DYNAMICS OF LINEAR SYSTEMS

Definitions

Order of the process:

$$G(s) = \frac{N(s)}{D(s)} = \frac{a_m s^m + a_{m-1} s^{m-1} + \dots\dots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \dots\dots + b_1 s + b_0}$$

$$= \frac{(s - z_1)(s - z_2)\dots\dots(s - z_m)}{(s - p_1)(s - p_2)\dots\dots(s - p_n)}$$

Transfer function is the ratio of two polynomials, n should be greater or equal to m

- **t domain: the order of the highest derivative term of the output**
- **s domain: The order of the process will be the highest power of s encountered in denominator. (n)**

Zeros of the system: The values of s that cause the numerator of transfer function to become zero ($z_1, z_2, \ldots, z_m$).

Poles of the system: The values of s that cause the denominator of transfer function to become zero ($p_1, p_2, \ldots, p_m$).

1. order system transfer function: $G(s) = \dfrac{K}{\tau s + 1}$

Time constant, τ: the constant represents the time it takes the system to respond. It gives information about how fast the response of the system is. *(unit: time)*

Steady State Gain, K: gives information about the response of the output variable when a change is introduced in the process.

THE DYNAMIC BEHAVIOUR OF FIRST ORDER SYSTEMS

- It is important how the processes respond to the variations
- How the variation in any input varible affects our controlled variables?
- These variations in input variables can occur in different ways.

INPUT VARIATIONS

- There are 6 important input variations in industrial applications

1) Step input (Basamak girdisi)
2) Ramp input (Rampa girdisi)
3) Rectangular pulse (Dikdörtgen puls)
4) Sinusoidal input (Sinüzoidal girdi)
5) Impulse input (Impuls girdisi)
6) Variable inputs

THE DYNAMIC BEHAVIOUR OF FIRST ORDER SYSTEMS

Typical inputs to determine process dynamics;

