Using Eqs. (1) and (2), the unknown quantities for each case can be determined.

(a)
$$\frac{dT}{dx} = \frac{(-20-50)\,K}{0.25\,m} = -280\ K/m$$
$$q_x'' = -50\,\frac{W}{m\cdot K} \times \left[-280\,\frac{K}{m}\right] = 14.0\ kW/m^2.$$



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(b)
$$\frac{dT}{dx} = \frac{(-10-(-30))\,K}{0.25\,m} = 80\ K/m$$
$$q_x'' = -50\,\frac{W}{m\cdot K} \times \left[80\,\frac{K}{m}\right] = -4.0\ kW/m^2.$$

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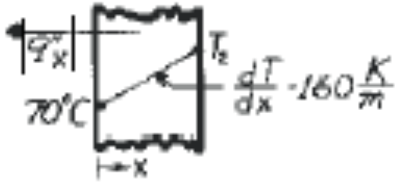
(c)
$$q_x'' = -50\,\frac{W}{m\cdot K} \times \left[160\,\frac{K}{m}\right] = -8.0\ kW/m^2$$
$$T_2 = L\cdot\frac{dT}{dx} + T_1 = 0.25\,m \times \left[160\,\frac{K}{m}\right] + 70^\circ C.$$
$$T_2 = 110^\circ C.$$



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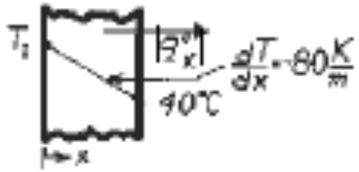
(d)
$$q_x'' = -50\,\frac{W}{m\cdot K} \times \left[-80\,\frac{K}{m}\right] = 4.0\ kW/m^2$$
$$T_1 = T_2 - L\cdot\frac{dT}{dx} = 40^\circ C - 0.25\,m\left[-80\,\frac{K}{m}\right]$$
$$T_1 = 60^\circ C.$$



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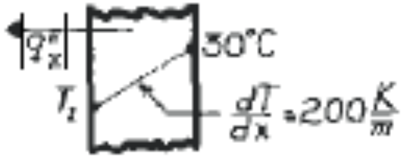
(e)
$$q_x'' = -50\,\frac{W}{m\cdot K} \times \left[200\,\frac{K}{m}\right] = -10.0\ kW/m^2$$
$$T_1 = T_2 - L\cdot\frac{dT}{dx} = 30^\circ C - 0.25\,m\left[200\,\frac{K}{m}\right] = -20^\circ C.$$



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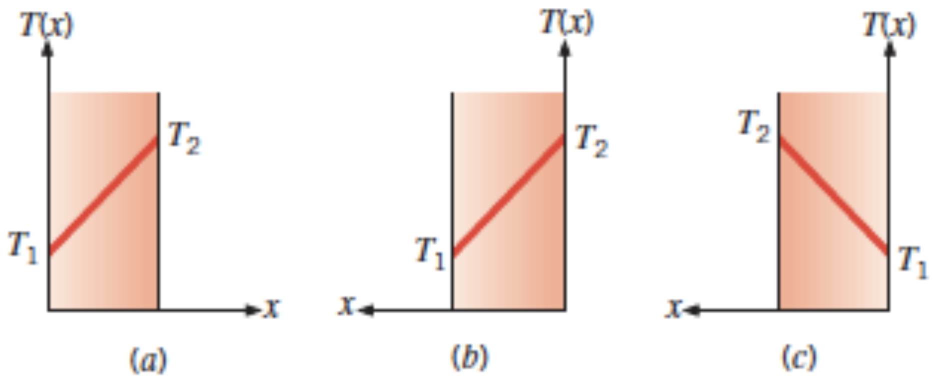
Problem - 3

Consider a plane wall 120 mm thick and of thermal conductivity 120 W/m · K. Steady-state conditions are known to exist with $T_1 = 500$ K and $T_2 = 700$ K. Determine the heat flux q_x'' and the temperature gradient dT/dx for the coordinate systems shown.



