Transfer Functions and Block Diagrams

TRANSFER FUNCTIONS

- The transfer function is an algebraic expression for the dynamic relation between the input and the output of the process model.

- Transfer function is the ratio of Laplace transform of output variable to Laplace transform of input variable.

- The input and output variables are expressed in the form of deviation variable while determining the transfer function.

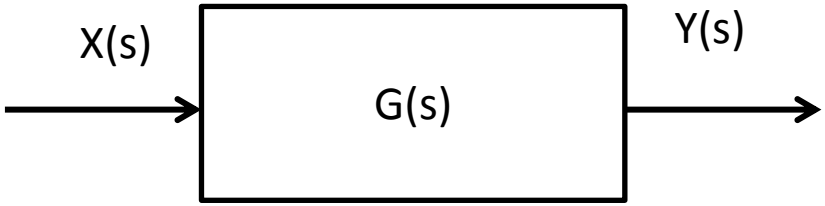
Deviation variable: It shows how much a variable deviates from its initial steady state conditions.
(Unsteady state value-Steady state value)

TRANSFER FUNCTIONS

$$G(s) = \frac{Y(s)}{X(s)}$$

- G(s) → Transfer function
- Y(s) → Laplace transform of output variable (in the form of deviation variable)
- X(s) → Laplace transform of input variable (in the form of deviation variable)

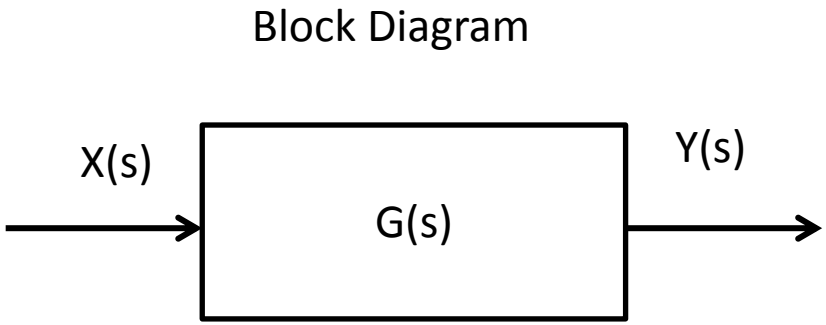
Block Diagram



$$G(s) = \frac{Y(s)}{X(s)}$$

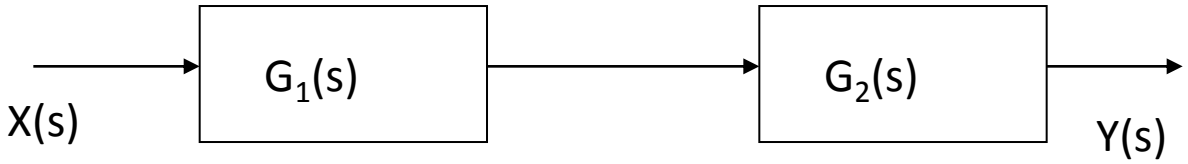
Block Diagrams

- It is always useful to express the system graphically while designing or analyzing it.
- Block diagrams helps us to visualize the system.



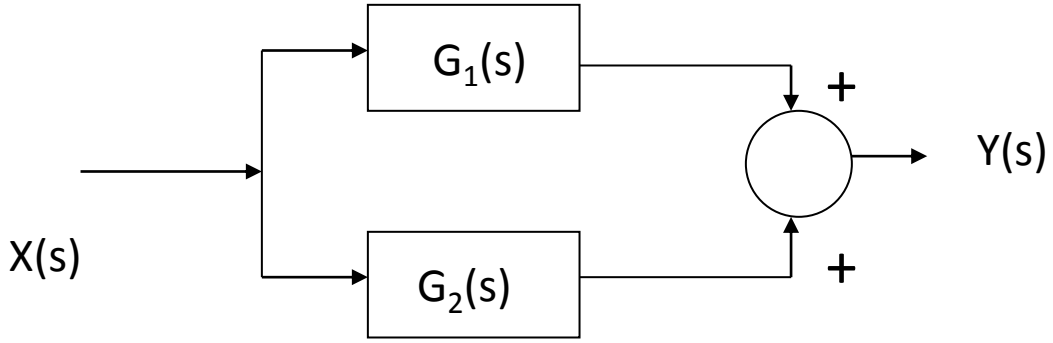
Properties of Block Diagrams

- If the blocks are in series, then the transfer functions are multiplied.

