HEAT EXCHANGERS

The process of heat exchange between two fluids that are at different temperatures and separated by a solid wall occurs in many engineering applications. The device used to implement this exchange is termed a heat exchanger, and specific applications may be found in space heating and air-conditioning, power production, waste heat recovery, and chemical processing.

HEAT EXCHANGER TYPES

Heat exchangers are typically classified according to flow arrangement and type of construction. The simplest heat exchanger is one for which the hot and cold fluids move in the same or opposite directions in a concentric tube (or double-pipe) construction. In the parallel-flow arrangement, the hot and cold fluids enter at the same end, flow in the same direction, and leave at the same end. In the counterflow arrangement, the fluids enter at opposite ends, flow in opposite directions, and leave at opposite ends. Alternatively, the fluids may move in cross flow (perpendicular to each other), as shown by the finned and unfinned tubular heat exchangers.

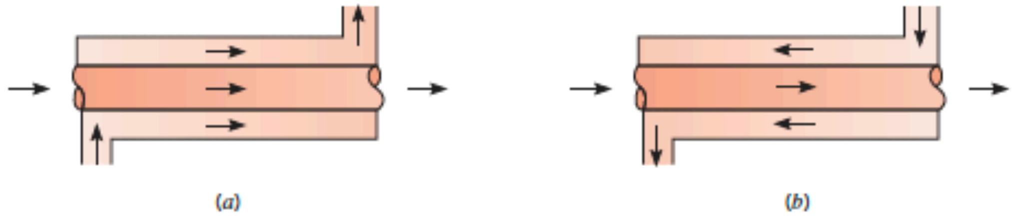


FIGURE 11.1 Concentric tube heat exchangers. (*a*) Parallel flow. (*b*) Counterflow.

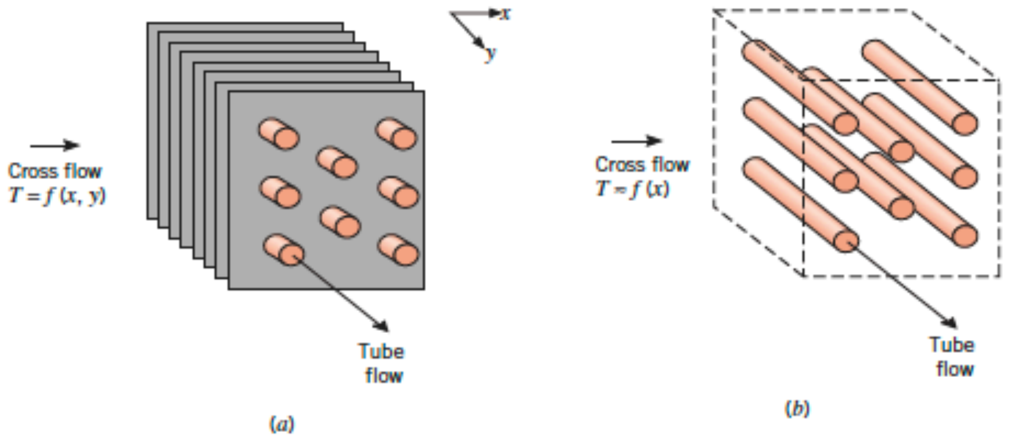


FIGURE 11.2 Cross-flow heat exchangers. (*a*) Finned with both fluids unmixed. (*b*) Unfinned with one fluid mixed and the other unmixed.

Another common configuration is the shell-and-tube heat exchanger. Specific forms differ according to the number of shell-and-tube passes, and the simplest form, which involves single tube and shell passes.

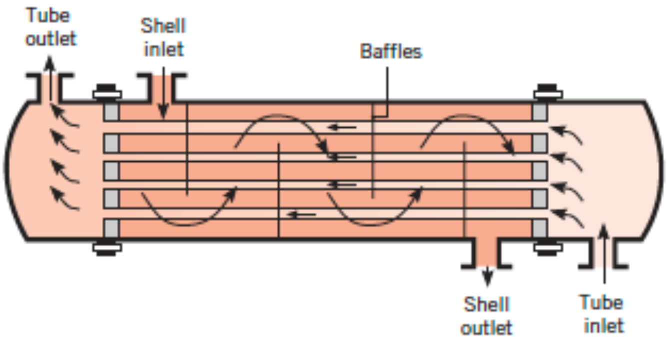


FIGURE 11.3 Shell-and-tube heat exchanger with one shell pass and one tube pass (cross-counterflow mode of operation).

