References:

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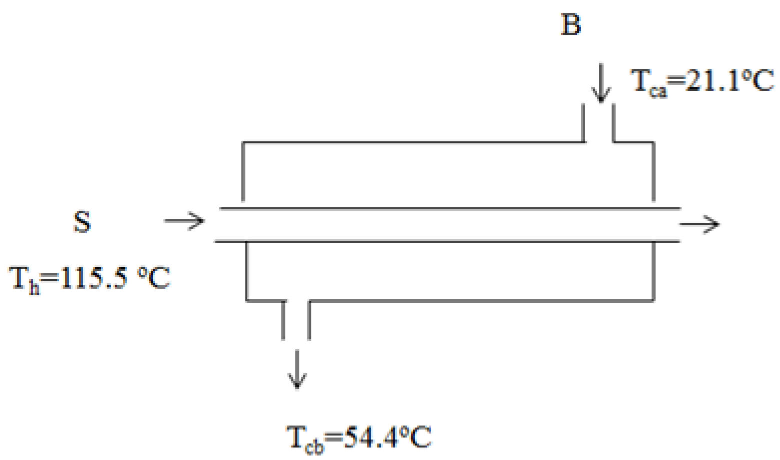
7. Heat transfer to fluids by forced convection in laminar and turbulent flows

Example:



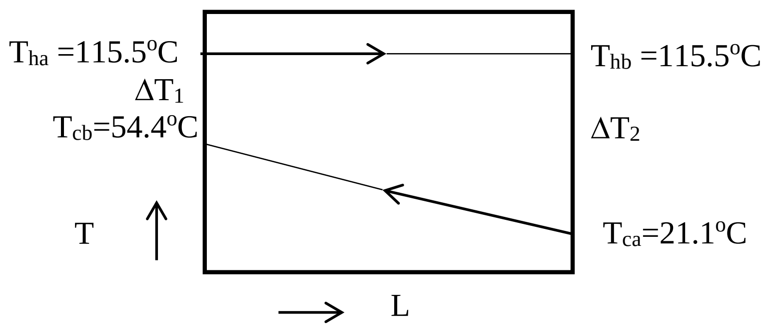


Table 10: Steel pipe dimensions

2 in sch 40 D_{ii}=2.067 in=52.5x10^{-3} m D_{io}==2.375 in=60.32x10^{-3} m

3 in sch 40 D_{oi}=3.068 in=77.92x10^{-3} m

D_e= D_{oi} - D_{io} =(77.92-60.32)x10^{-3} m=17.6x10^{-3} m

$$\mathrm{Re} = \frac{D_e v \rho}{\mu} = \frac{(D_{oi} - D_{io}).\dot{m}.\rho.}{\rho\left[\frac{\pi}{4}D_{oi}{}^2 - \frac{\pi}{4}D_{io}{}^2\right]\mu} = \frac{\dot{m}}{\frac{\pi}{4}(D_{oi} + D_{io})\mu} = \frac{0.554}{\frac{\pi}{4}(77.92 + 60.32)x10^{-3}x0.51x10^{-3}} = 10005$$

Re>10^4 Turbulent flow

For turbulent flow; since $\frac{L}{D} > 150$ we ignore the effect of pipe length.

$$\frac{4.572}{17.6x10^{-3}} = 259.7 > 150$$

$$Nu = \frac{h_o D_e}{k} = 0.023(\mathrm{Re})^{0.8}(\mathrm{Pr})^{0.4}$$ (n=0.4, B is heating)

$$\mathrm{Pr} = \frac{C_p \mu}{k}$$

$$h_o = \frac{0.156}{17.6x10^{-3}}x0.023x(10005)^{0.8}(\frac{2424.4x0.51x10^{-3}}{0.156})^{0.4} = 739.8\frac{W}{m^2\,{}^oC}$$

Viscosity value of B is small, so we ignored viscosity correction factor (ϕ_v)

For B; μ=0.51x 10^{-3} kg/(m.s)

$$\Delta T_i = \frac{\frac{1}{h_i}}{\frac{1}{h_i} + \frac{D_i}{D_o}h_o}x\Delta T = \frac{\frac{1}{11338.2}}{\frac{1}{11338.2} + \frac{0.052}{0.0603}x739.8}(115.5 - 37.75) = 1.06x10^{-5}$$

For heating T_w=T+ΔT_i

ΔT_i is very low. So T_w=T and viscosity does'nt change.