Problem -4

A cylindrical rod of stainless steel is insulated on its exterior surface except for the ends. The steady-state temperature distribution is $T(x) = a - bx/L$, where $a =$ 305 K and $b = 10$ K. The diameter and length of the rod are $D = 20$ mm and $L = 100$ mm, respectively. Determine the heat flux along the rod, q''_x. *Hint:* The mass of the rod is $m = 0.248$ kg.

ANALYSIS: The heat flux can be found from Fourier's law,

$$q_x'' = -k\frac{dT}{dx}$$

Table A.1 gives values for the thermal conductivity of stainless steels, however we are not told which type of stainless steel the rod is made of, and the thermal conductivity varies between them. We do know the mass of the rod, and can use this to calculate its density:

$$\rho = \frac{M}{V} = \frac{M}{\pi D^2 L/4} = \frac{0.248 \text{ kg}}{\pi \times (0.02 \text{ m})^2 \times 0.1 \text{ m}/4} = 7894 \text{ kg/m}^3$$

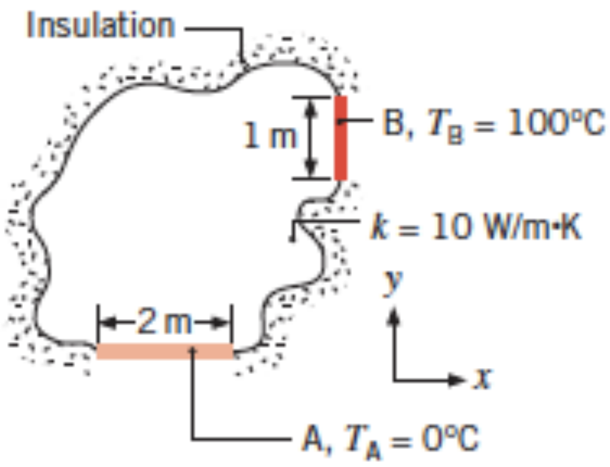
From Table A.1, it appears that the material is AISI 304 stainless steel. The temperature of the rod varies from 295 K to 305 K. Evaluating the thermal conductivity at 300 K, $k = 14.9$ W/m·K. Thus,

$$q_x'' = -k\frac{dT}{dx} = -k(-b/L) = 14.9 \text{ W/m}\cdot\text{K} \times 10 \text{ K}/0.1 \text{ m} = 1490 \text{ W/m}^2 \qquad <$$

COMMENTS: If the temperature of the rod varies significantly along its length, the thermal conductivity will vary along the rod as much or more than the variation in thermal conductivities between the different stainless steels.

Problem -5

In the two-dimensional body illustrated, the gradient at surface A is found to be $\partial T/\partial y = 30$ K/m. What are $\partial T/\partial y$ and $\partial T/\partial x$ at surface B?



Assumptions: thickness of the body is constant
The body is at S.S.

Problem -6

The steady-state temperature distribution in a one-dimensional wall of thermal conductivity 50 W/m · K and thickness 50 mm is observed to be $T(°C) = a + bx^2$, where $a = 200°C$, $b = -2000°C/m^2$, and x is in meters.

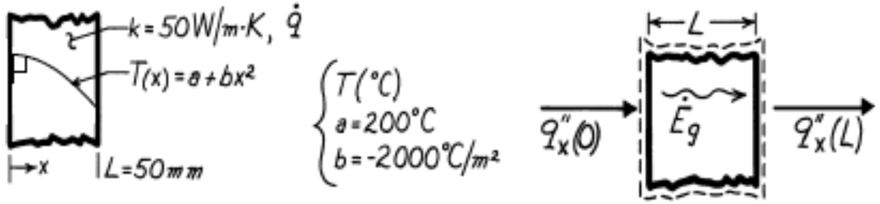
(a) What is the heat generation rate $\dot{q}$ in the wall?

(b) Determine the heat fluxes at the two wall faces. In what manner are these heat fluxes related to the heat generation rate?

KNOWN: Temperature distribution in a one-dimensional wall with prescribed thickness and thermal conductivity.

