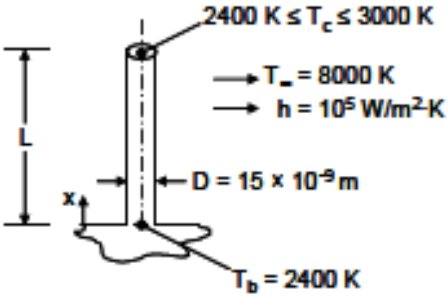
PROBLEM 3.109

KNOWN: Diameter and base temperature of a silicon carbide nanowire, required temperature of the catalyst tip.

FIND: Maximum length of a nanowire that may be grown under specified conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Nanowire stops growing when $T_c = T(x = L) = 3000$ K, (2) Constant properties, (3) One-dimensional heat transfer, (4) Convection from the tip of the nanowire, (5) Nanowire grows very slowly, (6) Negligible impact of nanoscale heat transfer effects.

PROPERTIES: Table A.2, silicon carbide (1500 K): $k = 30$ W/m·K.

ANALYSIS: The tip of the nanowire is initially at T = 2400 K, and increases in temperature as the nanowire becomes longer. At steady-state, the tip reaches T = 3000 K. The temperature distribution at steady-state is given by Eq. 3.75:

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L - x) + (h/mk)\sinh m(L - x)}{\cosh mL + (h/mk)\sinh mL} \tag{1}$$

where

$$m = \left(\frac{hP}{kA_c}\right)^{1/2} = \left(\frac{4h}{kD}\right)^{1/2} = \left(\frac{4 \times 10^5 \text{ W/m}^2 \cdot \text{K}}{30 \text{ W/m} \cdot \text{K} \times 15 \times 10^{-9} \text{ m}}\right)^{1/2} = 943 \times 10^3 \text{ m}^{-1}$$

and

$$\frac{h}{mk} = \frac{10^5 \text{ W/m}^2 \cdot \text{K}}{943 \times 10^3 \text{ m}^{-1} \times 30 \text{ W/m} \cdot \text{K}} = 3.53 \times 10^{-3}$$

Equation 1, evaluated at x = L, is

$$\frac{\theta}{\theta_b} = \frac{(3000 - 8000) \text{ K}}{(2400 - 8000) \text{ K}} = 0.893 = \frac{1}{\cosh(943 \times 10^3 \times L) + 3.53 \times 10^{-3} \sinh(943 \times 10^3 \times L)}$$

A trial-and-error solution yields $L = 510 \times 10^{-9}$ m = 510 nm <

Continued…

PROBLEM 3.109 (Cont.)

COMMENTS: (1) The importance of radiation heat transfer may be ascertained by evaluating Eq. 1.9. Assuming large surroundings at a temperature of T_{sur} = 8000 K and an emissivity of unity, the radiation heat transfer coefficient at the fin tip is

$$h_r = \varepsilon\sigma(T(x = L) + T_{sur})\left[T^2(x{=}L) + T_{sur}^2\right]$$

$$= 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4 \times (3000\ \text{K} + 8000\ \text{K}) \times \left[(3000\ \text{K})^2 + (8000\ \text{K})^2\right] = 4.5 \times 10^4\ \text{W/m}^2 \cdot \text{K}$$

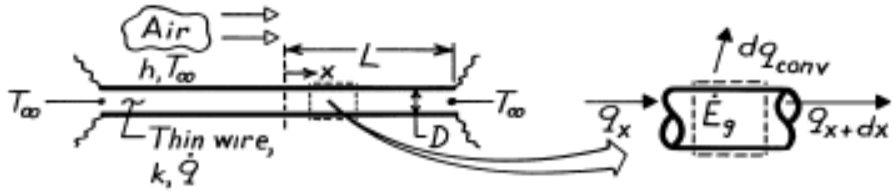
We see that $h_r < h$, but radiation may be important. (2) The thermal conductivity has been evaluated at 1500 K and extrapolated to a much higher temperature. More accurate values of the thermal conductivity, accounting for the high temperature and possible nanoscale heat transfer effects, are desirable. (3) If the nanowire were to grow rapidly, the transient temperature distribution within the nanowire would need to be evaluated.

PROBLEM 3.116

KNOWN: Thermal conductivity, diameter and length of a wire which is annealed by passing an electrical current through the wire.

FIND: (a) Steady-state temperature distribution along wire, (b) Maximum wire temperature, (c) Average wire temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction along the wire, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient h.