- $x_s(t) = \begin{cases} 0 & t < 0 \\ A & t \geq 0 \end{cases}$

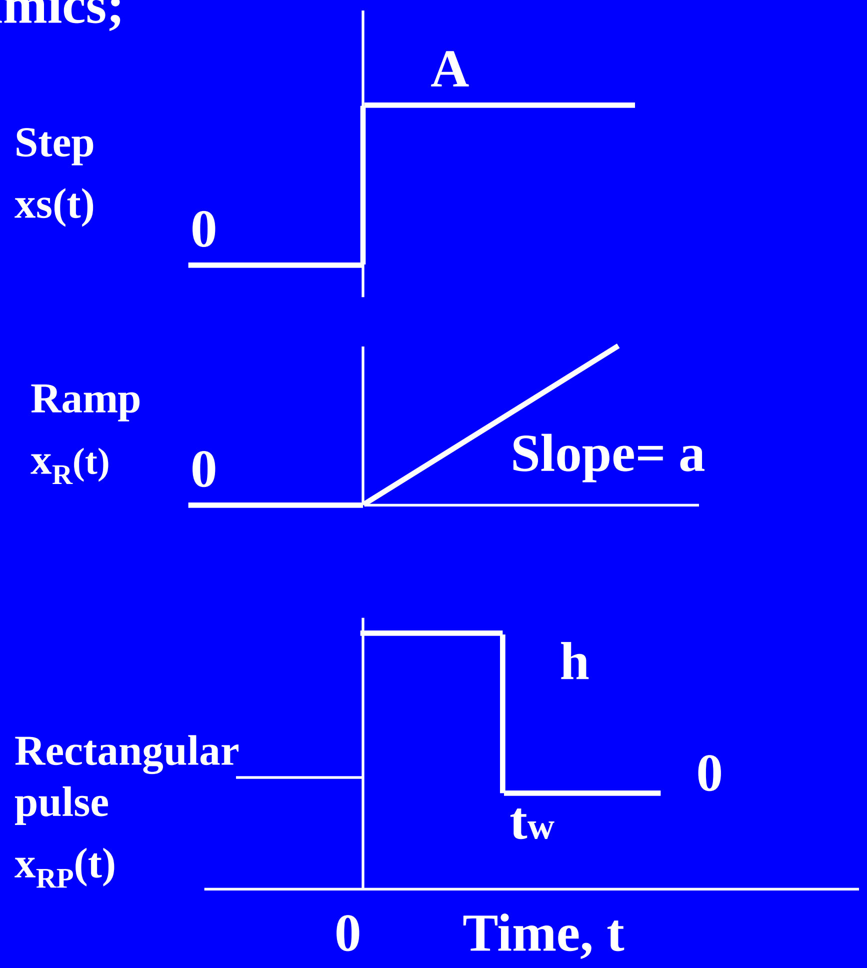
  $x_s(s) = A/s$

- $x_R(t) = \begin{cases} 0 & t < 0 \\ at & t \geq 0 \end{cases}$

  $x_R(s) = a / s^2$

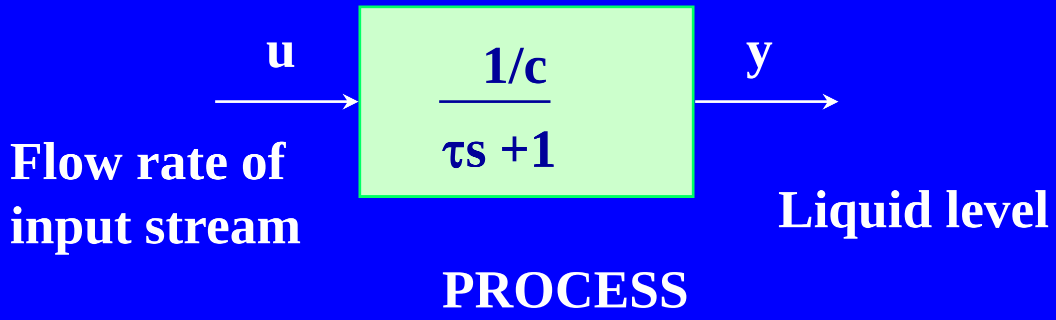
- $x_{RP}(t) = \begin{cases} 0 & t < 0 \\ h & 0 \leq t < t_w \\ 0 & t > t_w \end{cases}$

  $x_{RP}(s) = \dfrac{h}{s}(1 - e^{-t_w s})$



The response of the system to a step input

$$G(s) = \frac{y(s)}{u(s)} = \frac{1/c}{\tau s + 1}$$ **(For the liquid level control system)**



Step Input:

$$u(t) = \begin{cases} 0 & t < 0 \\ A & t \geq 0 \end{cases} \quad ; \quad u(s) = \frac{A}{s}$$

$$y(s) = \frac{K}{\tau s + 1} \frac{A}{s}$$

Partial fractionation;

$$y(s) = AK\left(\frac{1}{s} - \frac{\tau}{\tau s + 1}\right)$$

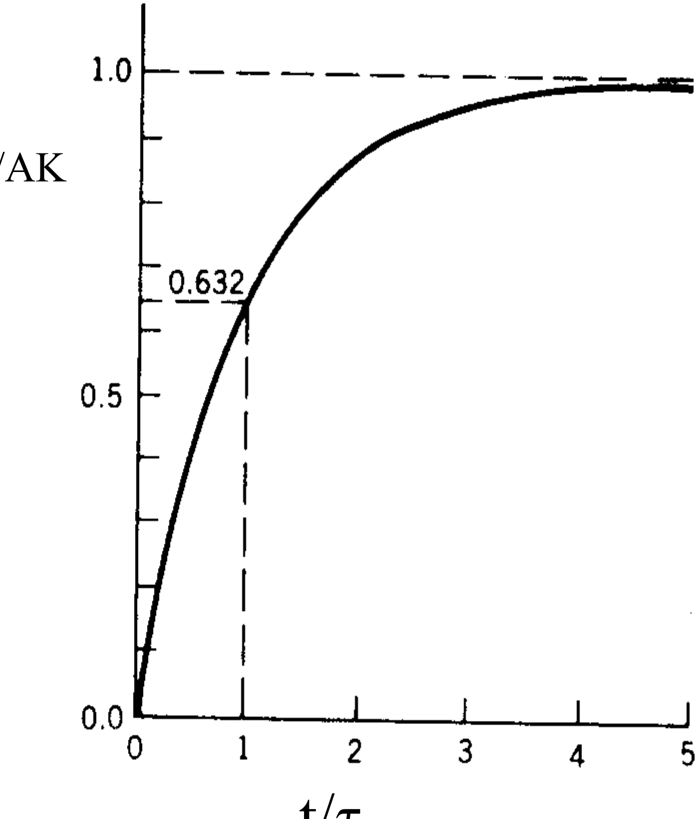
By taking the inverse laplace of each side;

