a- Proportional Control (P)

$$\theta_0(t) = k_1\theta_e(t)$$

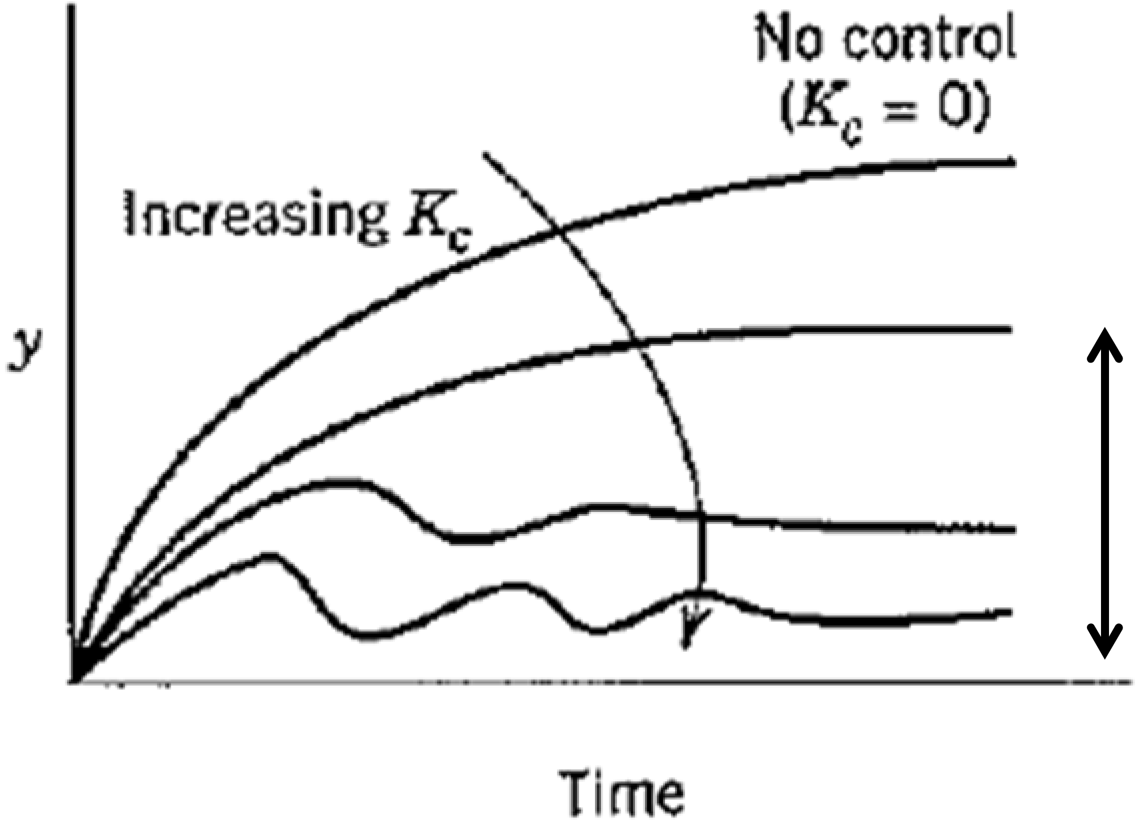
Taking Laplace, $\longrightarrow$ $\theta_0(s) = k_1\theta_e(s)$

$$G_c = k_1 = K_C$$

$k_1 = K_C$ proportional gain

- It is the simplest mode of control
- It eliminates the osciallation in On/off control.
- The difference between set point and the value of the controlled variable is called **offset.**
- It is the characteristic of proportional control.