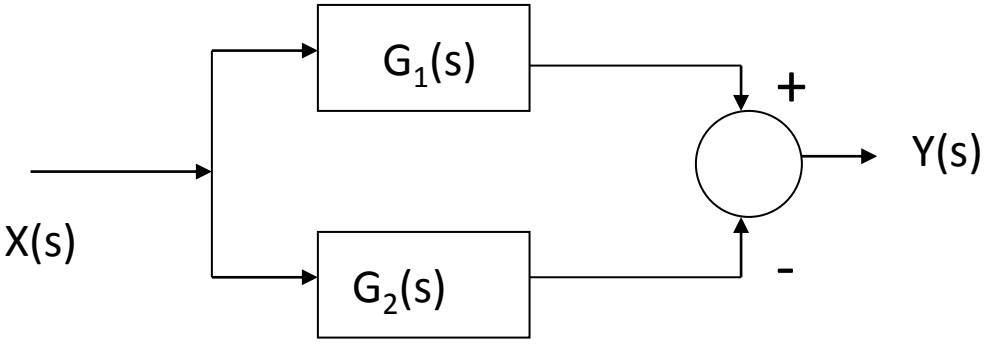


$$\frac{Y(s)}{X(s)} = G_1(s).G_2(s)$$

- If the blocks are in parallel, then the transfer functions are summed up or subtracted.

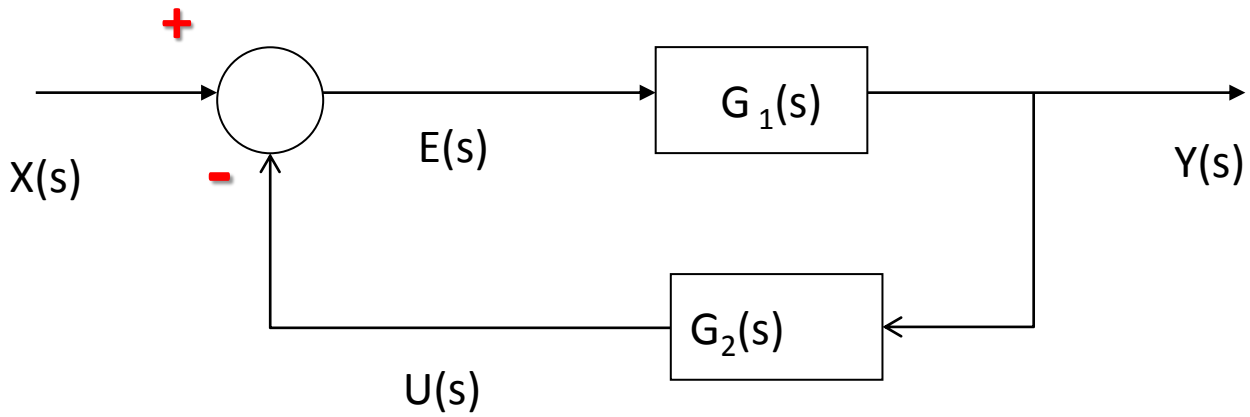


$$\frac{Y(s)}{X(s)} = [G_1(s) + G_2(s)]$$



$$\frac{Y(s)}{X(s)} = [G_1(s) - G_2(s)]$$

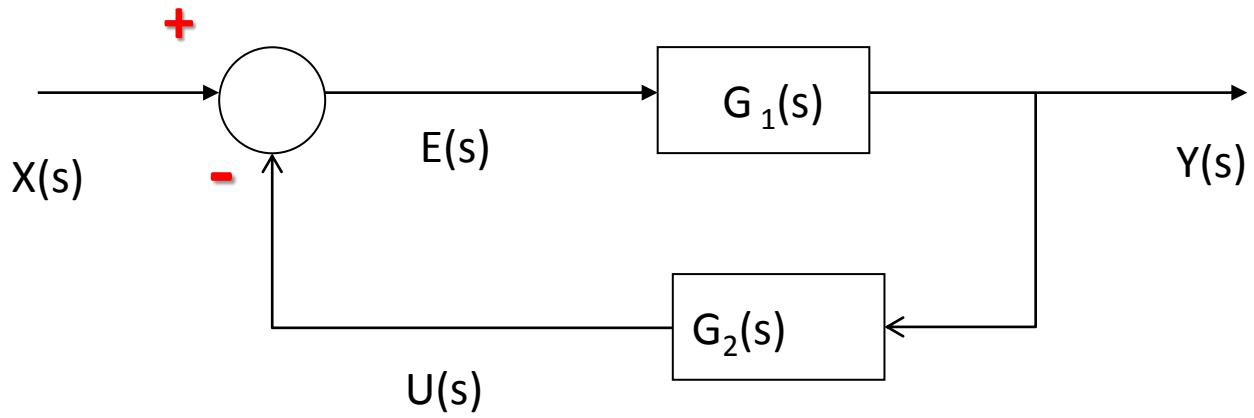
- Transfer function of a negative feedback system:



(Negative feedback)

$$\frac{Y(s)}{X(s)} = ?$$

- Transfer function of a negative feedback system:



$$\frac{Y(s)}{X(s)} = ?$$

$$\frac{Y(s)}{E(s)} = G_1(s) \longrightarrow \quad E(s) = \frac{Y(s)}{G_1(s)}$$

$$\frac{U(s)}{Y(s)} = G_2(s) \longrightarrow \quad U(s) = G_2(s).Y(s)$$

$$E(s) = X(s) - U(s)$$

- Transfer function of a negative feedback system:

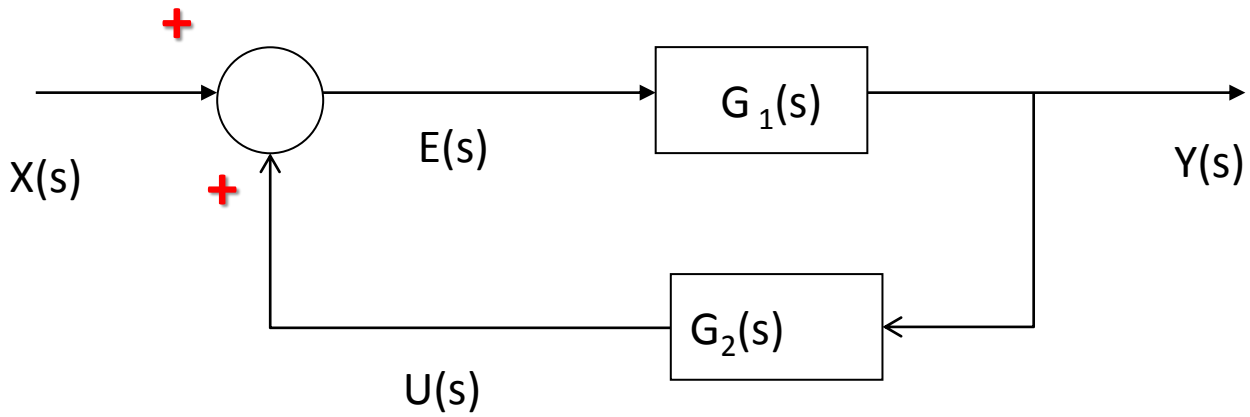
$$\frac{Y(s)}{G_1(s)} = X(s) - G_2(s).Y(s)$$

$$Y(s)\left[\frac{1}{G_1(s)} + G_2(s)\right] = X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{1}{\left[\frac{1}{G_1(s)} + G_2(s)\right]}$$

$$\frac{Y(s)}{X(s)} = \frac{G_1(s)}{1 + G_1(s)G_2(s)}$$

- Transfer function of a positive feedback system:

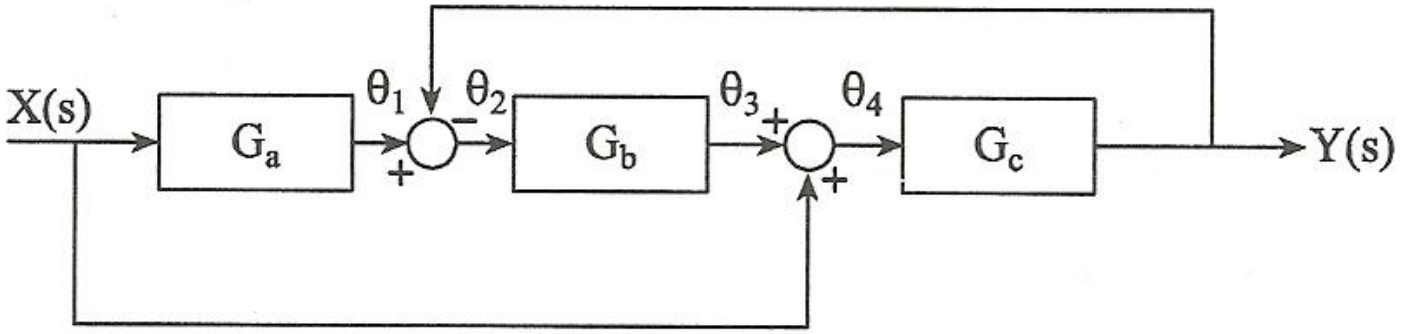


(Positive feedback)

$$\frac{Y(s)}{X(s)} = \frac{G_1(s)}{1 - G_1(s)G_2(s)}$$

Example:

Find the transfer function of the system. $\frac{Y(s)}{X(s)}$



$$\frac{Y(s)}{X(s)} = ?$$