ANALYSIS: (a) Applying conservation of energy to a differential control volume,

$$q_x + \dot{E}_g - dq_{conv} - q_{x+dx} = 0$$

$$q_{x+dx} = q_x + \frac{dq_x}{dx}dx \qquad q_x = -k\left(\pi D^2/4\right)dT/dx$$

$$dq_{conv} = h\left(\pi D\, dx\right)\left(T - T_\infty\right) \qquad \dot{E}_g = \dot{q}\left(\pi D^2/4\right)dx.$$

Hence,

$$k\left(\pi D^2/4\right)\frac{d^2T}{dx^2}dx + \dot{q}\left(\pi D^2/4\right)dx - h\left(\pi Ddx\right)\left(T - T_\infty\right) = 0$$

or, with $\theta \equiv T - T_\infty$,

$$\frac{d^2\theta}{dx^2} - \frac{4h}{kD}\theta + \frac{\dot{q}}{k} = 0$$

The solution (general and particular) to this nonhomogeneous equation is of the form

$$\theta = C_1\, e^{mx} + C_2\, e^{-mx} + \frac{\dot{q}}{km^2}$$

where $m^2 = (4h/kD)$. The boundary conditions are:

$$\left.\frac{d\theta}{dx}\right|_{x=0} = 0 = m\, C_1\, e^0 - mC_2\, e^0 \rightarrow \qquad C_1 = C_2$$

$$\theta(L) = 0 = C_1\left(e^{mL} + e^{-mL}\right) + \frac{\dot{q}}{km^2} \rightarrow \qquad C_1 = \frac{-\dot{q}/km^2}{e^{mL} + e^{-mL}} = C_2$$

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The temperature distribution has the form

$$T = T_\infty - \frac{\dot{q}}{km^2}\left[\frac{e^{mx}+e^{-mx}}{e^{mL}+e^{-mL}}-1\right] = T_\infty - \frac{\dot{q}}{km^2}\left[\frac{\cosh mx}{\cosh mL}-1\right]. \quad <$$

(b) The maximum wire temperature exists at $x = 0$. Hence,

$$T_{max} = T(x=0) = T_\infty - \frac{\dot{q}}{km^2}\left[\frac{\cosh(0)}{\cosh(mL)}-1\right] = T_\infty - \frac{\dot{q}}{km^2}\left[\frac{1}{\cosh(mL)}-1\right] \quad <$$

(c) The average wire temperature may be obtained by evaluating the expression

$$\overline{T} = \frac{1}{L}\int_{x=0}^{L} T(x)dx = \frac{1}{L}\int_{x=0}^{L}\left[T_\infty - \frac{\dot{q}}{km^2}\left[\frac{\cosh(mx)}{\cosh(mL)}-1\right]\right]dx$$

$$= T_\infty + \frac{\dot{q}}{km^2} - \tanh(mL)\frac{\dot{q}}{Lkm^3} \quad <$$

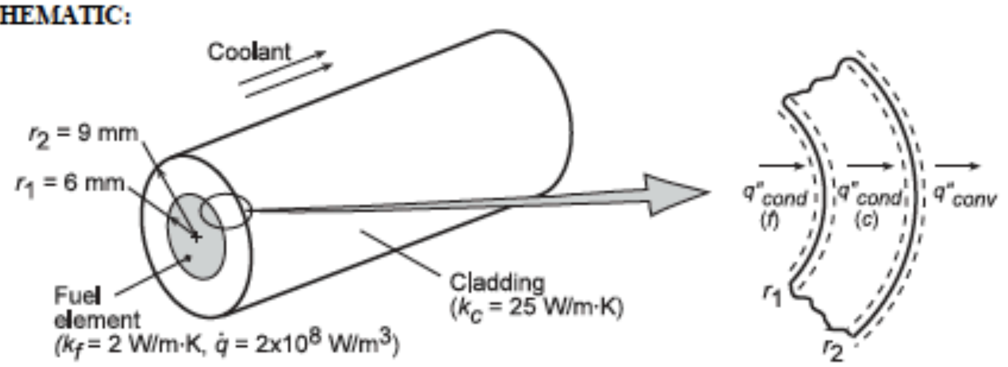
COMMENTS: (1) This process is commonly used to anneal wire and spring products. It is also used for flow measurement based upon the principle that the maximum or average wire temperature varies with the value of m and, hence, the convective heat transfer coefficient h and, ultimately, the fluid velocity. (2) To check the result of part (a), note that $T(L) = T(-L) = T_\infty$.

PROBLEM 3.98

KNOWN: Radii and thermal conductivities of reactor fuel element and cladding. Fuel heat generation rate. Temperature and convection coefficient of coolant.