Proportional control (P)



Offset

Set point

b- Proportional + Integral Control : (P+I)

$$G_c = \frac{\theta_0\,(s)}{\theta_e\,(s)} \Rightarrow G_c = k_1\left(1+\frac{1}{\tau_R s}+\tau_D s\right)$$

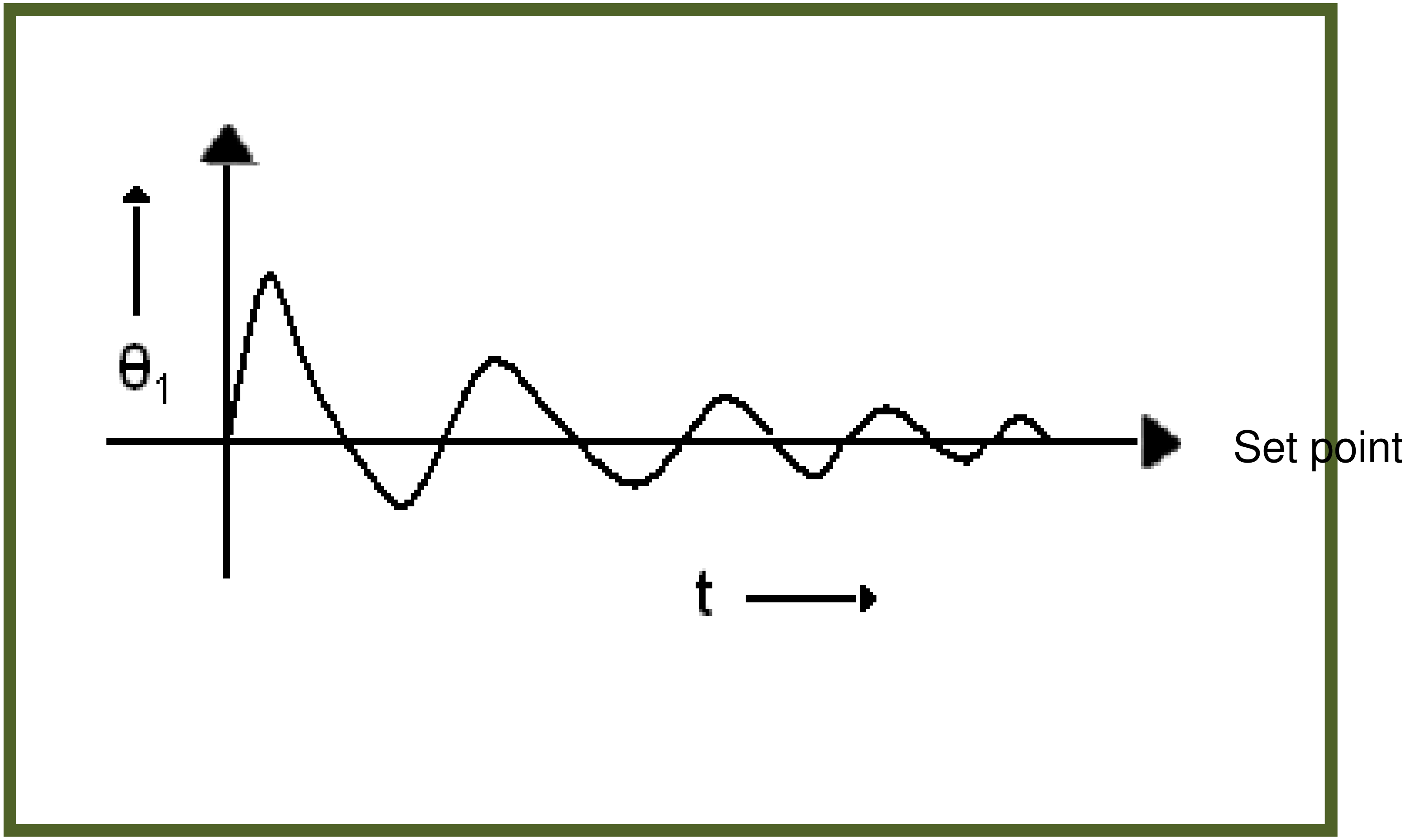
$$\tau_D = 0$$

$$G_c = k_1\left[1+\frac{1}{\tau_R s}\right] = \frac{k_1(1+\tau_R s)}{\tau_R s}$$

-the addition of integral control eliminates the offset in controlled variable.