$$y(t) = AK\left(1 - e^{-t/\tau}\right)$$

K = Process gain

A = the magnitude of the step input

τ = time constant

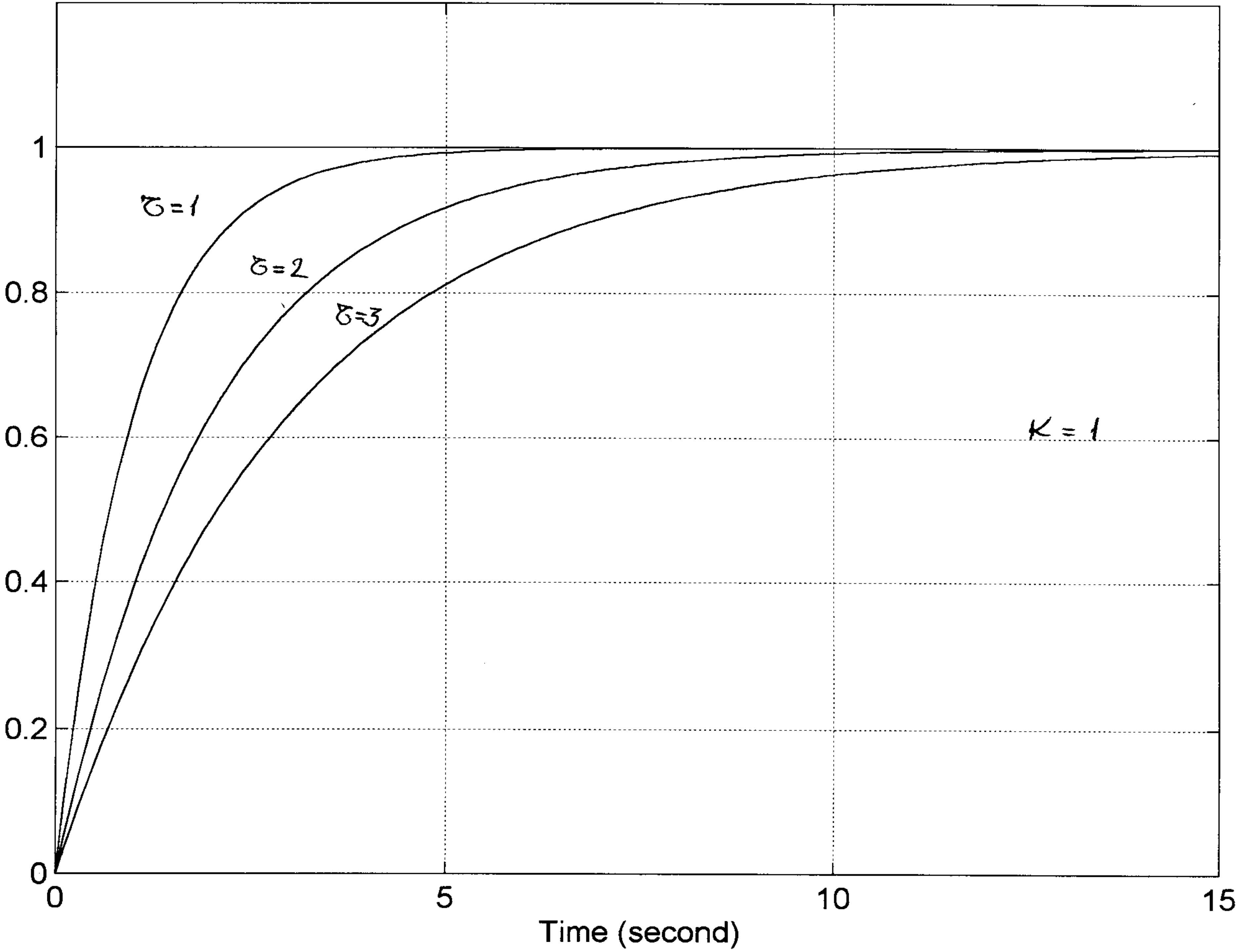


igure 5.4. Step response of a first-order process.

