ALPHA DECAY

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SPONTANEOUS DECAY

Alpha particles were first identified as the least penetrating of the radiations emitted by naturally occurring materials.

Alpha decay is a process in which the parent nucleus disintegrates into a daughter nucleus and an alpha particle. Alpha particles are doubly ionized helium ions, He^{++}, moving with very high speeds.

In the process of spontaneous alpha decay, therefore, the parent nucleus loses two protons and two neutrons, so that its mass decreases by four unites ($\Delta A = -4$) and its charge by two units ($\Delta Z = -2$).

SPONTANEOUS DECAY

The nuclear disintegration may be represented by the equation

$$_{Z}X^{A} \rightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4}\left(\alpha\right) \qquad (1)$$

Why α Decay Occurs

Alpha emission is a Coulomb repulsion effect. It becomes increasingly important for heavy nuclei because the disruptive Coulomb force increases with size at a faster rate (namely, as Z^2) than does the specific nuclear binding force, which increases approximately as A.

Why is the α particle chosen as the agent for the spontaneous carrying away of positive charge?

SPONTANEOUS DECAY

Why α Decay Occurs

When we call a process *spontaneous*, we mean that some kinetic energy has suddenly appeared in the system for no apparent cause; this energy must come from a decrease in the mass of the system. The α particle, because it is a very stable and tightly bound structure, has a relatively small mass compared with the mass of its separate constituents. It is particularly favored as an emitted particle if we hope to have the disintegration products as light as possible and thus get the largest possible release of kinetic energy.

CONDITION FOR SPONTANEOUS DECAY

Consider a nucleus $_{Z}X^{A}$ of mass M_p that decays into another nucleus $_{Z-2}Y^{A-4}$ of mass M_d and an alpha particle of mass m_α. Because the parent nucleus is at rest before the decay, the daughter and the α-particle must leave in opposite directions after the decay in order to conserve momentum.

Let E_i and E_f be the total energies of the system before and after decay. From the conservation of energy principle

$$E_i = E_f \qquad (2)$$

or

$$M_p c^2 = M_d c^2 + K_d + m_\alpha c^2 + K_\alpha \qquad (3)$$

CONDITION FOR SPONTANEOUS DECAY

The disintegration energy, Q, of this process is given by the following:

$$Q = K_d + K_\alpha = \left(M_p - M_d - m_\alpha\right)c^2 \qquad (4)$$

For a spontaneous decay, Q must be positive.

The usual practice is to express the Q-values in terms of atomic masses instead of nuclear masses. By adding and subtracting Zm_e (where me is the mass of the electron) from the right-hand side of Eq. 4, we get

$$Q = \left[M\left(A,Z\right) - M\left(A-4,Z-2\right) - m\left(4,2\right)\right]c^2 \qquad (5)$$

KINETIC ENERGY OF THE ALPHA PARTICLE

From the conservation of momentum and energy, we have

$$m_\alpha v_\alpha = M_d V_d \qquad (6)$$

and

$$Q = K_d + K_\alpha = \frac{1}{2} M_d V_d^2 + \frac{1}{2} m_\alpha v_\alpha^2 \qquad (7)$$

where v_α and V_d are the velocities of the alpha particle and daughter nucleus, respectively. Substituting for V_d from Eq. 6 into Eq. 7, we get

$$Q = \frac{1}{2} M_d \left(m_\alpha v_\alpha / M_d \right)^2 + \frac{1}{2} m_\alpha v_\alpha^2 = K_\alpha \left(\frac{m_\alpha}{M_d} + 1 \right) \quad \rightarrow \quad K_\alpha = \frac{A-4}{A} |Q| \qquad (8)$$

For a large A, therefore (A-4)/A is almost unity, and the alpha particle, as a result, will take most of the disintegration energy, Q, but not all.

