$$\hat{\theta} \cdot \hat{z} = \hat{z} \cdot \hat{\theta} = \cos(90+\theta) = -\sin(\theta)$$

$$\hat{\varphi} \cdot \hat{z} = \hat{z} \cdot \hat{\varphi} = \cos(90) = 0$$

Silindirik-Küresel

$$\hat{r} \cdot \hat{R} = \hat{R} \cdot \hat{r} = \cos(90-\theta) = \sin(\theta)$$

$$\hat{\varphi} \cdot \hat{R} = \hat{R} \cdot \hat{\varphi} = \cos(90) = 0$$

$$\hat{z} \cdot \hat{R} = \hat{R} \cdot \hat{z} = \cos(\theta)$$

$$\hat{r} \cdot \hat{\theta} = \hat{\theta} \cdot \hat{r} = \cos(\theta)$$

$$\hat{\varphi} \cdot \hat{\theta} = \hat{\theta} \cdot \hat{\varphi} = \cos(90) = 0$$

$$\hat{z} \cdot \hat{\theta} = \hat{\theta} \cdot \hat{z} = \cos(90+\theta) = -\sin(\theta)$$

$$\hat{r} \cdot \hat{\varphi} = \hat{\varphi} \cdot \hat{r} = \cos(90) = 0$$

$$\hat{\varphi} \cdot \hat{\varphi} = \hat{\varphi} \cdot \hat{\varphi} = \cos(0) = 1$$

$$\hat{z} \cdot \hat{\varphi} = \hat{\varphi} \cdot \hat{z} = \cos(90) = 0$$

Herhangi bir koordinat sisteminde (kaynak) ifade edilmiş bir vektörün başka koordinat sisteminde (hedef) ifade edilmesi için gereken bileşenler, kaynak vektörünün hedef koordinat sistemindeki birim vektörler ile nokta çarpımından bulunur.

$$\vec{\mathbf{A}} = \mathbf{A_x\hat{x} + A_y\hat{y} + A_z\hat{z}} \quad \textbf{(Kaynak olsun)}$$

$$\vec{\mathbf{A}} = \mathbf{A_u\hat{u} + A_v\hat{v} + A_w\hat{w}} \quad \textbf{(Hedef olsun)}$$

$$A_u = \vec{A} \cdot \hat{u} = \left(A_x\hat{x} + A_y\hat{y} + A_z\hat{z}\right) \cdot \hat{u}$$

$$A_u = A_x(\hat{x}\cdot\hat{u}) + A_y(\hat{y}\cdot\hat{u}) + A_z(\hat{z}\cdot\hat{u})$$

$$\mathbf{A_v}=\vec{\mathbf{A}}\cdot\hat{\mathbf{v}}= \left(\mathbf{A_x\hat{x}} + \mathbf{A_y\hat{y}} + \mathbf{A_z\hat{z}}\right)\cdot\hat{\mathbf{v}}$$

$$\mathbf{A_v} = \mathbf{A_x}(\hat{\mathbf{x}}\cdot\hat{\mathbf{v}}) + \mathbf{A_y}(\hat{\mathbf{y}}\cdot\hat{\mathbf{v}}) + \mathbf{A_z}(\hat{\mathbf{z}}\cdot\hat{\mathbf{v}})$$

$$A_w=\vec{A}\cdot\hat{w}= \left(A_x\hat{x} + A_y\hat{y} + A_z\hat{z}\right)\cdot\hat{w}$$

$$A_w = A_x(\hat{x}\cdot\hat{w}) + A_y(\hat{y}\cdot\hat{w}) + A_z(\hat{z}\cdot\hat{w})$$

$\vec{A}=A_x\hat{x}+ A_y\hat{y}+ A_z\hat{z}$ (Kaynak olsun)

