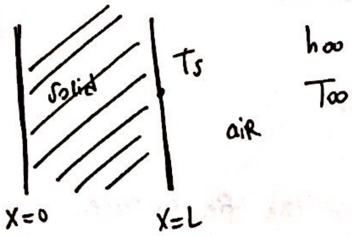
CHAPTER 3 - Heat From Extended Surfaces

Term extended surface is commonly used to depict an important special case involving heat transfer by conduction within a solid and heat transfer by convection (and/or radiation) from the boundaries of the solid.

Extended surfaces are used to increase heat transfer rate.



Plane wall

at $x=L$ [conv. heat t.r = cond. heat t.r]

$$q = hA(T_s - T_\infty) = -kA \left.\frac{dT}{dx}\right|_{x=L}$$

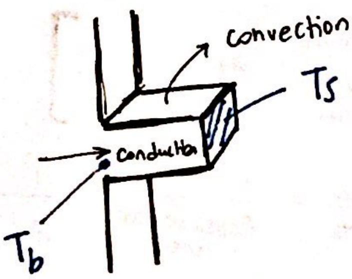
q can be increased by the following strategies:

- h can be increased by increasing air (fluid) velocity
- T_∞ can be reduced by using a coolant.
- A can be increased → Employing extended surfaces.

By attaching extended surfaces on the original surface the area across which convection occurs can be increased.

* Extended surfaces ⇒ **FINS**

* **Fins** are extended surfaces from the wall to the fluid.

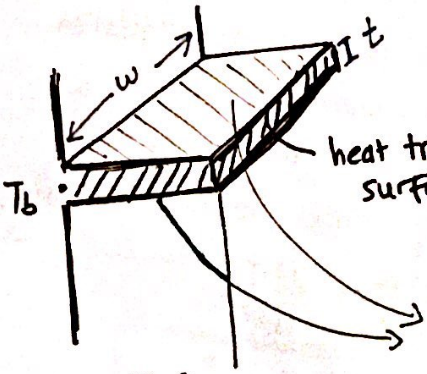


Rectangular fin

T_b : base temperature

T_s : tip temperature

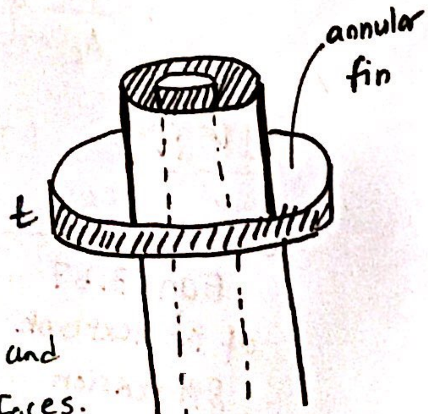
Ideally the fin material should have large k to maintain a uniform temperature along the fin (to minimize temperature variations from its base to its tip)



heat transfer from these surfaces is negligible.

for fins, heat transfer is considered for only upper and lower larger surfaces.

Straight fin
thickness : t
width : w] Rectangular fins



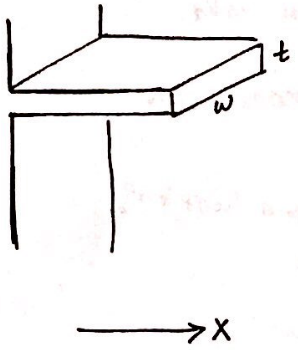
thickness : t
width : $2\pi r$

General Conduction Analysis for Fins

Normally we have 3-D conduction within the fin.

But 1-D conduction is assumed to obtain $T = f(x)$ because the associated distances are not long enough.

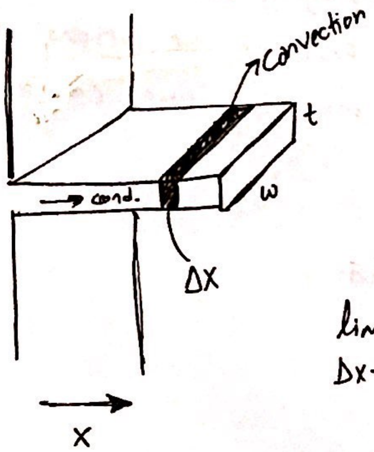
Wall



ΔT in y- and z-directions are small compared to ΔT change in x-direction. } $T = f(x)$ only.

Assume steady-state conduction
- constant k
- negligible Radiation
- no heat generation
- h is uniform over the fin surface.

Doing an energy balance for a differential volume element of fin:



$$\left[q_x''\big|_x - q_x''\big|_{x+\Delta x} \right] w.t - \left[q_{conv}'' \right] 2.w.\Delta x = 0$$

Divide the whole equation by $w.t.\Delta x$ and take $\lim_{\Delta x \to 0}$

$$\lim_{\Delta x \to 0} \frac{q_x''|_x - q_x''|_{x+\Delta x}}{\Delta x} - \frac{2}{t} q''_{conv} = 0$$

$$-\frac{dq_x''}{dx} - \frac{2}{t} q''_{conv} = 0 \qquad \left[\text{From the Fourier's Law} \quad q_x'' = -k\frac{dT}{dx} \right]$$

$$\frac{d^2T}{dx^2} - \frac{2}{t} q''_{conv} = 0 \qquad \left[\frac{2}{t} = \frac{\text{Perimeter}}{\text{Cross sectional area}} = \frac{P}{A_c} \right]$$

$$\frac{d^2T}{dx^2} - \frac{P}{A_c} q''_{conv} = 0$$

$$\underbrace{\frac{d^2T}{dx^2} - \frac{P.h}{A_c}(T - T_\infty) = 0}_{\text{Eqn 3.67 of the textbook 8th version}}$$

→ To simplify the form of this equation, we transform the dependent variable by defining an excess temperature θ

$\boxed{\theta = T - T_\infty}$ since T_∞ is constant

$$\frac{d\theta}{dx} = \frac{dT}{dx}, \quad \frac{d^2\theta}{dx^2} = \frac{d^2T}{dx^2}$$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \qquad \boxed{m^2 = \frac{P.h}{A_c}}$$