MEASUREMENT OF THE ENERGY OF ALPHA PARTICLES

Accurate determination of the energies of alpha particles is important in two respects;

- To improve the theories governing alpha decay
- To construct exact nuclear energy-level schemes

Many techniques have been employed for the measurement of the energies of the alpha particles. These methods can be described under the following three categories:

- Magnetic deflection
- Range –energy relationship
- Pulse-height analysis

MEASUREMENT OF THE ENERGY OF ALPHA PARTICLES

- ***Magnetic deflection***

One of the oldest and most precise methods for the determination of energies is the measurement of the deflection of the path of alpha particles under the influence of a magnetic field. When a charged particle moves in a plane at right angles to the direction of a magnetic field, it describes a circular path of radius r given by,

$$qvB = mv^2 / r \qquad (9)$$

Where B is the strength of the magnetic field, and q and m are the charge and mass of the particle, respectively. The velocity is given by

$$v = \frac{q}{m}(Br) \qquad (10)$$

MEASUREMENT OF THE ENERGY OF ALPHA PARTICLES

- ***Magnetic deflection***

The kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{q}{m}Br\right)^2 \qquad (11)$$

Knowing m, B, r and the charge to mass ratio, q/m, K can be calculated from Eq. 11. If the relativistic effects are taken into consideration, Eq. 10 and 11 take the following forms,

$$v = \frac{q}{m_0}Br\left(1-\frac{v^2}{c^2}\right)^{1/2} \qquad (12)$$

$$K = m_0c^2\left[\frac{1}{\sqrt{1-\left(v^2/c^2\right)}}-1\right] \qquad (13)$$

MEASUREMENT OF THE ENERGY OF ALPHA PARTICLES

- ***Range-Energy Relationship***

The ranges of alpha particles can be measured by making use of a cloud chamber, nuclear emulsion plates, or ion chamber. If the tracks of the alpha particles in a cloud chamber or path of an alpha particle in a nuclear emulsion plate are measured, it is possible to obtain energy of alpha particles from the range-energy relationship.

MEASUREMENT OF THE ENERGY OF ALPHA PARTICLES

- ***Pulse-Height Analysis***

The principle of this method is based on the fact that the size of the electrical pulse produced is proportional to the energy of the alpha particle. This can be accomplished in three different ways:

- by using a total-ionization method in which the particle is made to lose all its energy in an ionization chamber or proportional counter
- by using a solid-state counter, and
- by using scintillation counter

RANGE AND IONIZATION

A charged particle moving through an absorber loses its kinetic energy by electromagnetic interaction with the electrons of the atoms of the absorbing material.

During the collision, if an electron gains enough energy, it may completely detach itself from the atom. Otherwise, the electron is left in an excited bound-state. The average energy needed for ionization is called the average ionization potential, and it is denoted by $\bar{I}$.

The range of an alpha particle may be defined as the distance it travels from the source to the point where its kinetic energy is zero.

RANGE AND IONIZATION

MEASUREMENT OF THE RANGE OF ALPHA PARTICLES

Depending on the method of measurement, the value of the range will differ slightly. Three types of range;

- Extrapolated range
- Mean range
- Ionization range

A modified form of an ionization chamber is a convenient device for accurate measurement of the ranges of alpha particles in air.

RANGE AND IONIZATION

MEASURMENT OF THE RANGE OF ALPHA PARTICLES

If the source of alpha particles is mounted perpendicular to the face of the chamber, individual alpha particles reaching the chamber produce ionization. An electrical pulse resulting from the ionization is communicated to an amplifier and then to a scaler. The counting rate is measured for different distances between the source and the chamber face (See Fig 7.10 in Fundamentals of Nuclear Physics by Atam P Arya for relative ionization versus distance curves for Po^{210} alpha particles).

Curve A in Fig 7.10 shows a number-distance curve for the case of Po^{210} alpha particles (only the end portion of the curve is shown).

RANGE AND IONIZATION

MEASURMENT OF THE RANGE OF ALPHA PARTICLES

It shows that the number of alpha particles reaching the chamber remains constant to a distance of about 3.7 cm, after which the counting rate falls very sharply to about 3.85 cm and then steadily goes to zero. Extrapolated range, R_e, is defined as the distance from the origin to the point where the tangent drawn to the curve A, at its point of inflection, intersects the distance axis.

Curve B in Fig. 7.10 is called the differential range curve and is obtained by taking the derivative of the number-distance curve A at different distances. The resulting curve shows a maximum at the point of inflection of A.