$\vec{A}=A_r\hat{r}+ A_\varphi\hat{\varphi}+ A_z\hat{z}$ (Hedef olsun)

$$A_r = \vec{A}\cdot\hat{R}= \left(A_x\hat{x} + A_y\hat{y} + A_z\hat{z}\right)\cdot\hat{r}$$

$$A_r = A_x(\hat{x}\cdot\hat{r}) + A_y(\hat{y}\cdot\hat{r}) + A_z(\hat{z}\cdot\hat{r})$$

$$A_r = A_x\big(\cos(\varphi)\big) + A_y\big(\sin(\varphi)\big)+ A_z(\cos(90))$$

$$A_r = A_x\big(\cos(\varphi)\big) + A_y\big(\sin(\varphi)\big)$$

$$\mathbf{A_\varphi} = \vec{\mathbf{A}}\cdot\hat{\boldsymbol{\varphi}} = \left(\mathbf{A_x\hat{x}} + \mathbf{A_y\hat{y}} + \mathbf{A_z\hat{z}}\right)\cdot\hat{\boldsymbol{\varphi}}$$

$$\mathbf{A_\varphi} = \mathbf{A_x}(\hat{\mathbf{x}}\cdot\hat{\boldsymbol{\varphi}}) + \mathbf{A_y}(\hat{\mathbf{y}}\cdot\hat{\boldsymbol{\varphi}}) + \mathbf{A_z}(\hat{\mathbf{z}}\cdot\hat{\boldsymbol{\varphi}})$$

$$\mathbf{A_\varphi} = \mathbf{A_x}\big(\mathbf{cos}(\mathbf{90}\boldsymbol{\varphi})\big)+\mathbf{A_y}\big(\mathbf{cos}(\boldsymbol{\varphi})\big)+\mathbf{A_z}(\mathbf{cos}(\mathbf{90}))$$

$$\mathbf{A_\varphi} = \mathbf{A_x}\big(-\mathbf{sin}(\boldsymbol{\varphi})\big) + \mathbf{A_y}\big(\mathbf{cos}(\boldsymbol{\varphi})\big)$$

$$A_z = \vec{A}\cdot\hat{z}= \left(A_x\hat{x} + A_y\hat{y} + A_z\hat{z}\right)\cdot\hat{z}=A_z\hat{z}\cdot\hat{z}=A_z$$

$$A_z = A_x(\hat{x}\cdot\hat{z}) + A_y(\hat{y}\cdot\hat{z}) + A_z(\hat{z}\cdot\hat{z})$$

$$A_z = A_x(\cos(90)) + A_y(\cos(90)) + A_z(\cos(0)) = A_z$$

$$\mathbf{\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}}\ \textbf{(Kaynak olsun)}$$

$$\mathbf{\vec{A} = A_R\hat{R} + A_\theta\hat{\theta} + A_\varphi\hat{\varphi}}\ \textbf{(Hedef olsun)}$$

$$A_R = \vec{A}\cdot\hat{R} = \left(A_x\hat{x} + A_y\hat{y} + A_z\hat{z}\right)\cdot\hat{R} = A_x(\hat{x}\cdot\hat{R}) + A_y(\hat{y}\cdot\hat{R}) + A_z(\hat{z}\cdot\hat{R})$$

$$A_R = A_x\left(\sin(\theta)\cos(\varphi)\right) + A_y\left(\sin(\theta)\sin(\varphi)\right) + A_z(\cos(\theta))$$

$$A_\theta = \vec{A}\cdot\hat{\theta} = \left(A_x\hat{x} + A_y\hat{y} + A_z\hat{z}\right)\cdot\hat{\theta} = A_x(\hat{x}\cdot\hat{\theta}) + A_y(\hat{y}\cdot\hat{\theta}) + A_z(\hat{z}\cdot\hat{\theta})$$

$$A_\theta = A_x\left(\cos(\theta)\cos(\varphi)\right) + A_y\left(\cos(\theta)\sin(\varphi)\right) + A_z(\cos(90+\theta))$$

$$A_\theta = A_x\left(\cos(\theta)\cos(\varphi)\right) + A_y\left(\cos(\theta)\sin(\varphi)\right) + A_z(-\sin(\theta))$$