-But the disadvantage is: longer response time

Proportional+ Integral Control : (P+I)



C- Proportional + Integral+Derivative Control :(P+I+D)

$$\theta_0(s) = k_1\left(1 + \frac{1}{\tau_R s} + \tau_D s\right)\theta_e(s)$$

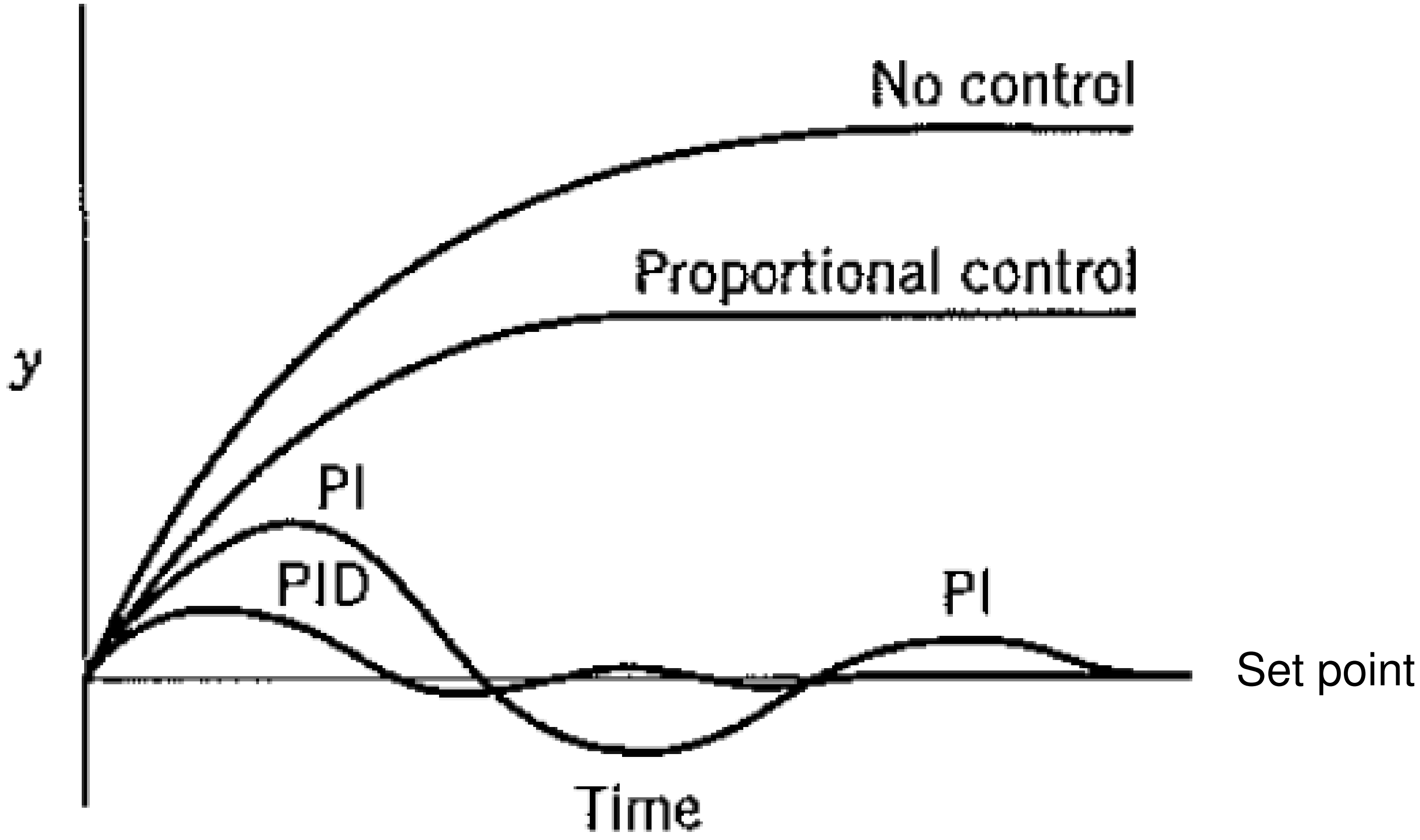
$$G_c = \frac{\theta_0(s)}{\theta_e(s)} \Rightarrow G_c = k_1\left(1 + \frac{1}{\tau_R s} + \tau_D s\right)$$

-It has the advantages and disadvantages of P+I control.