RANGE AND IONIZATION

MEASURMENT OF THE RANGE OF ALPHA PARTICLES

The mean range , $\bar{R}$, is defined as the distance from the origin to the maximum of the differential range curve. In this case

$$\bar{R} = 3.842\ cm$$

An important significance of the mean range is that half of the alpha particles have ranges more than $\bar{R}$, and half less than $\bar{R}$.

RANGE AND IONIZATION

STRAGGLING

Alpha particles lose their energy by the process of ionization and excitation. Therefore, the energy loss occurs in discrete amounts and will show statistical fluctuations about a mean, or most probable range.

Curve A and B in Fig 7.10 show that all alpha particles do not have the same range.

If all alpha particles had exactly the same range, there would have been a very sharp drop at the end. This fluctuation in the range is called range straggling.

RANGE AND IONIZATION

IONIZATION RANGE

The measure of the range and the ionization along the path of an alpha particle can be used to calculate its initial energy. Specific ionization is defined as the amount of ionization per unit length of the path of the beam.

The relative specific-ionization produced by a beam of alpha particles at different distances from the source can be measured with the help of the shallow ionization chamber. For this purpose, the amplifier of the ionization chamber is designed so that the height of the output-voltage pulse is proportional to the number of ion pairs formed in the chamber.

A plot of specific ionization versus distance from the end of the range is called a *Bragg curve*.

RANGE AND IONIZATION

IONIZATION RANGE

Two such curves for Po^{210} and Po^{214} alpha particles are shown in Fig. 7.11 in Fundamentals of Nuclear Physics by Atam P. Arya.

These curves show that the relative specific-ionization remains constant up to a certain distance, then rises rapidly and is followed by a sharp drop.

Such a behavior can be explained by the fact that as the alpha particles reach the end of the range, they spend more time near the atoms because of their lower velocities, and, thus, they cause increased ionization. A further decrease in velocity results in the capture of the electrons by the alpha particles forming helium atoms.

RANGE AND IONIZATION

IONIZATION RANGE

The ionization extrapolated range, R_i , is defined as the distance from the origin to the point where the tangent to the ionization curve, at the point of inflection, intersects the distance axis. From curve D in Fig 7.10 we get R_i=3.870 cm.

STOPPING POWER AND RANGE

Another quantity of importance in dealing with the absorption of charged particles is the stopping power, which is defined as the amount of energy lost per unit length by a particle in a given material.

$$S(E) = -dE / dx = wI \qquad (14)$$

where S(E) is a function of the kinetic energy, *E*, of the particle and is different for different materials. *I* is the average specific-ionization in terms of the number of ion pairs formed per unit length, and *w* is the energy required to produce an ion pair. If the value of the stopping power is known, the mean range can be calculated.

$$\bar{R} = \int_0^{\bar{R}} dx = \int_0^E \left(-\frac{dE}{dx} \right)^{-1} dE = \int_0^E \frac{dE}{S(E)} \qquad (15)$$

STOPPING POWER AND RANGE

On the other hand, if the mean range $\bar{R}$ of the alpha particle in a medium of stopping power S(E) is known, its energy may be calculated

$$E = \int_0^{\bar{R}} wI\ dR = \int_0^{\bar{R}} \left(-\frac{dE}{dx} \right) dR \qquad (16)$$

It is also possible to find the stopping power of a given material, if one knows the range as a function of energy in that material.

$$\frac{dR}{dE} = \frac{1}{S(E)} \qquad (17)$$

The importance of the stopping power lies in the fact that it is not necessary to measure it experimentally for different absorbers, because it can be calculated theoretically either from classical mechanics or quantum mechanics.

STOPPING POWER AND RANGE

The energy lost by a nonrelativistic particle per unit length of its path is given by,

$$S\left(E\right)=-\frac{dE}{dx}=\frac{4\pi z^{2}e^{4}}{mv^{2}}NZ\ln\left(\frac{2mv^{2}}{\bar{I}}\right) \qquad (18)$$

where v is the velocity of the particle, ze its charge, and m is the mass of the electron. N, Z and $\bar{I}$ are the number of atoms per unit volume, atomic number, and average ionization energy of the absorber, respectively.