FIND: (a) The heat generation rate, $\dot{q}$, in the wall, (b) Heat fluxes at the wall faces and relation to $\dot{q}$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat flow, (3) Constant properties.

ANALYSIS: (a) The appropriate form of the heat equation for steady-state, one-dimensional conditions with constant properties is Eq. 2.21 re-written as

$$\dot{q}=-k\frac{d}{dx}\left[\frac{dT}{dx}\right]$$

Substituting the prescribed temperature distribution,

$$\dot{q}=-k\frac{d}{dx}\left[\frac{d}{dx}\left(a+bx^2\right)\right]=-k\frac{d}{dx}[2bx]=-2bk$$

$$\dot{q}=-2\left(-2000^\circ C/m^2\right)\times 50\ W/m\cdot K=2.0\times 10^5\ W/m^3.$$ <

(b) The heat fluxes at the wall faces can be evaluated from Fourier's law,

$$q''_x(x)=-k\left.\frac{dT}{dx}\right]_x.$$

Using the temperature distribution T(x) to evaluate the gradient, find

$$q''_x(x)=-k\frac{d}{dx}\left[a+bx^2\right]=-2kbx.$$

The fluxes at x = 0 and x = L are then

$$q''_x(0)=0$$ <

$$q''_x(L)=-2kbL=-2\times 50W/m\cdot K\left(-2000^\circ C/m^2\right)\times 0.050m$$

$$q''_x(L)=10,000\ W/m^2.$$ <

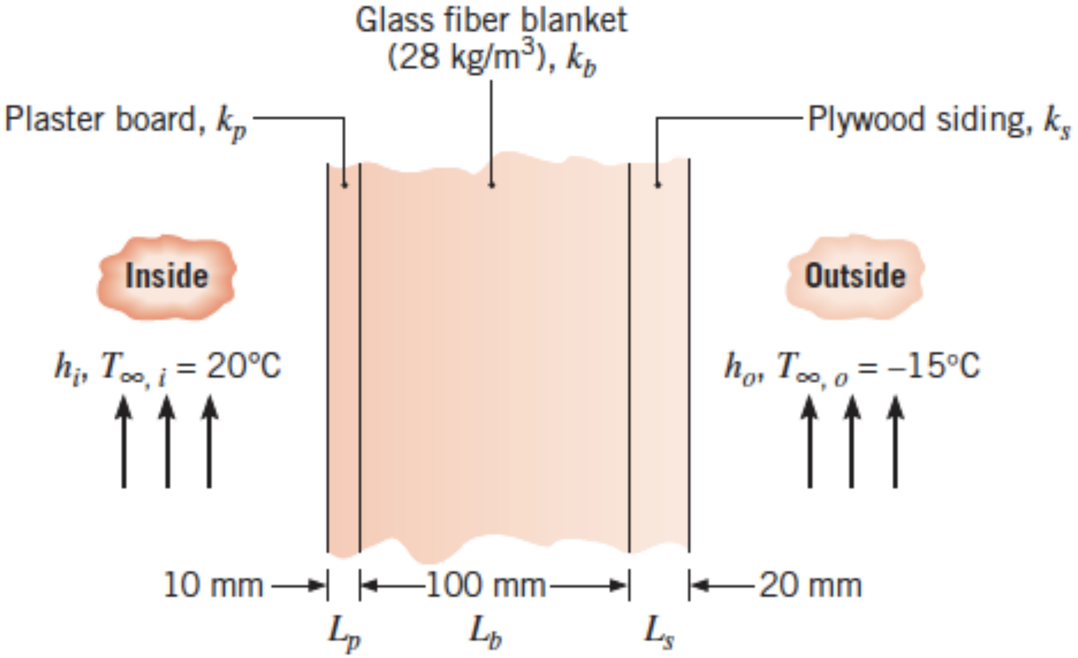
COMMENTS: From an overall energy balance on the wall, it follows that, for a unit area,

$$\dot{E}_{in}-\dot{E}_{out}+\dot{E}_g=0 \qquad q''_x(0)-q''_x(L)+\dot{q}L=0$$

$$\dot{q}=\frac{q''_x(L)-q''_x(0)}{L}=\frac{10,000\ W/m^2-0}{0.050m}=2.0\times 10^5\ W/m^3.$$

Problem -7

A house has a composite wall of wood, fiberglass insulation, and plaster board, as indicated in the sketch. On a cold winter day, the convection heat transfer coefficients are $h_o = 60\ \text{W/m}^2 \cdot \text{K}$ and $h_i = 30\ \text{W/m}^2 \cdot \text{K}$. The total wall surface area is $350\ \text{m}^2$.