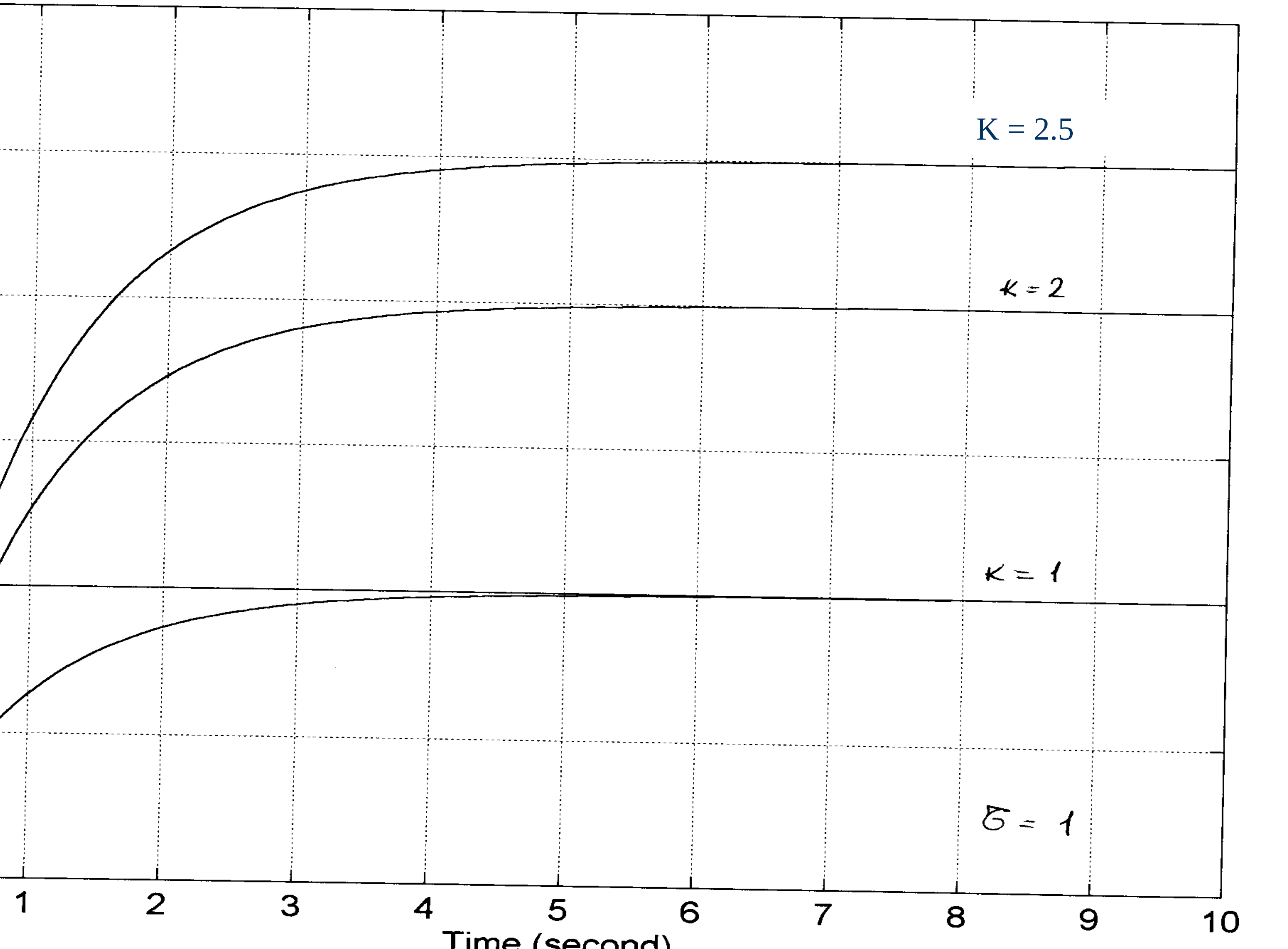
$$y = AK(1 - e^{-\frac{t}{\tau}})$$

Table 5.1 Response of a First-Order Process to a Step Input

t	$\frac{y}{AK} = (1 - e^{-\frac{t}{\tau}})$
0	0
τ	0.6321
2τ	0.8647
3τ	0.9502
4τ	0.9817
5τ	0.9933