(See Section of Theory of Stopping Power in page 192 in Fundamentals of Nuclear Physics Book by Atam P. Arya for derivation of stopping power expression given by Eq. 18)

THEORY OF STOPPING POWER

A heavy charged particle passing through an absorber loses most of its energy in ionizing the atoms of the absorber. The energy loss per unit length, the stopping power, can be calculated theoretically. The expression will be derived from the classical viewpoint and then discuss the changes which must be incorporated from the use of quantum mechanical treatments. An incident charged particle of mass M has a charge ze and velocity v.

(See Fig 7.13 in Fundamentals of Nuclear Physics by Atam P. Arya for interaction of an alpha particle with an electron of an atom)

THEORY OF STOPPING POWER

Let A, Z and ρ be the mass number, atomic number, and density, respectively, of the absorber. Consider an electron of mass *m* at a distance *b*, the impact parameter, from the path of the charged particle as shown in Fig 7.13.

t=0 represents the time when the charged particle is at the origin. To make the derivation simple, we make the following assumptions.

- The charged particles are heavy and, because of high velocities, their paths in the absorbers are straight lines. They lose their energy only in ionizing and excitation the atoms of the absorber along their path. We also assume that the motion of the charged particles is governed by classical mechanics, and no relativistic corrections are needed. This is true for alpha particles of energies less than 10 MeV.

THEORY OF STOPPING POWER

- The electron in the absorber is free and is initially at rest during the collision. Also the motion of the electron during the collision is so small that the electric field may be calculated as if the electron were not displaced from its position. This is true only if the velocity of the charged particle is much greater than the electronic velocities in the atoms.

From the symmetrical nature of the problem the net x-component of the impulse given to the electron is zero.

$$\int_{-\infty}^{0} F_x dt = \int_{0}^{\infty} F_x dt \qquad (19)$$

Where F_x is the x-component of the force $F=ze^2/r^2$.

THEORY OF STOPPING POWER

The y-component of the impulse (momentum) given to the electron is

$$P_y = \int_{-\infty}^{\infty} F_y dt = \int_{-\infty}^{\infty} \left(ze^2 / r^2 \right) \sin\theta \, dt \qquad (20)$$

Introducing a change of variables (from Fig. 7.13)

$$\sin\theta = b / r \qquad -vt / b = \cot\theta$$

$$dt = (b / v) \csc^2 \theta d\theta$$

into Eq. 20 and integrating, we get

$$P_y = 2ze^2 / bv \qquad (21)$$

THEORY OF STOPPING POWER

The energy given to a single electron at a distance b, therefore, is

$$E_e = \frac{P_y^2}{2m} = \frac{2z^2e^4}{mb^2v^2} \qquad (22)$$

If N_A is Avogadro's number, there are $(Z\rho N_A)/A$ number of electrons per unit volume of the absorber. Because of the cylindrical symmetry of the problem, the number of electrons in a shell of radii b and $b+db$ and length dx as shown in Fig 7.14 is (See Fig 7.14 in Fundamentals of Nuclear Physics by Atam P. Arya)

$$dN = 2\pi b\, db\, dx\left(Z\rho N_A / A\right) \qquad (23)$$

THEORY OF STOPPING POWER

Combining Eq. 22 and 23, the energy loss to the shell of length *dx* at *b* and of thickness *db* is

$$-dE(b) = 2\pi b\, db\, dx \frac{Z\rho N_A}{A} \cdot \frac{2z^2e^4}{mb^2v^2} \qquad (24)$$

Therefore, the total loss in energy per unit length to the electrons in all the shells bounded by the minimum impact parameter, b_{min}, and the maximum impact parameter, b_{max}, is

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4 N_A \rho Z}{mv^2 A} \int_{b_{\min}}^{b_{\max}} \frac{db}{b} = \frac{4\pi z^2 e^4 NZ}{mv^2} \ln \frac{b_{\max}}{b_{\min}} \qquad (25)$$

where $N=\rho N_A/A$, the number of atoms per unit volume of the absorber.