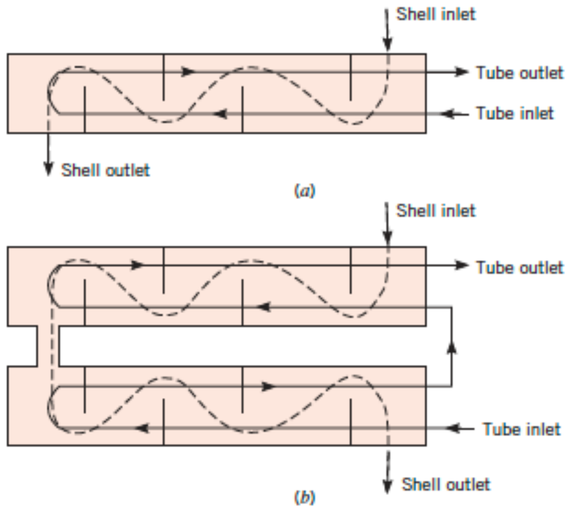
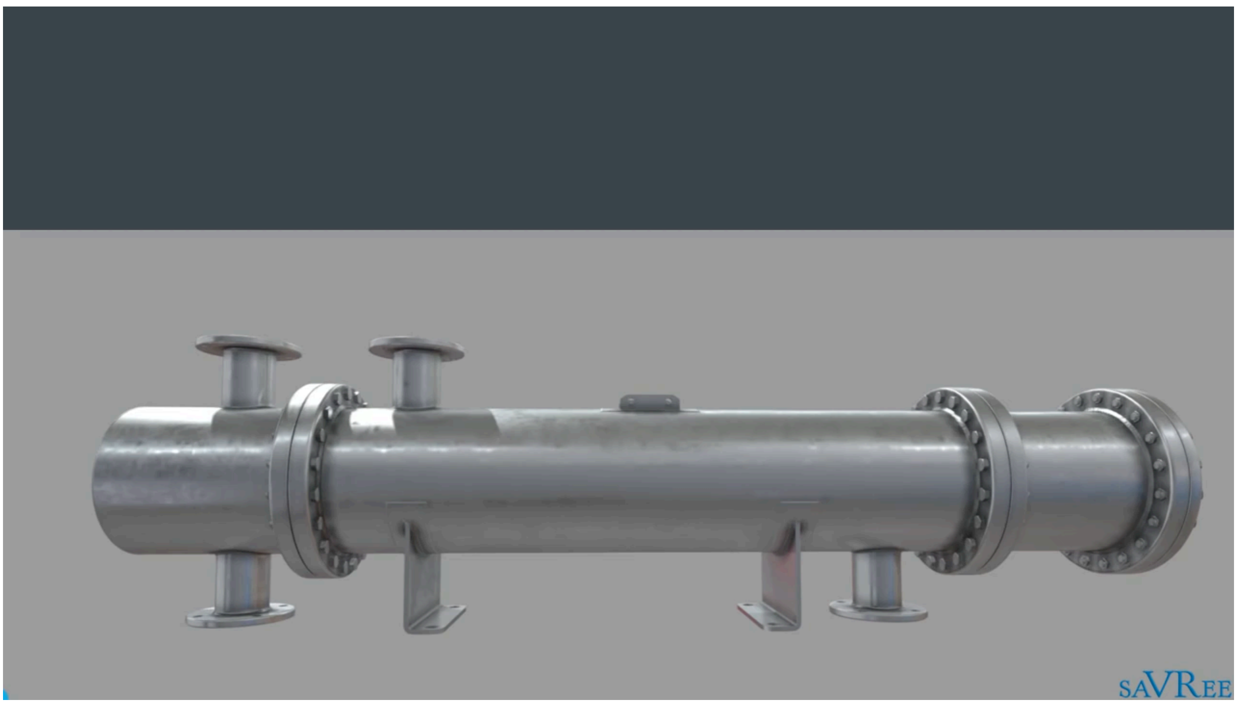


FIGURE 11.4 Shell-and-tube heat exchangers. (*a*) One shell pass and two tube passes. (*b*) Two shell passes and four tube passes.



