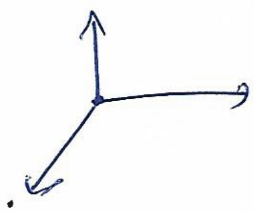
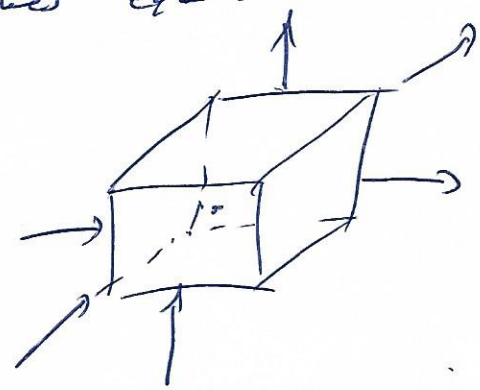
Differential analysis of fluid flow:

- continuity (conservation of mass)
- Navier – Stokes equations.





$$(\rho U) = \frac{kg}{m^3} \cdot \frac{m}{s} = \frac{kg}{m^2 - s}$$

mass per unit area time.

$$(\rho U)_{x+\Delta x} = (\rho U)_x + \frac{\partial(\rho U)}{\partial x} \cdot \Delta x + \frac{\partial^2(\rho U)}{\partial x^2} \cdot \frac{1}{2!} \Delta x^2 + \cdots$$

x – direction

$$(\rho U)_{x+\Delta x} = (\rho U)_x + \frac{\partial(\rho U)}{\partial x} \cdot \Delta x$$

$$(\rho U)_{y+\Delta y} = (\rho U)_y + \frac{\partial(\rho U)}{\partial y} \cdot \Delta y$$

$$(\rho U)_{z+\Delta z} = (\rho U)_z + \frac{\partial(\rho U)}{\partial z} \cdot \Delta z$$

In - out

$$[\Sigma m]_x = \left\{(\rho u)_x - \left[(\rho u)_x + \frac{\partial(\rho u)}{\partial x}\cdot\Delta x\right]\right\}\Delta y\Delta z$$

$$+\left\{(\rho v)_y - \left[(\rho v)_y + \frac{\partial(\rho v)}{\partial y}\Delta y\right]\right\}\Delta x\Delta z$$

$$+\left\{(\rho w)_z - \left[(\rho w)_z + \frac{\partial(\rho w)}{\partial z}\Delta z\right]\right\}\Delta y\Delta z = \frac{\partial\rho}{\partial x}\Delta x\Delta y\Delta z$$

$$\frac{\partial\rho}{\partial t}\cancel{\Delta x\Delta y\Delta z} = -\frac{\partial(\rho u)}{\partial x}\cancel{\Delta x\Delta y\Delta z} - \frac{\partial(\rho v)}{\partial y}\cancel{\Delta x\Delta y\Delta z} - \frac{\partial(\rho w)}{\partial z}\cancel{\Delta x\Delta y\Delta z}$$

$$\frac{\partial\rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0.$$

Continuity equation in Cartesian coordinates!

$$\frac{\partial\rho}{\partial t} + \frac{1}{r}\cdot\frac{\partial(r\rho u_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho u_\theta)}{\partial\theta} + \frac{\partial(\rho u_z)}{\partial z} = 0$$ in cylindrical coordinates.

Special cases:

- steady - compressible flow: $\frac{\partial\rho}{\partial t} = 0$
- Incompressible flow - steady, [ρ : not a function of time or space :

$$\cancel{\frac{\partial(\rho u)}{\partial x} = \rho\frac{\partial u}{\partial x} + u\frac{\partial\rho}{\partial x}} \Rightarrow \approx 0$$

$\partial\rho$ ... ρ : constant.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Incompressible flow: changes in the density are not to be account significant!

$$\frac{1}{r}\cdot\frac{\partial v_r}{\partial r} + \frac{1}{r}\cdot\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0.$$

Navier-Stokes equations:

Cartesian coordinates

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \frac{\partial p}{\partial z} + \rho g_z + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

Cylindrical coordinates:

$r-$
$$\rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} + \frac{u_\theta^2}{r} + u_z\frac{\partial u_r}{\partial z}\right) = -\frac{\partial p}{\partial r} +$$
$$\rho g_r + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right) - \frac{u_r}{r^2} + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2}\right.$$

$\theta-$
$$\rho\left(\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} + u_z\frac{\partial u_\theta}{\partial z}\right) =$$
$$-\frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\theta}{\partial r}\right) - \frac{u_\theta}{r^2} + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} +\right.$$
$$\left.\frac{2}{r^2}\frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_\theta}{\partial z^2}\right]$$

$z-$
$$\rho\left(\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \rho g_z +$$
$$+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right]$$