THEORY OF STOPPING POWER

The minimum value of *b* can be calculated from the fact that classically the maximum velocity that can be imparted to an electron in a head-on collision is *2v*. Its energy, therefore, is given by

$$E_e \leq \frac{1}{2} m(2v)^2 = 2mv^2 \qquad (26)$$

From Eq. 22 and 26

$$b_{\min} = \frac{ze^2}{mv^2} \qquad (27)$$

The maximum value of *b* can be calculated from the nonvalidity of the assumption that the electron is free during the collision. The electrons are essentially bound, and there is some average minimum excitation energy, $\bar{I}$. Thus b_{max} is not infinity but is given by (from Eq. 22)

$$\bar{I} = \frac{2z^2e^4}{mb_{\max}^2 v^2} \qquad or \qquad b_{\max} = \frac{ze^2}{v}(2/m\bar{I})^{1/2} \qquad (28)$$

THEORY OF STOPPING POWER

By combining Eqs. 25, 27 and 28, we get

$$S = -\frac{dE}{dx} = \frac{4\pi z^2 e^4}{mv^2} NZ \ln\left(\frac{2mv^2}{\bar{I}}\right)^{1/2} \qquad (29)$$

The more complete quantum-mechanical treatments give different values of the limits of *b*, and the expression obtained for the stopping power is given by Eq. 30 (which is essentially the same except for the ln term).

$$S = -\frac{dE}{dx} = \frac{4\pi z^2 e^4}{mv^2} NZ \ln\left(\frac{2mv^2}{\bar{I}}\right) \qquad (30)$$

If the relativity correction that occurs at high energies is taken into consideration, the following expression is obtained

$$S = -\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_0 v^2} NZ \left[\ln\left(\frac{2m_0 v^2}{\bar{I}}\right) - \ln\left(1 - \frac{v^2}{c^2}\right) - \frac{v^2}{c^2}\right] \qquad (31)$$

RANGE-ENERGY RELATIONSHIP

We have already seen that the range and energy of an alpha particle are related by Eqs.

$$\bar{R} = \int_0^{\bar{R}} dx = \int_0^{E} \left(-dE/dx\right)^{-1} dE$$

and

$$E = \int_0^{\bar{R}} wI \; dR = \int_0^{\bar{R}} \left(-dE/dx\right) dR$$

It is easier to measure a range of an alpha particle and then calculate its energy from the range-energy curves. For approximate calculations the energies of the alpha particles with ranges between 3 cm and 7 cm can be calculated from the following empirical relationship

$$\bar{R} = 0.318\, E^{3/2}$$

where E is the energy in MeV and $\bar{R}$ is the mean range in cm in air at 15 ^{0}C and 760 mm Hg.

THEORY OF ALPHA DECAY

One of the first successes of quantum mechanics was its application to the theory of alpha decay. The explanation of alpha emission by a quantum mechanically theory was given by Gamow and by Gurney and Condon.

In this theory an α particle is assumed to move in a spherical region determined by the *daughter* nucleus. The central feature of this *one-body model* is that the α particle is preformed inside the parent nucleus.

Figure 8.3 shows a plot, suitable for purposes of the theory, of the potential energy between the α particle and the residual nucleus for various distances between their centers (See Figure 8.3 in Introductory Nuclear Physics by Kenneth S. Krane)

THEORY OF ALPHA DECAY

The horizontal line Q is the disintegration energy. Note that the Coulomb potential is extended inward to a radius *a* and then arbitrarily cut off. The radius *a* can be taken as the sum of the radius of the residual nucleus and of the α particle. There are three regions of interest. In the spherical region $r < a$ we are inside the nucleus and speak of a potential well of depth $-V_0$, where V_0 is taken as a positive number. Classically the α particle can move in this region, with a kinetic energy $Q + V_0$ but it cannot escape from it. The annularshell region $a < r < b$ forms a potential barrier because here the potential energy is more than the total available energy Q.

Classically the α particle cannot enter this region from either direction, just as a tennis ball dropped from a certain height cannot rebound higher; in each case the kinetic energy would have to be negative.

THEORY OF ALPHA DECAY

The region $r > b$ is a classically permitted region outside the barrier. From the classical point of view, an α particle in the spherical potential well would sharply reverse its motion every time it tried to pass beyond $r = a$.