1
0.8
0.6
0.4
0.2
0
0
5
10
15
Time (second)
τ = 1
τ = 2
τ = 3
K = 1


K = 2.5
K = 2
K = 1
τ = 1
1
2
3
4
5
6
7
8
9
10
Time (second)

Example 1

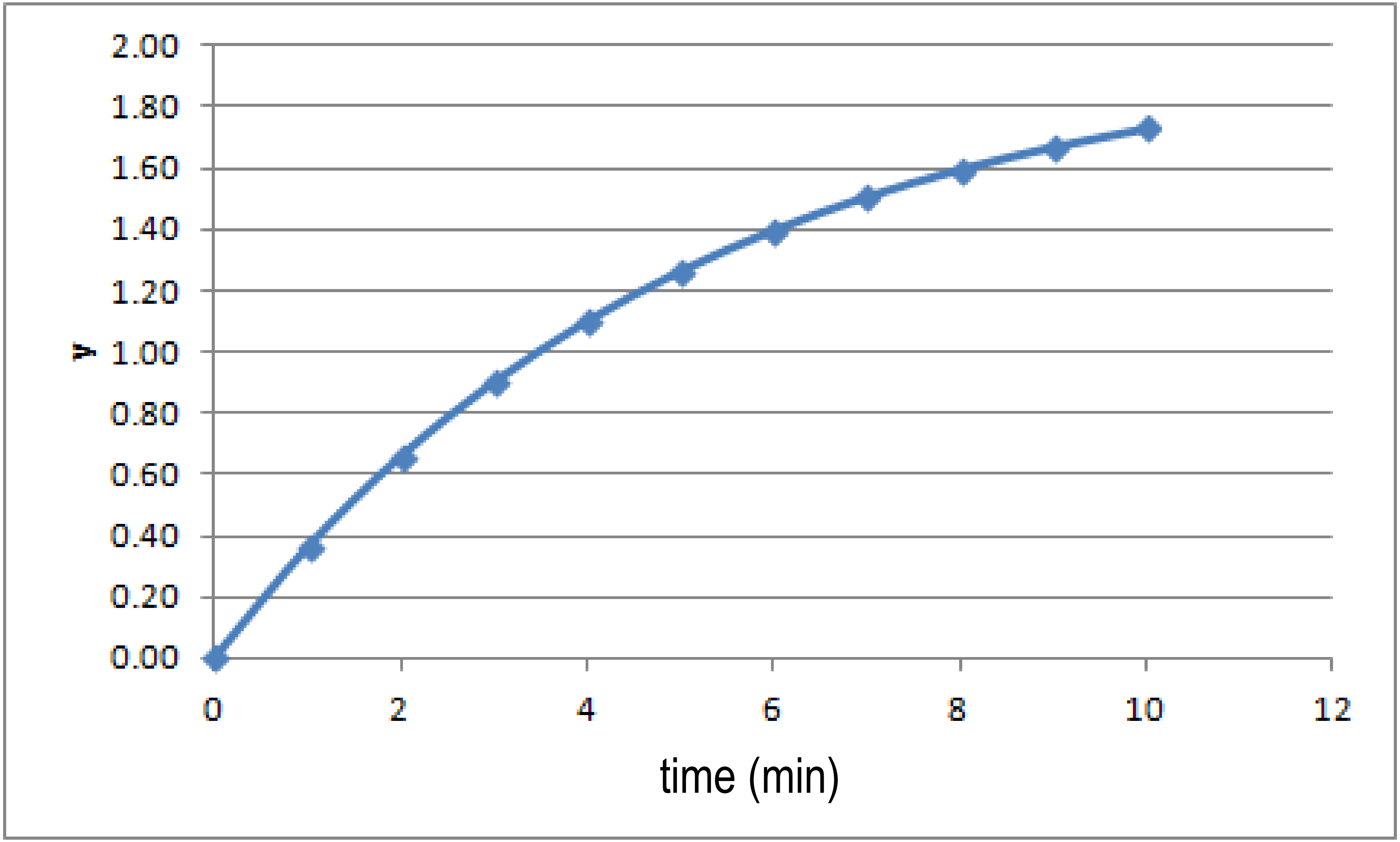
For a first order process with a time constant of 5 min and process gain of 1, a step input was adapted to the input variable with a magnitude of 2. How is the output variable effected from this step input? Draw the graph of output variable vs time.