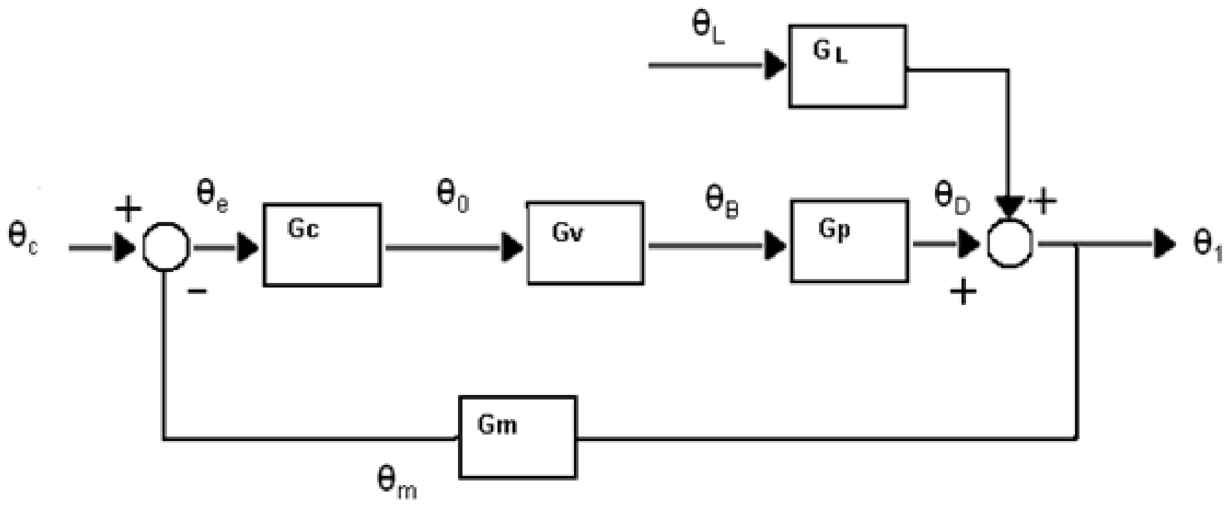
-Due to Integral control it eliminates the offset and due to derivative control it decreases the response time and the degree of oscillation

-It is expensive

C- Proportional + Integral+Derivative Control :(P+I+D)



FEEDBACK CONTROL SYSTEM



- Two input variables; θ_L , θ_c

One output variable; θ_1

Two important characteristics of controller

-Regulatory control: eliminates the disturbances
-Servo control: monitors the variations in set point

θ_L → Disturbance

θ_c → Variation in set point.

In a control system the controlled variable can be affected from disturbances and variations in set point.

Transfer function of feedback control system;

$$\theta_e = \theta_c - \theta_m$$

$$\theta_D = G_p G_v G_c \theta_e = G\theta_e$$

$$G_p G_v G_c = \frac{\theta_D}{\theta_B}\frac{\theta_B}{\theta_0}\frac{\theta_0}{\theta_e} = \frac{\theta_D}{\theta_e}$$

$$\theta_e = \theta_c - \theta_1 G_m$$

$$\theta_1 = \theta_D + G_L \theta_L$$

$$\theta_1 = \underset{\substack{\Downarrow \\ \textbf{I. Term}}}{\boxed{\frac{G_L}{1 + GG_m}\theta_L}} + \underset{\substack{\Downarrow \\ \textbf{II. Term}}}{\boxed{\frac{G}{1 + GG_m}\theta_c}}$$

- ***For regulatory control first term should be used.***
- ***For servo control second term should be used.***

Proportional (P) control of a first order system

- θ_1 can be calculated by using the transfer function of each factor in the system

$G_c = k_1$ Proportional control

$G_v = K_v$ Valve

$G_m = K_m$ Measuring device

$G_p = \frac{K_p}{Ls + 1}$ Process

$G_L = \frac{K_L}{Ls + 1}$ Disturbance

$$G = G_c G_v G_p$$

$$G = \frac{k_1 K_v K_p}{Ls + 1}$$

A step input as a disturbance;

$$\frac{\theta_1(s)}{\theta_L(s)} = \frac{G_L}{1 + GG_m}$$

$$\frac{\theta_1(s)}{\theta_L(s)} = \frac{K_L/(Ls+1)}{1 + \underbrace{k_1 K_v K_p\, K_m}_{K}/(Ls+1)}$$

$$\frac{\theta_1(s)}{\theta_L(s)} = \frac{K_L/(Ls+1)}{1 + \dfrac{K}{Ls+1}} = \frac{K_L}{(Ls+1) + K}$$