Quantum mechanically, however, there is a chance of "leakage" or "tunnelling" through such a barrier. This barrier accounts for the fact that α -unstable nuclei do not decay immediately. The α particle within the nucleus must present itself again and again at the barrier surface until it finally penetrates.

In order to resolve this paradox, it is necessary that we understand the problem of potential barrier from a quantum mechanical point of view.

THEORY OF ALPHA DECAY

ONE-DIMENSIONAL POTENTIAL BARRIER

Let a particle of rest mass m and kinetic energy E be incident (from left) on a potential barrier of height V_0 such that $E<V_0$ (See Fig. 7.24 in Fundamentals of Nuclear Physics by Atam P. Arya for one-dimensional rectangular potential barrier).

$$V(x) = \begin{cases} 0 & for\ x < 0 \\ V_0 & for\ 0 < x < a \\ 0 & for\ x > a \end{cases}$$

THEORY OF ALPHA DECAY

ONE-DIMENSIONAL POTENTIAL BARRIER

Classically, a particle with kinetic energy $E<V_0$ can never penetrate the potential barrier. But quantum-mechanically we shall show that even though $E<V_0$, the particle has some probability of crossing the potential barrier. This probability of penetration of the barrier, or transparency, is defined as

$$Transparency = \frac{transmitted\ \mathrm{int}\, ensity}{incident\ \mathrm{int}\, ensity} = \frac{(transmitted\ amplitude)^2}{(incident\ amplitude)^2} \qquad (32)$$

THEORY OF ALPHA DECAY

ONE-DIMENSIONAL POTENTIAL BARRIER

For regions I and III (Fig. 7.24), the time-independent Schrödinger wave equation is

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0 \qquad (33)$$

The solution of this equation is

$$\psi_I = Ie^{ikx} + \mathrm{Re}^{-ikx}$$

$$\psi_{III} = Te^{ikx} \qquad (34)$$

Where $k = \sqrt{2mE} / \hbar = p / \hbar$ and I, R and T are the incident, reflected and transmitted amplitudes, respectively.

THEORY OF ALPHA DECAY

ONE-DIMENSIONAL POTENTIAL BARRIER

In region II, the time-independent Schrödinger wave equation is

$$\frac{d^2\psi}{dx^2} - \frac{2m}{\hbar^2}\left(V_0 - E\right)\psi = 0 \qquad (35)$$

The solution of this equation is

$$\psi_{II} = Ae^{k'x} + B\,\mathrm{e}^{-k'x} \qquad (36)$$

Where $k' = \sqrt{2m\left(V_0 - E\right)} / \hbar = q / \hbar$ and A and B are constant. In terms of these constants ,

$$Transparency = \frac{|T|^2}{|I|^2} \qquad (37)$$

THEORY OF ALPHA DECAY

ONE-DIMENSIONAL POTENTIAL BARRIER

In order to calculate this we have to evaluate the constants, which can be accomplished by making use of the fact that the wave function must be well-behaved. Thus, applying the boundary conditions that ψ and dψ/dx (denoted by ψ') must be continuous at x=0 and x=a.

$$\begin{aligned}
\psi_I(0) &= \psi_{II}(0) \\
\psi_I'(0) &= \psi_{II}'(0) \\
\psi_{II}(a) &= \psi_{III}(a) \\
\psi_{II}'(a) &= \psi_{III}'(a) \qquad (38)
\end{aligned}$$

THEORY OF ALPHA DECAY

ONE-DIMENSIONAL POTENTIAL BARRIER

We get the following set of equations,

$$I + R = A + B \qquad (39a)$$

$$ik(I - R) = k'(A - B) \qquad (39b)$$

$$Ae^{k'a} + Be^{-k'a} = Te^{ika} \qquad (39c)$$

$$k'(Ae^{k'a} - Be^{-k'a}) = ikTe^{ika} \qquad (39d)$$

Eliminating R from Eqs. 39a and 39b, and evaluating A and B from Eqs. 39c and 39d

