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11. Heat transfer by natural convection

Basic Definitions

Buoyancy effect:

Hot plate

Warm, ρ

Surrounding fluid, cold, ρ_2, T_2

ρ_1

Net force = $(\rho_2 - \rho_1)gV$

The density difference is due to the temperature difference and it can be characterized by their volumetric thermal expansion coefficient, β :

$$\beta = \frac{(\partial v / \partial T)_p}{v} = \frac{\frac{1}{\rho_2} - \frac{1}{\rho_1}}{(T_2 - T_1) \frac{(\frac{1}{\rho_1} + \frac{1}{\rho_2})}{2}} = \frac{\rho_1 + \rho_2}{\rho_1 \rho_2 (T_2 - T_1)}$$

$\rho_1 = \frac{\rho_1 + \rho_2}{2}$

For an ideal gas, since $v = RT/p$, $\beta = \frac{(\partial v / \partial T)_p}{v} = \frac{R/p}{RT/p} = \frac{1}{T}$

v : Specific volume of fluid
 $(\partial v / \partial T)_p$ = rate of change of specific volume with temperature at constant pressure
 ρ_1 = Density of fluid at temperature T_1
 ρ_2 = Density of fluid at temperature T_2

For single horizontal cylinders, the heat transfer coefficient can be correlated by an equation containing three dimensionless groups, the Nusselt number, the Prandtl number and the Grashof number or specifically,

$$Nu = 0.53(Gr * Pr)^{0.25}$$

Naturel convection to air from vertical shapes and horizontal planes

Equations for heat transfer in naturel convection between fluids and solids of definite

geometric shape are of the form

$$\frac{hL}{k} = b \left(\frac{C_p \mu_f}{k_f}, \frac{L^3 \rho_f^2 \beta_f \Delta T}{\mu_f^2} \right)^n$$

$$Nu_f = b(N_{Pr}, N_{Gr})^n$$

Where b,n are const

L= height of vertical surface or length of horizontal square surface, ft

Example:

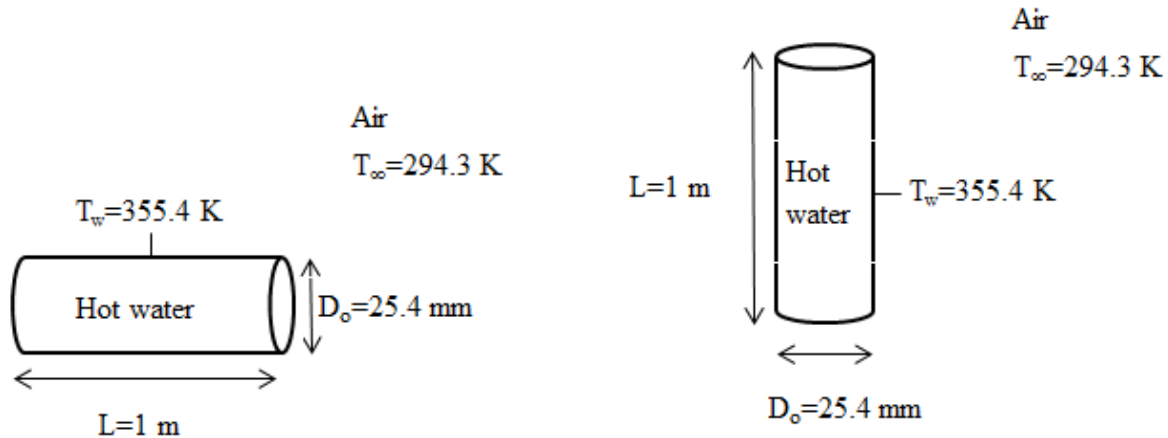


Figure 1. Heat Loss from a horizontal tube and vertical tube

$$Nu = b(Gr.Pr)_f^n$$

Flow around a mere pipe

$$Gr = \frac{D_o^3 \cdot \rho_f^2 \cdot g \cdot \beta \cdot \Delta T_o}{\mu_f^2}$$

$$T_f = \frac{T_w + T_\infty}{2} = \frac{355.4 + 294.3}{2} = 324.85 \text{ K}$$

$$D_o = 0.0254 \text{ m}$$

$$g = 9.81 \text{ m/s}^2$$

$$\beta = 1/T = 1/(324.85 \text{ K}) = 3.07 \times 10^{-3}$$

$$\Delta T_o = T_w - T_\infty = 355.4 - 294.3 = 61.1 \text{ K}$$

The properties of air at $T_f = 324.85 \text{ K}$ are read from Table 5

$$\rho_f = 1.088 \text{ kg/m}^3, C_{pf} = 1007 \text{ J/(kg} \cdot \text{°C)}, \mu_f = 1.96 \times 10^{-5} \text{ kg/(m} \cdot \text{s)}, k_f = 0.028 \text{ W/(m} \cdot \text{°C)}, Pr_f = 0.702$$

Horizontal case

$$Gr = \frac{D_o^3 \cdot \rho_f^2 \cdot g \cdot \beta \cdot \Delta T_o}{\mu_f^2}$$

$$Gr = \frac{(0.0254)^3 \cdot (1.088)^2 \cdot (9.81) \cdot (3.07 \times 10^{-3}) \cdot 61.1}{(1.96 \times 10^{-5})^2} = 92917.04$$

$$(Gr.Pr) = 92917.04 \times 0.702 = 65227.7$$

$\log(Gr.Pr) = 4.81 > 4$ Flow around a mere horizontal pipe or

From Table 15 for horizontal cylinder $10^4 < Gr.Pr = 65227.7 < 10^9$ $b=0.53$ $n=0.25$

$$Nu = 0.53(Gr.Pr)_f^{0.25}$$

$$\frac{hD_o}{k} = \frac{hx0.0254}{0.028} = 0.53(Gr.Pr)_f^{0.25} = 0.53(65227.7)^{0.25}$$

$$h=9.33 \text{ W/(m}^2\text{C)}$$

Vertical case

$$Gr = \frac{(1)^3 \cdot (1.088)^2 \cdot (9.81) \cdot (3.07 \times 10^{-3}) \cdot 61.1}{(1.96 \times 10^{-5})^2} = 5.67 \times 10^9$$

$$(Gr.Pr) = 5.67 \times 10^9 \times 0.702 = 3.98 \times 10^9$$

From Table 15 for vertical cylinder $10^9 < Gr.Pr = 3.98 \times 10^9 < 10^{12}$ $b=0.13$ $n=0.33$

$$Nu = 0.13(Gr.Pr)_f^{0.33}$$

$$\frac{hL}{k} = \frac{hx1}{0.028} = 0.13(Gr.Pr)_f^{0.33} = 0.13(3.98 \times 10^9)^{0.33}$$

$$h=5.35 \text{ W/(m}^2\text{C)}$$

$$\frac{q_{horizontal}}{q_{vertical}} = \frac{h_{horizontal}}{h_{vertical}} \cdot \frac{\pi D_o L (T_w - T_\infty)}{\pi D_o L (T_w - T_\infty)} = \frac{9.33}{5.35} = 1.74$$