$\theta_L(s) \;=\; F/s$ A step input as a disturbance;

$$\theta_1(s) = \frac{F}{s}\,\frac{K_L}{1+K}\left[\frac{1}{(L/K+1)s+1}\right]$$

Taking the inverse laplace

$$\theta_1(t) = \frac{FK_L}{1+K}\,(1 - \;exp\,(-t(1+K)/L))$$

$$\theta_1(t)_\infty = \frac{FK_L}{1+K}$$

……………………………………………………………………………………………………

- ❑ ***when a step input is applied to a first order process without proportional control;***

$$\theta_1(t) = FK_L(1 - \;exp\,(-t/L))$$

$$\theta_1(t)_\infty = FK_L$$

- Proportional control approximates the output variable θ_1 to set point.

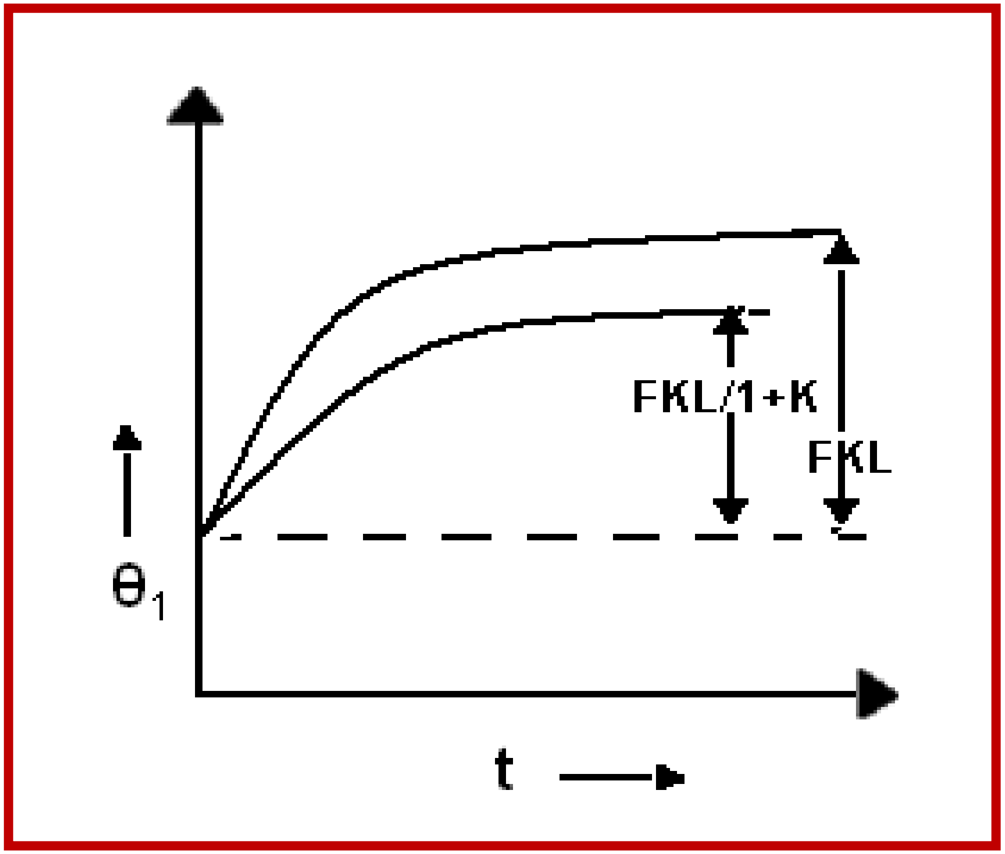


Figure Proportional control of a first order system and its dynamic response

Proportional +Derivative (PD) control of a first order system

- θ_1 can be calculated by using the transfer function of each factor in the system