FIND: (a) Expressions for temperature distributions in fuel and cladding, (b) Maximum fuel element temperature for prescribed conditions, (c) Effect of h on temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible contact resistance, (4) Constant properties.

ANALYSIS: (a) From Eqs. 3.54 and 3.28, the heat equations for the fuel (f) and cladding (c) are

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT_f}{dr}\right)=-\frac{\dot{q}}{k_f} \qquad (0 \le r \le r_1) \qquad \frac{1}{r}\frac{d}{dr}\left(r\frac{dT_c}{dr}\right)=0 \qquad (r_1 \le r \le r_2)$$

Hence, integrating both equations twice,

$$\frac{dT_f}{dr}=-\frac{\dot{q}r}{2k_f}+\frac{C_1}{k_f r} \qquad T_f=-\frac{\dot{q}r^2}{4k_f}+\frac{C_1}{k_f}\ln r+C_2 \qquad (1,2)$$

$$\frac{dT_c}{dr}=\frac{C_3}{k_c r} \qquad T_c=\frac{C_3}{k_c}\ln r+C_4 \qquad (3,4)$$

The corresponding boundary conditions are:

$$dT_f/dr)_{r=0}=0 \qquad T_f(r_1)=T_c(r_1) \qquad (5,6)$$

$$-k_f\frac{dT_f}{dr}\bigg)_{r=r_1}=-k_c\frac{dT_c}{dr}\bigg)_{r=r_1} \qquad -k_c\frac{dT_c}{dr}\bigg)_{r=r_2}=h\left[T_c(r_2)-T_\infty\right] \qquad (7,8)$$

Note that Eqs. (7) and (8) are obtained from surface energy balances at r_1 and r_2, respectively. Applying Eq. (5) to Eq. (1), it follows that $C_1 = 0$. Hence,

$$T_f=-\frac{\dot{q}r^2}{4k_f}+C_2 \qquad (9)$$

From Eq. (6), it follows that

$$-\frac{\dot{q}r_1^2}{4k_f}+C_2=\frac{C_3\ln r_1}{k_c}+C_4 \qquad (10)$$

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Also, from Eq. (7),

$$\frac{\dot{q}r_1}{2} = -\frac{C_3}{r_1} \qquad \text{or} \qquad C_3 = -\frac{\dot{q}r_1^2}{2} \tag{11}$$

Finally, from Eq. (8), $-\frac{C_3}{r_2} = h\left[\frac{C_3}{k_c}\ln r_2 + C_4 - T_\infty\right]$ or, substituting for C_3 and solving for C_4

$$C_4 = \frac{\dot{q}r_1^2}{2r_2 h} + \frac{\dot{q}r_1^2}{2k_c}\ln r_2 + T_\infty \tag{12}$$

Substituting Eqs. (11) and (12) into (10), it follows that

$$C_2 = \frac{\dot{q}r_1^2}{4k_f} - \frac{\dot{q}r_1^2 \ln r_1}{2k_c} + \frac{\dot{q}r_1^2}{2r_2 h} + \frac{\dot{q}r_1^2}{2k_c}\ln r_2 + T_\infty$$

$$C_2 = \frac{\dot{q}r_1^2}{4k_f} + \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r_1} + \frac{\dot{q}r_1^2}{2r_2 h} + T_\infty \tag{13}$$

Substituting Eq. (13) into (9),

$$T_f = \frac{\dot{q}}{4k_f}\left(r_1^2 - r^2\right) + \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r_1} + \frac{\dot{q}r_1^2}{2r_2 h} + T_\infty \tag{14}$$ <

Substituting Eqs. (11) and (12) into (4),

$$T_c = \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r} + \frac{\dot{q}r_1^2}{2r_2 h} + T_\infty . \tag{15}$$ <

(b) Applying Eq. (14) at r = 0, the maximum fuel temperature for $h = 2000\ \text{W/m}^2\cdot\text{K}$ is

$$T_f(0) = \frac{2\times10^8\ \text{W/m}^3 \times (0.006\,\text{m})^2}{4\times 2\ \text{W/m}\cdot\text{K}} + \frac{2\times10^8\ \text{W/m}^3 \times (0.006\,\text{m})^2}{2\times 25\ \text{W/m}\cdot\text{K}}\ln\frac{0.009\,\text{m}}{0.006\,\text{m}}$$

$$+\frac{2\times10^8\ \text{W/m}^3\,(0.006\,\text{m})^2}{2\times(0.009\,\text{m})\,2000\ \text{W/m}^2\cdot\text{K}} + 300\,\text{K}$$

$$T_f(0) = (900 + 58.4 + 200 + 300)\,\text{K} = 1458\,\text{K}.$$ <