THEORY OF ALPHA DECAY

ONE-DIMENSIONAL POTENTIAL BARRIER

$$I = \frac{1}{2}\left(1 + \frac{k'}{ik}\right)A + \frac{1}{2}\left(1 - \frac{k'}{ik}\right)B \tag{40}$$

$$A = Te^{ika}\left(1 + \frac{ik}{k'}\right) / 2e^{k'a} \tag{41}$$

$$B = Te^{ika}\left(1 - \frac{ik}{k'}\right) / 2e^{-k'a} \tag{42}$$

Substituting the values of A and B in Eq. 40 and letting

$$k = p / \hbar \quad and \quad k' = q / \hbar$$

we get

THEORY OF ALPHA DECAY

ONE-DIMENSIONAL POTENTIAL BARRIER

$$I = \frac{T}{4} e^{ika} \left\{ \left(1 + \frac{q}{ip}\right)\left(1 + \frac{ip}{q}\right) e^{-(qa/\hbar)} + \left(1 - \frac{q}{ip}\right)\left(1 - \frac{ip}{q}\right) e^{(qa/\hbar)} \right\} \qquad (43)$$

Hence the transparency $|T|^2 / |I|^2$ can be calculated from this equation, and it is not zero as predicted by the classical theory. The expression of Eq. 43 can be simplified if we assume

$$\frac{p}{q} = \frac{\sqrt{2mE}}{\sqrt{2m(V_0 - E)}} = \sqrt{\frac{E}{V_0 - E}} \sim 1$$

and also $qa/\hbar >> 1$. This reduces Eq. 43 to

$$I \approx T e^{ika} e^{qa/\hbar} \qquad (44)$$

THEORY OF ALPHA DECAY

ONE-DIMENSIONAL POTENTIAL BARRIER

Hence,

$$Transparency = \frac{|T|^2}{|I|^2} \approx e^{-(2qa)/\hbar} \qquad (45)$$

$$Transparency \sim \exp\left\{-2\sqrt{2m\left(V_0 - E\right)/\hbar^2}\,a\right\} \qquad (46)$$

For a barrier of arbitrary shape, the order of the magnitude of transparency is given by

$$Transparency \sim \exp\left\{-2\int_a^b \sqrt{\frac{2m\left[V\left(x\right) - E\right]}{\hbar^2}}\,dx\right\} \qquad (47)$$

THEORY OF ALPHA DECAY

ONE-DIMENSIONAL POTENTIAL BARRIER

or,

$$P = e^{-2\gamma} \qquad (48)$$

where P is the transparency and

$$\gamma = \int_a^b \sqrt{\frac{2m\left[V(x) - E\right]}{\hbar^2}}\, dx \qquad (49)$$

it is quite clear from Eq. 47 that P depends strongly on the mass of the particle and the width of the barrier. The disintegration constant of an α emitter is given in the one-body theory by

$$\lambda = fP \qquad (50)$$

where f is the frequency with which the α particle presents itself at the barrier and P is the probability of transmission through the barrier.

ALPHA-RAY SPECTRA

FINE STRUCTURE

Prior to 1930, it was assumed that all the alpha particles given out by an isotope have the same energy. By very careful ionization measurements, Bragg showed that there are four different energy groups of alpha particles emitted by a source of radium. S. Rosenblum demonstrated by the use of a magnetic spectrograph that many alpha emitters give out more than one alpha group with different energies in 1930. In such cases alpha emission is always followed by gamma emission.

The observation of alpha-particle groups and the associated gamma radiation is a good demonstration of the existence of ‘discrete nuclear energy levels’.

ALPHA-RAY SPECTRA

FINE STRUCTURE

Whenever some of the emitted alpha particles have less kinetic energy than the maximum available energy, the daughter nucleus is left in the excited state. The daughter nucleus decays to the ground state by the emission of one or more photons. The energies of these gamma rays have been measured and have been found to be equal to the differences in the kinetic energies of the alpha particles of different groups. For example, if E_0, E_1 and E_2 are the ground state and the excited energy levels of the daughter nucleus, respectively, then the energies of the gamma rays are given by,

$$h\nu_1 = E_1 - E_0 \quad , \quad h\nu_2 = E_2 - E_0 \qquad h\nu_3 = E_2 - E_1 \tag{51}$$

REFERENCES

1. Introductory Nuclear Physics. Kenneth S. Krane

2. Fundamentals of Nuclear Physics. Atam. P. Arya