$$G_c = K_1(1 + \tau_D s)$$ PD Control

$$G_v = K_v$$ Valve

$$G_m = K_m$$ Measuring device

$$G_p = \frac{K_p}{Ls + 1}$$ Process

$$G_L = \frac{K_L}{Ls + 1}$$ Disturbance

$$G = G_c G_v G_p$$

$$GG_m = \frac{K_m K_1 K_v K_p (1 + \ \tau_D s)}{Ls + 1}$$

$$\frac{\theta_1(s)}{\theta_L(s)} = \frac{G_L}{1 + GG_m}$$

$$\theta_1(s) = \frac{\frac{K_L}{Ls + 1}\theta_L(s)}{1 + \frac{K(1 + \ \tau D s)}{Ls + 1}}$$

$$\theta_L(s) = F/s$$

$$\theta_1(s) = \frac{FK_L}{1 + K}\left[\frac{1}{s\left(\frac{L + K\,\tau_D}{1 + K}s + 1\right)}\right]$$

$$\theta_1(t) = \frac{FK_L}{1 + K}(1 - \ exp(-(1 + K)/(L + K\,\tau_D)t)$$

- ***A step input applied to disturbance of a first order system under <u>Proportional+Derivative</u> (PD) control***

$$\theta_1(t) = \frac{FK_L}{1+K}(1 - \; exp(-(1+K)/(L+K\,\tau_D)t)$$

A step input applied to disturbance of a first order system under <u>Proportional</u> (P) control

$$\theta_1(t) = \frac{FK_L}{1+K}(1 - \; exp(-t(1+K)/L)$$

- ***A step input applied to disturbance of a first order system without any control***