A special and important class of heat exchangers is used to achieve a very large (400 m^2/m^3 for liquids and 700 m^2/m^3 for gases) heat transfer surface area per unit volume.

The Overall Heat Transfer Coefficient, U

An essential, and often the most uncertain, part of any heat exchanger analysis is determination of the overall heat transfer coefficient. Recall from Equation 3.19 that this coefficient is defined in terms of the total thermal resistance to heat transfer between two fluids. In Equations 3.18 and 3.36, the coefficient was determined by accounting for conduction and convection resistances between fluids separated by composite plane and cylindrical walls, respectively. For a wall separating two fluid streams, the overall heat transfer coefficient may be expressed as

$$\frac{1}{UA} = \frac{1}{U_c A_c} = \frac{1}{U_h A_h} = \frac{1}{(hA)_c} + R_w + \frac{1}{(hA)_h}$$

$$\frac{1}{UA} = \frac{1}{(\eta_o hA)_c} + \frac{R''_{f,c}}{(\eta_o A)_c} + R_w + \frac{R''_{f,h}}{(\eta_o A)_h} + \frac{1}{(\eta_o hA)_h}$$

$$q = \eta_o hA(T_b - T_\infty)$$

$$\eta_o = 1 - \frac{A_f}{A}(1 - \eta_f)$$

TABLE 11.1 Representative Fouling Factors [1]

Fluid	R'' ($m^2 \cdot K/W$)
Seawater and treated boiler feedwater (below 50°C)	0.0001
Seawater and treated boiler feedwater (above 50°C)	0.0002
River water (below 50°C)	0.0002–0.001
Fuel oil	0.0009
Refrigerating liquids	0.0002
Steam (nonoil bearing)	0.0001

$$\eta_f = \frac{\tanh(mL)}{mL}$$

where $m=(2h/kt)^{1/2}$ and t is the fin thickness

For the unfinned, tubular heat exchangers:

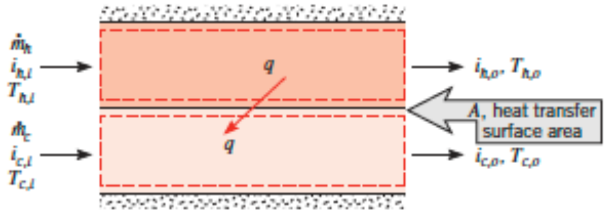
$$\frac{1}{UA} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{R''_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{R''_{f,o}}{A_o} + \frac{1}{h_o A_o}$$

where subscripts i and o refer to inner and outer tube surfaces which may be exposed to either the hot or the cold fluid.

Heat Exchanger Analysis: Use of the Log Mean Temperature Difference

To design or to predict the performance of a heat exchanger, it is essential to relate the total heat transfer rate to quantities such as the inlet and outlet fluid temperatures, the overall heat transfer coefficient, and the total surface area for heat transfer.

$$q = UA\Delta T_m$$



since T varies with position in the heat exchanger, it is necessary to work with a rate equation of the form where T_m is an appropriate mean temperature difference.

The Parallel-Flow Heat Exchanger

The hot and cold mean fluid temperature distributions associated with a parallel-flow heat exchanger are shown in Figure.

