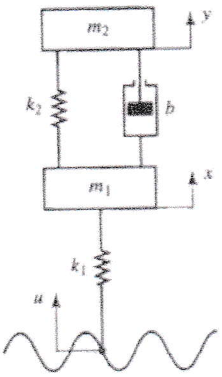
25p

1. The figure is the simplified version of an automobile or motorcycle suspension system. Obtain the transfer function Y(s)/U(s)



$$m_1\ddot{x} = k_2(y-x) + b(\dot{y}-\dot{x}) + k_1(u-x)$$

$$m_2\ddot{y} = -k_2(y-x) - b(\dot{y}-\dot{x})$$

$$m_1\ddot{x} + b\dot{x} + (k_1+k_2)x = b\dot{y} + k_2 y + k_1 u$$

$$m_2\ddot{y} + b\dot{y} + k_2 y = b\dot{x} + k_2 x$$

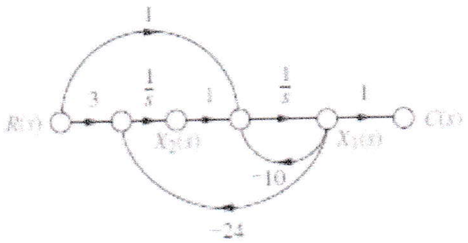
$$[m_1 s^2 + bs + (k_1+k_2)]\,X(s) = (bs+k_2)\,Y(s) + k_1 U(s)$$

$$[m_2 s^2 + bs + k_2]\,Y(s) = (bs+k_2)\,X(s)$$

$$(m_1 s^2 + bs + k_1 + k_2)\,\frac{m_2 s^2 + bs + k_2}{bs + k_2}\,Y(s) = (bs+k_2)\,Y(s) + k_1 U(s)$$

$$\frac{Y(s)}{U(s)} = \frac{k_1(bs+k_1)}{m_1 m_2 s^4 + (m_1+m_2)b s^3 + [k_1 m_2 + (m_1+m_2)k_2]s^2 + k_1 b s + k_1 k_2}$$

2. Obtain transfer functions from given diagrams by using Mason's rule 18p



$P_1 = 3\,\frac{1}{s}\,\frac{1}{s} = \frac{3}{s^2}$ - 3p

$P_2 = \frac{1}{s}$ -3p

We have only two loops

$L_1 = -\frac{10}{s}$ -3p

$L_2 = -\frac{24}{s^2}$ -3p

$\Delta = 1 - \left(-\frac{10}{s} - \frac{24}{s^2}\right)$

$= \frac{s^2 + 10s + 24}{s^2}$ - 3p

$\Delta_1 = \Delta\Big|_{L_1=0,\ L_2=0} = 1$

$\frac{C(s)}{R(s)} = \frac{s+3}{s^2 + 10s + 24}$ - 3p

$\Delta_2 = \Delta\Big| = 1$

3. Tell how many roots of the following polynomial are in the rhp, in the lhp, and on jw-axis **10p**

$$T(s) = \frac{s^3 + 2s^2 + 7s + 21}{s^5 - 2s^4 + 3s^3 - 6s^2 + 2s + 4}$$

s^5	1	3	2
s^4	-2	-6	4
s^3	$\cancel{0}$ ε	4	
s^2	$\frac{-6\varepsilon+8}{\varepsilon} = b_1$	4	
s^1	$\frac{4b_1 - 4\varepsilon}{b_1} = c_1$	0	
s^0	4		

$$c_1 = \frac{4\left(\frac{-6\varepsilon+8}{\varepsilon}\right) - 4\varepsilon}{\frac{-6\varepsilon+8}{\varepsilon}} = \frac{-24\varepsilon + 32 - 4\varepsilon^2}{-6\varepsilon+8}$$

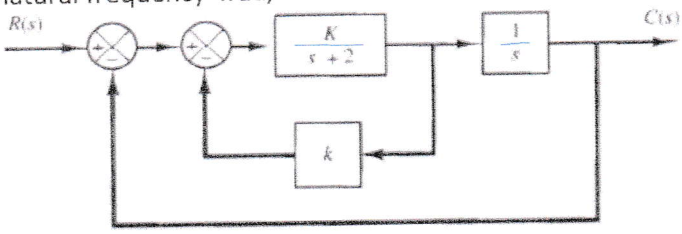
s^5	1	+
s^4	-2	−
s^3	ε	+
s^2	$\frac{-6\varepsilon+8}{\varepsilon}$	+
s^1	$\frac{4\varepsilon^2+24\varepsilon-32}{6\varepsilon-8}$	+
s^0	4	+