$$\theta_1(t) \; = \; FK_L(1 - \; exp(-t/L)$$

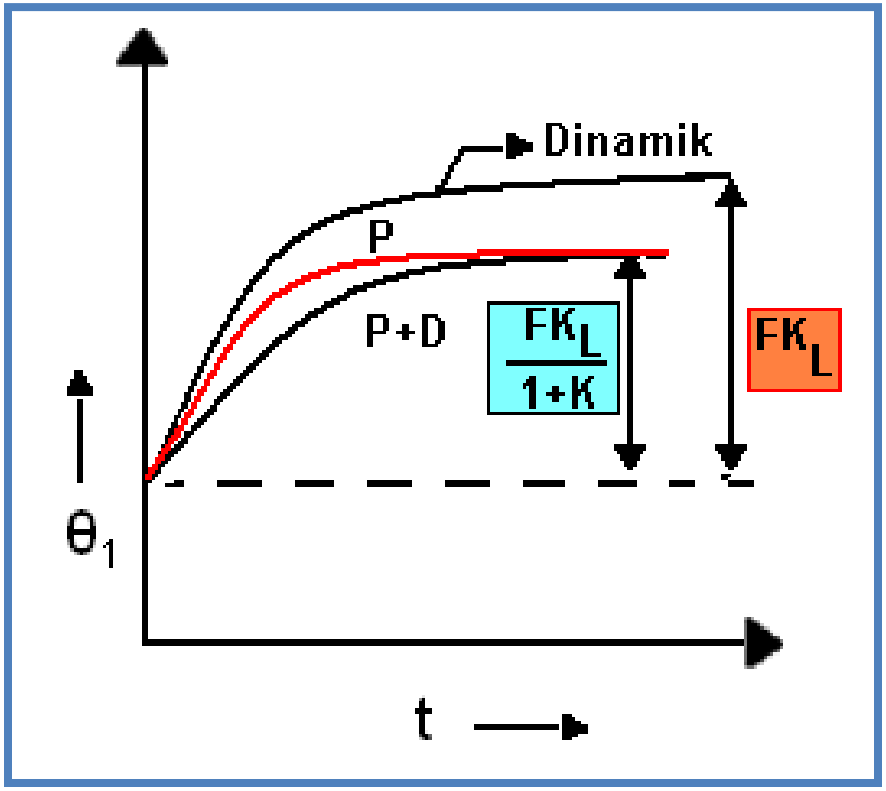


Figure Comparison of P and P+D control with dynamic results

Proportional +Integral+Derivative (PID) control of a first order system

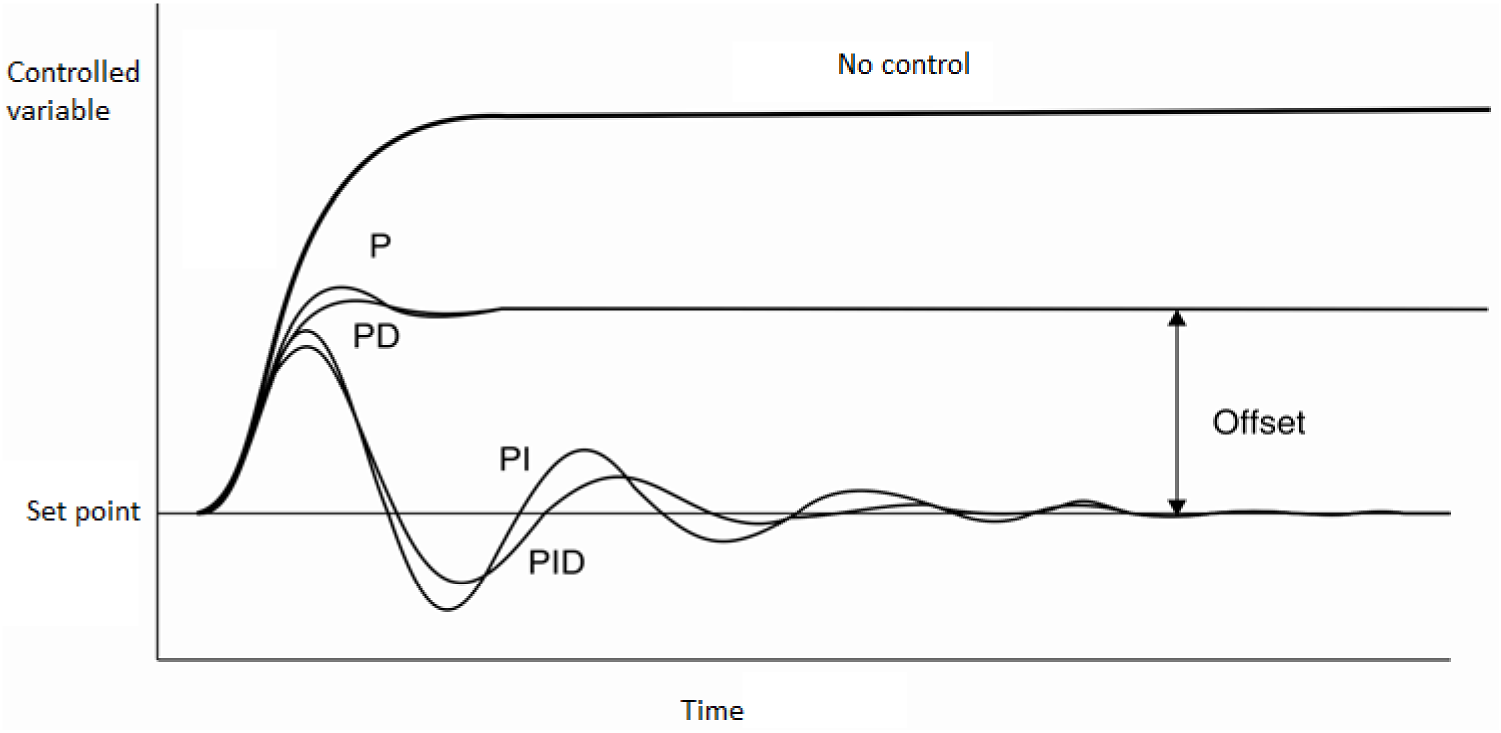
$$G_c = \frac{\theta_0(s)}{\theta_e(s)} \Rightarrow G_c = k_1\left(1+\frac{1}{\tau_R s}+\tau_D s\right)$$

$$G_c = k_1(\tau_R\tau_D s^2 + \tau_R s + 1)/\tau_R s$$

$$1 + GG_m = 1 + \frac{K_m K_v K_p k_1(\tau_R\tau_D s^2 + \tau_R s + 1}{(Ls + 1)\tau_R s}$$