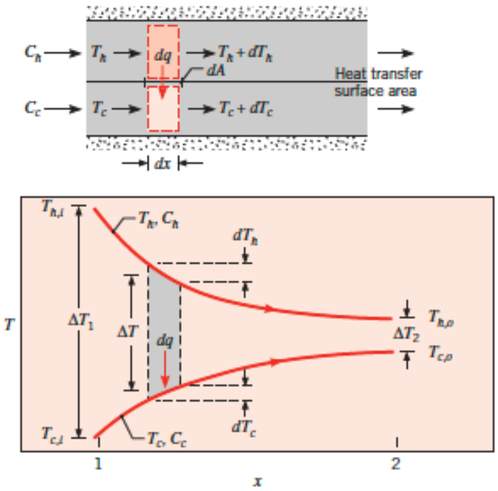


Figure 11.7 Temperature distributions for a parallel-flow heat exchanger.

The energy balances and the subsequent analysis are subject to the following assumptions:

1. The heat exchanger is insulated from its surroundings, in which case the only heat exchange is between the hot and cold fluids.
2. Axial conduction along the tubes is negligible.
3. Potential and kinetic energy changes are negligible.
4. The fluid specific heats are constant.
5. The overall heat transfer coefficient is constant.

$$q = UA\,\Delta T_{\text{lm}} \tag{11.14}$$

where

$$\Delta T_{\text{lm}} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} \tag{11.15}$$

Remember that, for the *parallel-ow exchanger* ,

$$\begin{bmatrix} \Delta T_1 \equiv T_{h,1} - T_{c,1} = T_{h,i} - T_{c,i} \\ \Delta T_2 \equiv T_{h,2} - T_{c,2} = T_{h,o} - T_{c,o} \end{bmatrix} \tag{11.16}$$

The Counterflow Heat Exchanger

The hot and cold fluid temperature distributions associated with a counterflow heat exchanger are shown in Figure 11.8. In contrast to the parallel-flow exchanger, this configuration provides for heat transfer between the hotter portions of the two fluids at one end, as well as between the colder portions at the other. For this reason, the change in the temperature difference, $\Delta T = T_h - T_c$, with respect to x is nowhere as large as it is for the inlet region of the parallel-flow exchanger.

The hot and cold fluid temperature distributions associated with a counterflow heat exchanger are shown in Figure

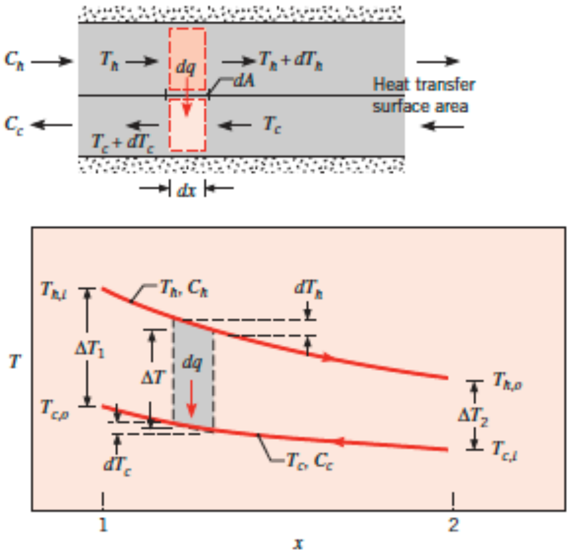


FIGURE 11.8 Temperature distributions for a counterflow heat exchanger.

$$\begin{bmatrix} \Delta T_1 \equiv T_{h,1} - T_{c,1} = T_{h,i} - T_{c,o} \\ \Delta T_2 \equiv T_{h,2} - T_{c,2} = T_{h,o} - T_{c,i} \end{bmatrix}$$

Heat Exchanger Analysis: The Effectiveness–NTU Method

It is a simple matter to use the log mean temperature difference (LMTD) method of heat exchanger analysis when the fluid inlet temperatures are known and the outlet temperatures are specified or readily determined from the energy balance expressions. The value of ΔT_m for the exchanger may then be determined. However, if only the inlet temperatures are known, use of the LMTD method requires a cumbersome iterative procedure. It is therefore preferable to employ an alternative approach termed the **effectiveness–NTU (or NTU) method.**