We neglect viscosity correction factor.

$$q = \dot{m}C_p\Delta T = 0.554x2424.4x(54.4 - 21.1) = 44725.8\frac{J}{s}$$

$$\Delta T_L = \frac{\Delta T_1 - \Delta T_2}{\ln\frac{\Delta T_1}{\Delta T_2}} = \frac{(T_{ha} - T_{cb}) - (T_{hb} - T_{ca})}{\ln\frac{T_{ha} - T_{cb}}{T_{hb} - T_{ca}}}$$

$$\Delta T_L = \frac{(115.5 - 54.4) - (115.5 - 21.1)}{\ln\frac{115.5 - 54.4}{115.5 - 21.1}} = 76.55^oC$$

$$U_o = \frac{q}{A_o \overline{\Delta T}_L} = \frac{44725.8}{\pi x 60.32 x 10^{-3} x 4.572 x 76.55} = 674.4 \frac{W}{m^2\,{}^oC}$$

$$U_o = \frac{1}{\cancel{\frac{D_o}{D_i h_{d\,i}}} + \frac{D_o}{D_i h_i} + \frac{x_w}{k_m}\frac{D_o}{\overline{D_L}} + \frac{1}{h_o} + \frac{1}{h_{d\,o}}}$$

$$\overline{D_L} = \frac{D_o - D_i}{\ln \frac{D_o}{D_i}} = \frac{0.0603 - 0.052}{\ln \frac{0.0603}{0.052}} = 0.056\ m$$

$$D_o = 0.0603\ m$$

$$D_i = 0.052\ m$$

$$\overline{D_L} = 0.056\ m$$

$$x_w = \frac{D_o - D_i}{2} = \frac{0.0603 - 0.052}{2} = 0.0041$$

k_m=386 W/m^{o}C Copper pure (Table 2)

$$674.4 = \frac{1}{\frac{0.0603}{0.052 x 11338.2} + \frac{0.0041}{386} x \frac{0.0603}{0.056} + \frac{1}{739.8} + \frac{1}{h_{d\,o}}}$$

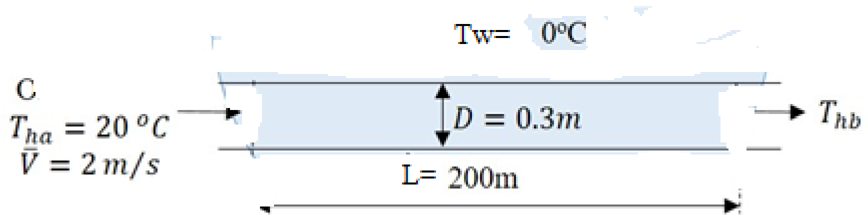
$$674.4 = \frac{1}{0.000102 + 0.000011 + 0.00135 + \frac{1}{h_{d\,o}}}$$

$$674.4 = \frac{1}{(0.00146 + \frac{1}{h_{d\,o}})}$$

$$(0.00146 + \frac{1}{h_{d\,o}}) = \frac{1}{674.4}$$

$$R = \frac{1}{h_{d\,o}} = 2.2 x 10^{-5} \frac{m^2\,{}^oC}{W}$$

Example :



$q = \dot{m}\, C_p\, (T_{ha} - T_{hb}) = h_i\, A_i\, \Delta T_i$

T_{hb} must be assumed.