-2,5p -2,5p 2 sign change

2 rhp -2,5p

3 lhp -2,5p

4. Determie the values of K and k such that the system has a damping ratio of 0.7, and undamped natural frequency 4rad/sec. **15p**



inner loop

$$\frac{\frac{K}{s+2}}{1 + \frac{kK}{s+2}} = \frac{K}{s+2+kK}$$ -3p

Outer loop

$$\frac{\frac{K}{s+2+kK}}{1 + \frac{K}{s+2+kK}\cdot\frac{1}{s}} = \frac{K}{s^2+2s+kKs+K}$$ -3p

$$\frac{C(s)}{R(s)} = \frac{K}{s^2 + 2s + kKs + K}$$ -3p

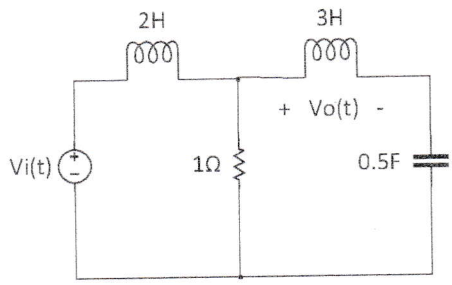
$K = w_n^2 = 4^2 = 16$ -3p

$2\xi w_n = 2 + kK$

$2 \cdot 0{,}7 \cdot 4 = 2 + kK = 2 + 16k$

$k = 0{,}225$ -3p

5. Find the transfer function Vo(s)/Vi(s) of the electrical system given below. (12p)



$$V_i(s) = 2s\, I_1(s) + (I_1(s) - I_2(s)) \quad -3p$$

$$V_o(s) = 3s\, I_2(s) \quad -3p$$

$$I_1(s) - I_2(s) = 3s\, I_2(s) + \frac{2}{s} I_2(s)$$

$$I_1(s) = \left(3s + 1 + \frac{2}{s}\right) I_2(s) \quad -3p$$

$$V_i(s) = (2s+1)\left(3s + \cancel{1} + \frac{2}{s}\right) I_2(s) - I_2(s)$$

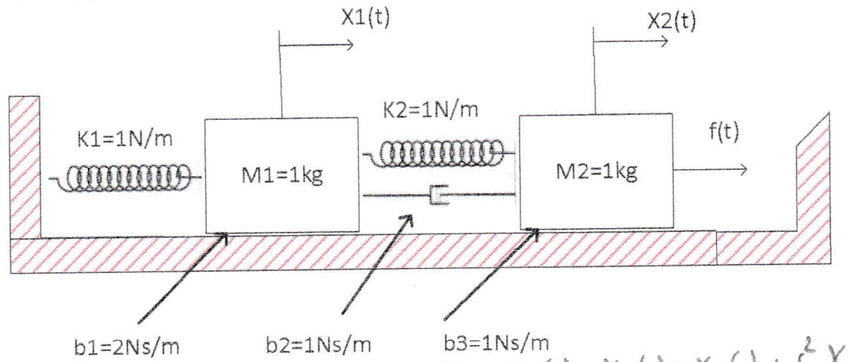
$$V_i(s) = \left(6s^2 + 2s + 4 + 3s + \cancel{1} + \frac{2}{s} - \cancel{1}\right) I_2(s) = \frac{6s^3 + 5s^2 + 4s + 2}{2}$$

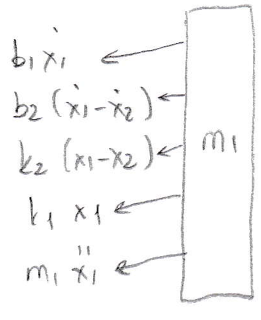
$$\frac{V_o(s)}{V_i(s)} = \frac{3s^2}{6s^3 + 5s^2 + 4s + 2} \quad -3p$$

6. Find the transfer function for the following mechanical systems. 20p

a) X1(s)/F(s)

b) X2(s)/F(s)



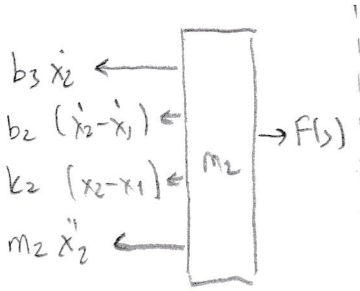


$$2s X_1(s) + s X_1(s) - s X_2(s) + X_1(s) - X_2(s) + X_1(s) + s^2 X_1(s) = 0 \quad -6p$$

$$X_1(s)\left[s^2 + 3s + 2\right] = \cancel{(s+1)} X_2(s)$$

$$(s+2)\cancel{(s+1)}$$

$$X_1(s)(s+2) = X_2(s)$$



$$F(s) = s X_2(s) + s X_2(s) - s X_1(s) + X_2(s) - X_1(s) + s^2 X_2(s) \quad -6p$$

$$F(s) = -(s+1) X_1(s) + (s^2 + 2s + 1) X_2(s)$$

$$F(s) = -(s+1) X_1(s) + (s^2 + 2s + 1)(s+2) X_1(s)$$

$$F(s) = (-s - 1 + s^3 + 2s^2 + 2s^2 + 4s + s + 2) X_1(s)$$

$$F(s) = (s^3 + 4s^2 + 4s + 1) X_1(s)$$

$$\frac{X_1(s)}{F(s)} = \frac{1}{s^3 + 4s^2 + 4s + 1} \quad -5p$$

$$\frac{X_2(s)}{F(s)} = \frac{s+2}{s^3 + 4s^2 + 4s + 1} \quad -3p$$