$$\frac{y(s)}{u(s)} = \frac{1}{[5s+1]}$$

$$y(s) = \frac{1}{[5s+1]}\frac{2}{s}$$

$$y = 2(1-e^{-\frac{t}{5}})$$

time(min)	y(t)
0	0.00
1	0.36
2	0.66
3	0.90
4	1.10
5	1.26
6	1.40
7	1.51
8	1.60
9	1.67
10	1.73



2.00
1.80
1.60
1.40
1.20
y
1.00
0.80
0.60
0.40
0.20
0.00
0
2
4
6
8
10
12
time (min)

Example 2 : Blending (Mixing) System

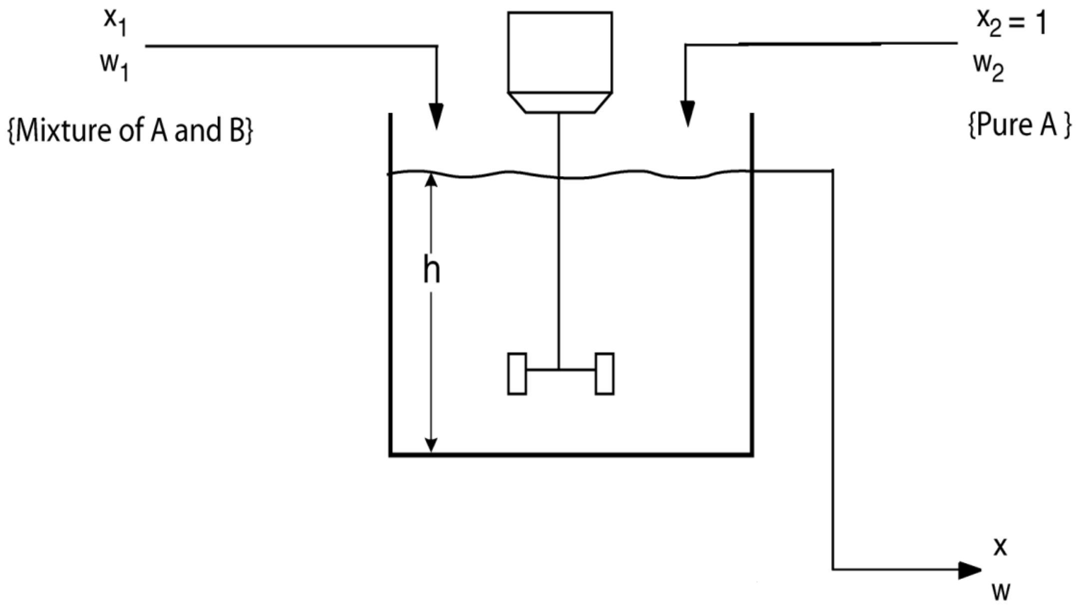


Figure 1.3. Stirred-tank blending system.

w_1, w_2 and w: mass flow rates

- x_1, x_2 and x: mass fraction of component A

- The blending system given in the figure is at steady state initially. The initial steady state values are given as; w_1=600 kg/min w_2= 2 kg/min x_1=0.05 x_2 =1. The density and volume of liquid are constant. (V=2 m^3 and ρ=900 kg/ m^3)

a) If x_2 and w_2 are constant through the whole process, calculate K and τ?

b) If x_1 is increased from 0.05 to 0.075 suddenley at t=0, what will be the response equation of the output variable, y(t)=?

c) What will be the concentration of outlet stream at 5. minute?

Dead time

➔**Dead Time:** The amount of time it takes for a process to start changing after a disturbance in the system.