Number of transfer units = NTU

To define the *effectiveness of a heat exchanger*, we must first determine the *maximum possible heat transfer rate*, q_{max}, for the exchanger. This heat transfer rate could, in principle, be achieved in a counterflow heat exchanger (Figure 11.8) of infinite length. In such an exchanger, one of the fluids would experience the maximum possible temperature difference, $T_{h,i} - T_{c,i}$. To illustrate this point, consider a situation for which $C_c < C_h$, in which case, from Equations 11.10 and 11.11, $|dT_c| > |dT_h|$. The cold fluid would then experience the larger temperature change, and since $L \rightarrow \infty$, it would be heated to the inlet temperature of the hot fluid ($T_{c,o} = T_{h,i}$). Accordingly, from Equation 11.7b,

$$C_c < C_h: \qquad q_{max} = C_c(T_{h,i} - T_{c,i})$$

Similarly, if $C_h < C_c$, the hot fluid would experience the larger temperature change and would be cooled to the inlet temperature of the cold fluid ($T_{h,o} = T_{c,i}$). From Equation 11.6b, we then obtain

$$C_h < C_c: \qquad q_{max} = C_h(T_{h,i} - T_{c,i})$$

From the foregoing results we are then prompted to write the general expression

$$q_{max} = C_{min}(T_{h,i} - T_{c,i}) \tag{11.18}$$

where C_{min} *is equal to* C_c or C_h, *whichever is smaller.* For prescribed hot and cold fluid inlet temperatures, Equation 11.18 provides the maximum heat transfer rate that could possibly be delivered by an exchanger. A quick mental exercise should convince the reader that the maximum possible heat transfer rate is *not* equal to $C_{max}(T_{h,i} - T_{c,i})$. If the fluid having the larger heat capacity rate were to experience the maximum possible temperature change, conservation of energy in the form $C_c(T_{c,o} - T_{c,i}) = C_h(T_{h,i} - T_{h,o})$ would require that the other fluid experience yet a larger temperature change. For example, if $C_{max} = C_c$ and one argues that it is possible for $T_{c,o}$ to be equal to $T_{h,i}$, it follows that $(T_{h,i} - T_{h,o}) = (C_c/C_h)(T_{h,i} - T_{c,i})$, in which case $(T_{h,i} - T_{h,o}) > (T_{h,i} - T_{c,i})$. Such a condition is clearly impossible.

It is now logical to define the *effectiveness*, ε, as the ratio of the actual heat transfer rate for a heat exchanger to the maximum possible heat transfer rate:

$$\varepsilon \equiv \frac{q}{q_{max}} \tag{11.19}$$

From Equations 11.6b, 11.7b, and 11.18, it follows that

$$\varepsilon = \frac{C_h(T_{h,i} - T_{h,o})}{C_{min}(T_{h,i} - T_{c,i})} \tag{11.20}$$

or

$$\varepsilon = \frac{C_c(T_{c,o} - T_{c,i})}{C_{\min}(T_{h,i} - T_{c,i})} \tag{11.21}$$

By definition the effectiveness, which is dimensionless, must be in the range $0 \leq \varepsilon \leq 1$. It is useful because, if ε, $T_{h,i}$, and $T_{c,i}$ are known, the actual heat transfer rate may readily be determined from the expression

$$q = \varepsilon C_{\min}(T_{h,i} - T_{c,i}) \tag{11.22}$$

For any heat exchanger it can be shown that [5]

$$\varepsilon = f\left(\mathrm{NTU}, \frac{C_{\min}}{C_{\max}}\right) \tag{11.23}$$

where $C_{\min}/C_{\max}$ is equal to C_c/C_h or C_h/C_c, depending on the relative magnitudes of the hot and cold fluid heat capacity rates. The *number of transfer units* (NTU) is a dimensionless parameter that is widely used for heat exchanger analysis and is defined as

$$\mathrm{NTU} \equiv \frac{UA}{C_{\min}} \tag{11.24}$$

TABLE 11.3 Heat Exchanger Effectiveness Relations [5]

