Example 3: Isothermal CSTR with 1. Order Chemical Reaction

Find the model and the transfer function of the process.

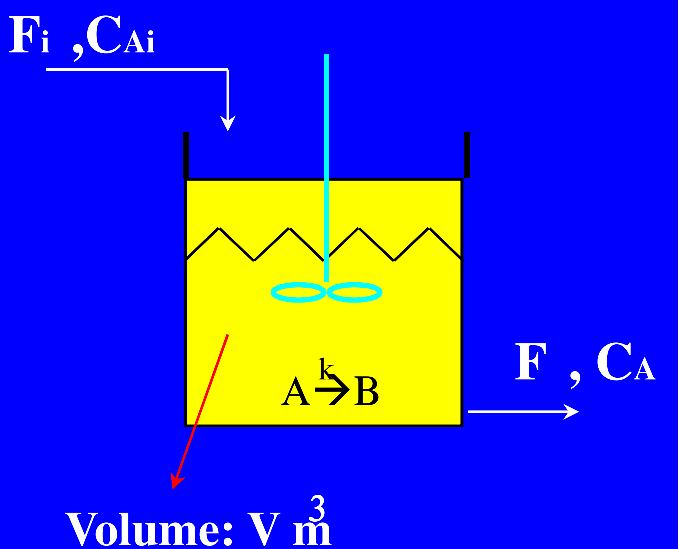
Mass balance for component A

Accumulation=In- Out + Generation

$$\frac{d(M_A V C_A)}{dt} = M_A F C_{Ai} - M_A F C_A - M_A V k C_A$$

M_A: molecular weight of A, V= constant

$$\frac{dC_A}{dt} = \frac{F}{V} C_{Ai} - \frac{F}{V} C_A - kC_A$$



Output var.: C_A (mol/m^3)

Input var.: C_{Ai} (mol/m^3)

$$\frac{dC_A}{dt} + \left(\frac{F + kV}{V}\right) C_A = \frac{F}{V} C_{Ai}$$

Model:

$$\tau \frac{dC_A}{dt} + C_A = \frac{F}{F + kV} C_{Ai}$$

$$\tau = \frac{V}{F + kV}$$

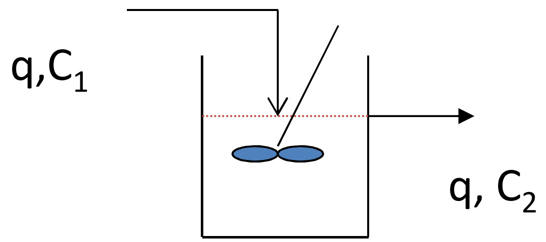
Take laplace , [C_A (t=0) = 0 initial condition]

$$\tau s C_A(s) + C_A(s) = \underbrace{\frac{F}{F + kV}}_{K} C_{Ai}(s)$$

Transfer Function:

$$G(s) = \frac{C_A(s)}{C_{Ai}(s)} = \frac{K}{\tau s + 1}$$

Example 4: Mixing



V: volume of tank(m^3)
q: volumetric flow rate(m^3 /s)
C_1 : inlet sugar conc.(kg/m^3)
C_2 : outlet sugar conc.(kg/m^3)

Find the model and the transfer function of this process

C_1 : input variable (in terms of deviation variable)
C_2 : output variable (in terms of deviation variable)

What is the transfer function of the process:

$$\frac{C_2(s)}{C_1(s)} = ?$$

Mass Balance:

Rate of mass in–Rate of mass out= Rate of Mass Accum.

$$\dot{m}_{in} - \dot{m}_{out} = \frac{d(m)}{dt}$$

$$qC_1 - qC_2 = \frac{d(VC_2)}{dt}$$

$$q(C_1 - C_2) = V\frac{d(C_2)}{dt}$$

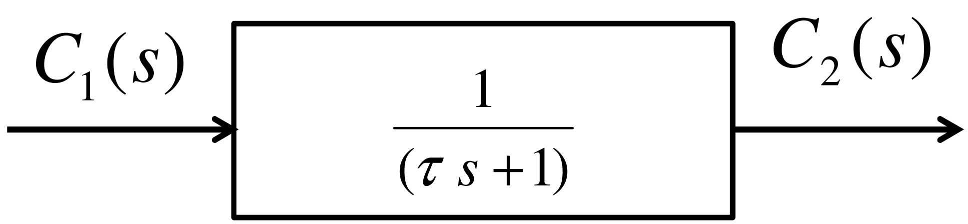
$$(C_1 - C_2) = \underbrace{\left(\frac{V}{q}\right)}_{\tau}\frac{d(C_2)}{dt}$$

$$C_1 = \tau\frac{d(C_2)}{dt} + C_2$$

$$C_1(s) = \tau \; s \, C_2(s) + C_2(s)$$

$$C_1(s) = C_2(s)\left[\tau \; s + 1\right]$$

$$\frac{C_2(s)}{C_1(s)} = \frac{1}{\left[\tau \; s + 1\right]}$$



Block Diagram

Example 5: If there are two input variables

In some cases there may be two input variables effecting the process. (Example: mixing tank with two inlet streams)

$$\tau \frac{dy}{dt} + y = x$$

Mathematical model of a process that is represented by a 1st order differential equation <u>(single input variable)</u>

$$\tau \frac{dy}{dt} + y = K_1 x_1 + K_2 x_2$$

Mathematical model of a process that is represented by a 1st order differential equation <u>(two input variables)</u>

y: output variable
x: input variable
τ : (tau) time constant
K: gain →constant

Example 5: If there are two input variables

Let's find the transfer function of this process. (take laplace)

$$\tau \frac{dy}{dt} + y = K_1 x_1 + K_2 x_2$$

$$\tau sY(s) - y(0) + Y(s) = K_1 X_1(s) + K_2 X_2(s)$$

$$Y(s)[\tau s + 1] = K_1 X_1(s) + K_2 X_2(s)$$

$$Y(s) = \frac{K_1}{[\tau s + 1]} X_1(s) + \frac{K_2}{[\tau s + 1]} X_2(s)$$

$$\frac{Y(s)}{X_1(s)} = \frac{K_1}{[\tau s + 1]}$$

$$\frac{Y(s)}{X_2(s)} = \frac{K_2}{[\tau s + 1]}$$

Example 5: If there are two input variables

$$\frac{Y(s)}{X_1(s)} = \frac{K_1}{[\tau s + 1]}$$ **Transfer function I**

$$\frac{Y(s)}{X_2(s)} = \frac{K_2}{[\tau s + 1]}$$ **Transfer function II**

Y(s) is equal to sum of these two transfer functions