(a) Determine a symbolic expression for the total thermal resistance of the wall, including inside and outside convection effects for the prescribed conditions.

(b) Determine the total rate of heat loss through the wall.

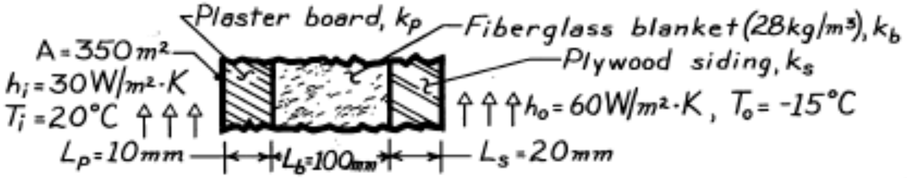
(c) If the wind were blowing violently, raising h_o to $300\ \text{W/m}^2 \cdot \text{K}$, determine the percentage increase in the rate of heat loss.

(d) What is the controlling resistance that determines the amount of heat flow through the wall?

KNOWN: Composite wall of a house with prescribed convection processes at inner and outer surfaces.

FIND: (a) Expression for thermal resistance of house wall, R_{tot}; (b) Total heat loss, q(W); (c) Effect on heat loss due to increase in outside heat transfer convection coefficient, h_o; and (d) Controlling resistance for heat loss from house.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) Negligible contact resistance.

PROPERTIES: *Table A-3*, $\left(\overline{T} = (T_i + T_o)/2 = (20-15)^\circ C/2 = 2.5^\circ C \approx 300K\right)$: Fiberglass blanket, 28 kg/m^3, k_b = 0.038 W/m·K; Plywood siding, k_s = 0.12 W/m·K; Plasterboard, k_p = 0.17 W/m·K.

ANALYSIS: (a) The expression for the total thermal resistance of the house wall follows from Eq. 3.18.

$$R_{tot} = \frac{1}{h_i A} + \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_s}{k_s A} + \frac{1}{h_o A}. \qquad <$$

(b) The total heat loss through the house wall is

$$q = \Delta T / R_{tot} = (T_i - T_o)/R_{tot}.$$

Substituting numerical values, find

$$R_{tot} = \frac{1}{30 W/m^2 \cdot K \times 350 m^2} + \frac{0.01m}{0.17 W/m \cdot K \times 350 m^2} + \frac{0.10m}{0.038 W/m \cdot K \times 350 m^2}$$
$$+ \frac{0.02m}{0.12 W/m \cdot K \times 350 m^2} + \frac{1}{60 W/m^2 \cdot K \times 350 m^2}$$
$$R_{tot} = [9.52 + 16.8 + 752 + 47.6 + 4.76] \times 10^{-5}\ ^\circ C/W = 831 \times 10^{-5}\ ^\circ C/W$$

The heat loss is then,

$$q = [20 - (-15)]^\circ C / 831 \times 10^{-5}\ ^\circ C/W = 4.21\ kW. \qquad <$$

(c) If h_o changes from 60 to 300 W/m^2·K, $R_o = 1/h_o A$ changes from 4.76 × 10^{-5} °C/W to 0.95 × 10^{-5} °C/W. This reduces R_{tot} to 826 × 10^{-5} °C/W, which is a 0.6% decrease and hence a 0.6% increase in q.

(d) From the expression for R_{tot} in part (b), note that the insulation resistance, $L_b/k_b A$, is 752/830 ≈ 90% of the total resistance. Hence, this material layer controls the resistance of the wall. From part (c) note that a 5-fold decrease in the outer convection resistance due to an increase in the wind velocity has a negligible effect on the heat loss.