Flow Arrangement	Relation		
Parallel ow	$\varepsilon = \dfrac{1 - \exp[-\mathrm{NTU}(1 + C_r)]}{1 + C_r}$		(11.28a)
Counterow	$\varepsilon = \dfrac{1 - \exp[-\mathrm{NTU}(1 - C_r)]}{1 - C_r \exp[-\mathrm{NTU}(1 - C_r)]}$	$(C_r < 1)$	
	$\varepsilon = \dfrac{\mathrm{NTU}}{1 + \mathrm{NTU}}$	$(C_r = 1)$	(11.29a)
Shell-and-tube			
One shell pass (2, 4, . . . tube passes)	$\varepsilon_1 = 2\left\{1 + C_r + (1 + C_r^2)^{1/2} \times \dfrac{1 + \exp[-(\mathrm{NTU})_1(1 + C_r^2)^{1/2}]}{1 - \exp[-(\mathrm{NTU})_1(1 + C_r^2)^{1/2}]}\right\}^{-1}$		(11.30a)
n shell passes ($2n$, $4n$, . . . tube passes)	$\varepsilon = \left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - 1\right]\left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - C_r\right]^{-1}$		(11.31a)
Cross-ow (single pass)			
Both fluids unmixed	$\varepsilon = 1 - \exp\left[\left(\dfrac{1}{C_r}\right)(\mathrm{NTU})^{0.22}\{\exp[-C_r(\mathrm{NTU})^{0.78}] - 1\}\right]$		(11.32)
$C_{\max}$ (mixed), $C_{\min}$ (unmixed)	$\varepsilon = \left(\dfrac{1}{C_r}\right)(1 - \exp\{-C_r[1 - \exp(-\mathrm{NTU})]\})$		(11.33a)
$C_{\min}$ (mixed), $C_{\max}$ (unmixed)	$\varepsilon = 1 - \exp(-C_r^{-1}\{1 - \exp[-C_r(\mathrm{NTU})]\})$		(11.34a)
All exchangers ($C_r = 0$)	$\varepsilon = 1 - \exp(-\mathrm{NTU})$		(11.35a)

TABLE 11.4 Heat Exchanger NTU Relations

Flow Arrangement	Relation		
Parallel ow	$\mathrm{NTU} = -\dfrac{\ln[1 - \varepsilon(1 + C_r)]}{1 + C_r}$		(11.28b)
Counterow	$\mathrm{NTU} = \dfrac{1}{C_r - 1} \ln\left(\dfrac{\varepsilon - 1}{\varepsilon C_r - 1}\right)$	$(C_r < 1)$	
	$\mathrm{NTU} = \dfrac{\varepsilon}{1 - \varepsilon}$	$(C_r = 1)$	(11.29b)
Shell-and-tube			
One shell pass (2, 4, . . . tube passes)	$(\mathrm{NTU})_1 = -(1 + C_r^2)^{-1/2} \ln\left(\dfrac{E - 1}{E + 1}\right)$		(11.30b)
	$E = \dfrac{2/\varepsilon_1 - (1 + C_r)}{(1 + C_r^2)^{1/2}}$		(11.30c)
n shell passes ($2n$, $4n$, . . . tube passes)	Use Equations 11.30b and 11.30c with		
	$\varepsilon_1 = \dfrac{F - 1}{F - C_r} \quad F = \left(\dfrac{\varepsilon C_r - 1}{\varepsilon - 1}\right)^{1/n} \quad \mathrm{NTU} = n(\mathrm{NTU})_1$		(11.31b, c, d)
Cross-ow (single pass)			
$C_{\max}$ (mixed), $C_{\min}$ (unmixed)	$\mathrm{NTU} = -\ln\left[1 + \left(\dfrac{1}{C_r}\right)\ln(1 - \varepsilon C_r)\right]$		(11.33b)
$C_{\min}$ (mixed), $C_{\max}$ (unmixed)	$\mathrm{NTU} = -\left(\dfrac{1}{C_r}\right)\ln[C_r \ln(1 - \varepsilon) + 1]$		(11.34b)
All exchangers ($C_r = 0$)	$\mathrm{NTU} = -\ln(1 - \varepsilon)$		(11.35b)