- Delay of response:

$$C_s(t) = C(t-\theta)$$

- Transfer Function:

$$G_p(s) = e^{-\theta s}$$

The response to a step input with a magnitude of A

If the *dead time* = 0;

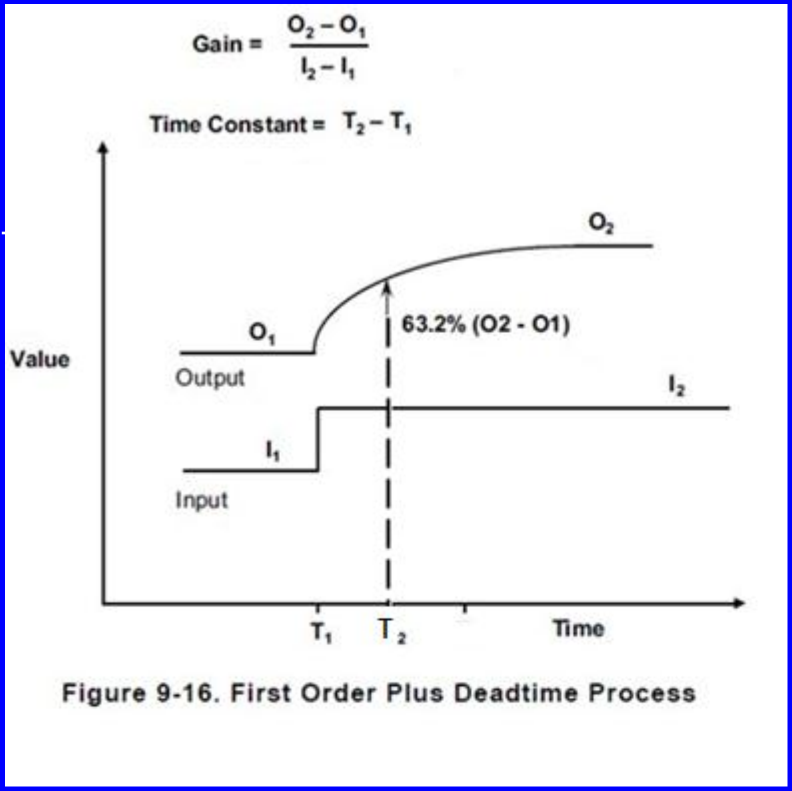


Figure 9-16. First Order Plus Deadtime Process

If the *dead time* ≠ 0;

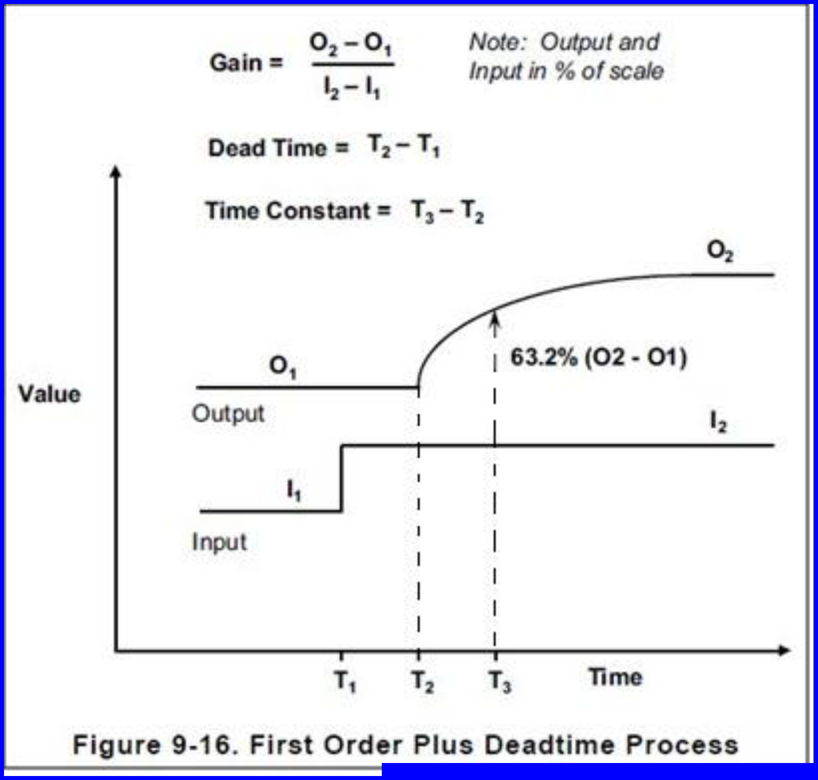


Figure 9-16. First Order Plus Deadtime Process

The time t when 63.2 % of (O_2-O_1) value is reached = τ

Model: $G(s) = \frac{K e^{-\theta s}}{\tau s + 1}$