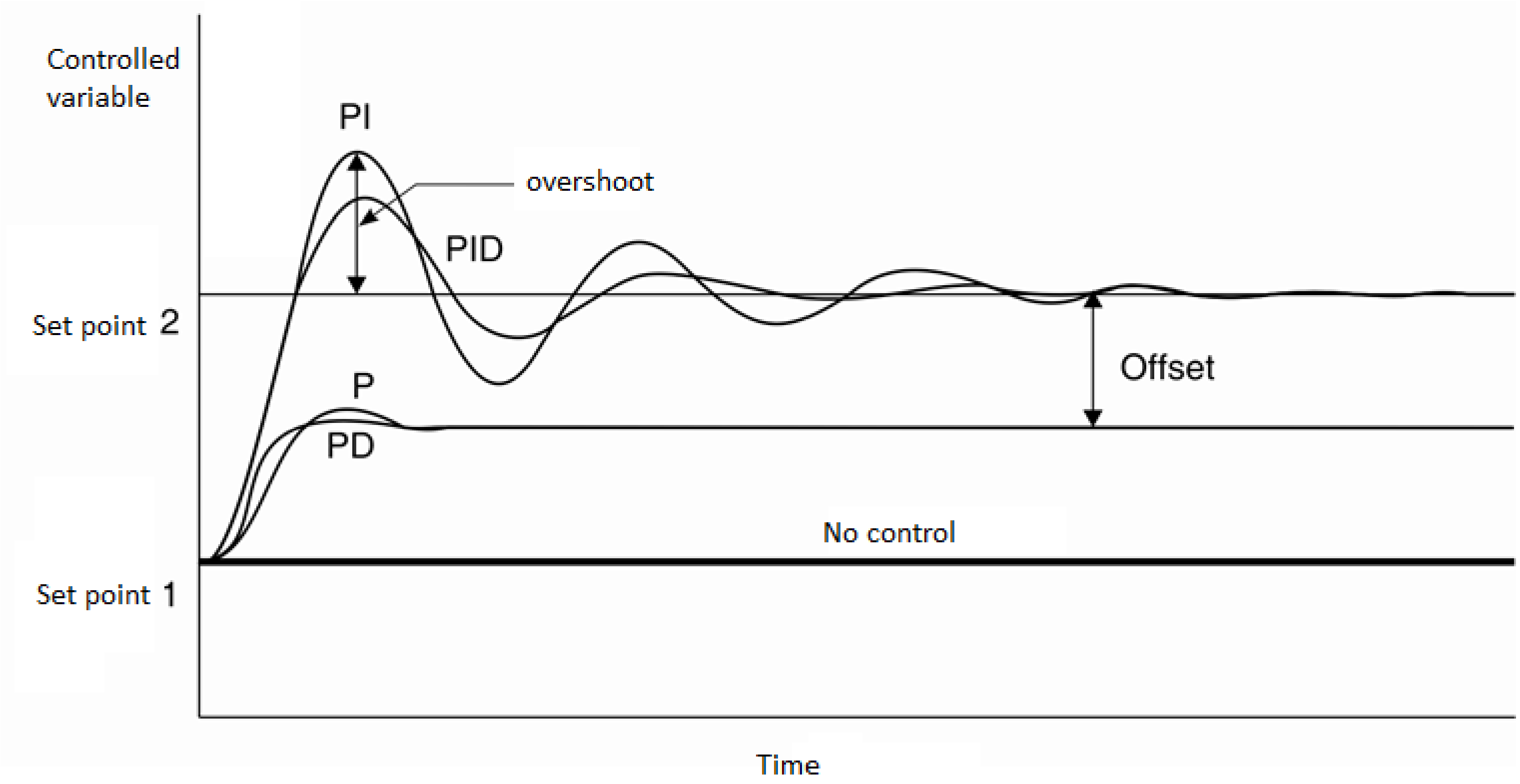
$$\frac{\theta_L(s)}{\theta_1(s)} = \frac{K_L\tau_R s}{\tau_R(L + K\tau_D)s^2 + \tau_R(1 + K)s + K}$$

The effects of control modes on the disturbance



P+I+D Control (Disturbance)

The effects of control modes on variations in set point



P+I+D Control (variations in set point)