<u>1. assumption;</u> $T_{hb} = 10\ °C$

$$\bar{T} = \frac{T_{ha} + T_{hb}}{2} = \frac{20 + 10}{2} = 15\ °C$$

Properties of engine oil at 15 °C

$\mu = 1.5815\ kg/ms$
$\rho = 890.825\ kg/m^3$
$\rho_{20\,^oC} = 888.1\ kg/m^3$
$C_p = 1860\ j/kg°C$
$k = 0.1455\ W/m°C$
$Pr = 20217$
$\mu_w = 3.814\ kg/ms$ (at 0°C)

$$Re = \frac{D\bar{V}\rho}{\mu} = \frac{0.3\ m \times\ 2\frac{m}{s} \times 890.825}{1.5815\frac{kg}{ms}} = 337.97 < 2100 \qquad \textit{Laminar flow}$$

$$\frac{L}{D} = \frac{200}{0.3} = 666.67 > 150 \qquad \textit{fully developed flow}$$

$$Nu = 2\ Gz^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14} = 2\left(\frac{\dot{m} \times C_p}{k \times L}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

Mass flow rate at the beginning of the pipeline.

$$\dot{m} = \rho_{20\,^oC} \times V \times S = (888.1)\frac{kg}{m^3} \times (2)\frac{m}{s} \times \left(\frac{\pi}{4}0.3^2\right)m^2 = 125.55\ \frac{kg}{s}$$

$$Nu = \frac{hD}{k} = \frac{h \times 0.3m}{0.1455\frac{W}{m°C}} = 2\left(\frac{125.55\frac{kg}{s} \times \frac{1860\ j}{kg°C}}{0.1455\frac{W}{m°C} \times 200m}\right)^{\frac{1}{3}}\left(\frac{1.5815\frac{kg}{ms}}{3.814\frac{kg}{ms}}\right)^{0.14} = 35.40$$

$$h = 17.169\frac{W}{m^2°C}$$

$\dot{m}\, C_p\, (T_b - T_a) = h_i\, \pi\, D_i\, L\, (T_w - \bar{T})$

$$\left(125.55\frac{kg}{s}\right)\left(1860\frac{j}{kg°C}\right)(20 - T_{hb}) = \left(17.169\frac{W}{m^2°C}\right)\pi\ (0.3\ m)\ (200\ m)\ (\frac{20°C + T_b}{2} - 0°C)$$

$T_{hb} = 19.72\ °C \neq 10°C$

<u>**2. assumption;**</u> $T_{hb} = 19.72\ °C$

$$\overline{T} = \frac{T_{ha} + T_{hb}}{2} = \frac{20 + 19.72}{2} = 19.86\ °C$$

Properties of engine oil at 19.86 °C

$\mu \quad = 0.8374$ kg/ms
$\rho \quad = 888.1$ kg/m^3
$C_p \quad = 1881$ j/kg°C
$k \quad = 0.1450$ W/m°C
$Pr \quad = 10863$
$\mu_w \quad = 3.814$ kg/ms (at 0°C)

$$Re = \frac{D\overline{V}\rho}{\mu} = \frac{0.3\ m \times 2\frac{m}{s} \times 888.1}{0.8374\frac{kg}{ms}} = 636.33 < 2100 \qquad \textit{Laminar flow}$$

$$Nu = 2\ Gz^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14} = 2\ \left(\frac{\dot{m} \times C_p}{k \times L}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

Mass flow rate will not change.

$$Nu = \frac{hD}{k} = \frac{h \times 0.3m}{0.1450\frac{W}{m°C}} = 2\left(\frac{125.55\ \frac{kg}{s} \times \frac{1881\ j}{kg°C}}{0.1450\frac{W}{m°C} \times 200m}\right)^{\frac{1}{3}}\left(\frac{0.8374\frac{kg}{ms}}{3.814\frac{kg}{ms}}\right)^{0.14} = 32.54$$

$$h = 15.72\frac{W}{m^2°C}$$

$$\dot{m}\ C_p\ (T_b - T_a) = h_i\ \pi\ D_i\ L\ (T_w - \overline{T})$$

$$\left(125.55\frac{kg}{s}\right)\left(1881\frac{j}{kg°C}\right)(20 - T_{hb}) = \left(15.72\frac{W}{m^2°C}\right)\pi\ (0.3\ m)\ (200\ m)\ (\frac{20°C + T_b}{2} - 0°C)$$

$T_{hb} = 19.75\ °C \cong 19.72°C$

Second Way:

<u>1. assumption</u> $T_{hb} = 20\ °C$

$$\overline{T} = \frac{T_{ha} + T_{hb}}{2} = \frac{20 + 20}{2} = 20\ °C$$

Properties of engine oil at 20 °C

$\mu \quad = 0.8374$ kg/ms
$\rho \quad = 888.1$ kg/m^3
$C_p \quad = 1881$ j/kg°C
$k \quad = 0.1450$ W/m°C

$\mathrm{Pr} \; = 10863$
$\mu_w \; = 3.814\ \mathrm{kg/ms} \quad$ (at 0°C)

$$\mathrm{Re} = \frac{D\bar{V}\rho}{\mu} = \frac{0.3\ \mathrm{m} \times 2\frac{m}{s} \times 888.1}{0.8374\frac{\mathrm{kg}}{\mathrm{ms}}} = 636.33 < 2100 \qquad \textit{Laminar flow}$$

$$\frac{L}{D} = \frac{200}{0.3} = 666.67 > 150 \qquad \textit{fully developed flow}$$

$$\mathrm{Nu} = 2\ \mathrm{Gz}^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14} = 2\ \left(\frac{\dot{m} \times C_p}{k \times L}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

Mass flow rate at the beginning of the pipeline.

$$\dot{m} = \rho_{20\,^\circ\mathrm{C}} \times v \times S = (888.1)\frac{\mathrm{kg}}{\mathrm{m}^3} \times (2)\frac{\mathrm{m}}{\mathrm{s}} \times \left(\frac{\pi}{4}0.3^2\right)\mathrm{m}^2 = 125.55\ \frac{\mathrm{kg}}{\mathrm{s}}$$

$$\mathrm{Nu} = 2\ \mathrm{Gz}^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14} = 2\ \left(\frac{\dot{\mathrm{m}} \times \mathrm{C_p}}{\mathrm{k} \times \mathrm{L}}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

$$\mathrm{Nu} = \frac{hD}{k} = \frac{h \times 0.3\mathrm{m}}{0.1450\frac{\mathrm{W}}{\mathrm{m^\circ C}}} = 2\left(\frac{125.55\frac{\mathrm{kg}}{\mathrm{s}} \times \frac{1881\ \mathrm{j}}{\mathrm{kg^\circ C}}}{0.1450\frac{\mathrm{W}}{\mathrm{m^\circ C}} \times 200\mathrm{m}}\right)^{\frac{1}{3}}\left(\frac{0.8374\frac{\mathrm{kg}}{\mathrm{ms}}}{3.814\frac{\mathrm{kg}}{\mathrm{ms}}}\right)^{0.14} = 32.54$$

$$h = 15.72\frac{\mathrm{W}}{\mathrm{m^2{}^\circ C}}$$

$$\dot{m}\ C_p\ (T_b - T_a) = h_i\ \pi\ D_i\ L\ (T_w - \bar{T})$$

$$\left(125.55\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(1881\frac{\mathrm{j}}{\mathrm{kg^\circ C}}\right)(20^\circ\mathrm{C} - T_{hb}) = \left(15.72\frac{\mathrm{W}}{\mathrm{m^2{}^\circ C}}\right)\pi\ (0.3\ \mathrm{m})\ (200\ \mathrm{m})\ (\frac{20^\circ\mathrm{C} + T_b}{2} - 0^\circ\mathrm{C})$$

$$T_{hb} = 19.75\ ^\circ\mathrm{C} \cong 20^\circ\mathrm{C}$$

b)

$$q = \dot{m}\ C_p\ (T_b - T_a) = \left(125.55\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(1881\frac{\mathrm{j}}{\mathrm{kg^\circ C}}\right)(20^\circ\mathrm{C} - 19.75^\circ\mathrm{C}) = 59040\mathrm{W} \cong 59\mathrm{kW}$$