$$A_\varphi = \vec{A}\cdot\hat{\varphi} = \left(A_x\hat{x} + A_y\hat{y} + A_z\hat{z}\right)\cdot\hat{\varphi} = A_x(\hat{x}\cdot\hat{\varphi}) + A_y(\hat{y}\cdot\hat{\varphi}) + A_z(\hat{z}\cdot\hat{\varphi})$$

$$A_\varphi = A_x\left(\cos(90+\varphi)\right) + A_y\left(\cos(\varphi)\right) + A_z(\cos(90))$$

$$A_\varphi = A_x\left(-\sin(\varphi)\right) + A_y\left(\cos(\varphi)\right) + A_z(0) = A_\varphi = A_x\left(-\sin(\varphi)\right) + A_y\left(\cos(\varphi)\right)$$

$$\mathbf{\vec{A} = A_r\hat{r} + A_\varphi\hat{\varphi} + A_z\hat{z}}\ \textbf{(Kaynak olsun)}$$

$$\mathbf{\vec{A} = A_R\hat{R} + A_\theta\hat{\theta} + A_\varphi\hat{\varphi}}\ \textbf{(Hedef olsun)}$$

$$A_R = \vec{A}\cdot\hat{R} = \left(A_r\hat{r} + A_\varphi\hat{\varphi} + A_z\hat{z}\right)\cdot\hat{R} = A_r(\hat{r}\cdot\hat{R}) + A_\varphi(\hat{\varphi}\cdot\hat{R}) + A_z(\hat{z}\cdot\hat{R})$$

$$A_R = A_r\left(\cos(90-\theta)\right) + A_\varphi\left(\cos(90)\right) + A_z(\cos(\theta)) = A_r\left(\sin(\theta)\right) + A_z(\cos(\theta))$$

$$A_\theta = \vec{A} \cdot \hat{\theta} = \left(A_r\hat{r} + A_\varphi\hat{\varphi} + A_z\hat{z}\right) \cdot \hat{\theta} = A_r(\hat{r} \cdot \hat{\theta}) + A_\varphi(\hat{\varphi} \cdot \hat{\theta}) + A_z(\hat{z} \cdot \hat{\theta})$$

$$A_\theta = A_r\left(\cos(\theta)\right) + A_\varphi\left(\cos(90)\right) + A_z(\cos(90 + \theta)) = A_r\left(\cos(\theta)\right) + A_z(-\sin(\theta))$$

$$A_\varphi = \vec{A} \cdot \hat{\varphi} = \left(A_r\hat{r} + A_\varphi\hat{\varphi} + A_z\hat{z}\right) \cdot \hat{\varphi} = A_r(\hat{r} \cdot \hat{\varphi}) + A_\varphi(\hat{\varphi} \cdot \hat{\theta}) + A_z(\hat{z} \cdot \hat{\theta})$$

$$A_\varphi = A_r\left(\cos(90)\right) + A_\varphi\left(\cos(0)\right) + A_z(\cos(90))$$

$$A_\varphi = A_r(0) + A_\varphi(1) + A_z(0) = A_\varphi$$

Genel olarak dik koordinat sistemlerinde metrik katsayılar:

$$\vec{R} = (\vec{R} \cdot \hat{u})\hat{u} + (\vec{R} \cdot \hat{v})\hat{v} + (\vec{R} \cdot \hat{w})\hat{w} = \cos(\propto)\hat{u} + \cos(\beta)\hat{v} + \cos(\gamma)\hat{w}$$

$$\frac{\partial \vec{R}}{\partial u} = h_u\hat{u} \Rightarrow \left|\frac{\partial \vec{R}}{\partial u}\right| = |h_u\hat{u}| = |h_u||\hat{u}| = |h_u|.1 = h_u = \left|\frac{\partial \vec{R}}{\partial u}\right|$$

$$\frac{\partial \vec{R}}{\partial v} = h_v\hat{v} \Rightarrow \left|\frac{\partial \vec{R}}{\partial v}\right| = |h_v\hat{v}| = |h_v||\hat{v}| = |h_v|.1 = h_v = \left|\frac{\partial \vec{R}}{\partial v}\right|$$

$$\frac{\partial \vec{R}}{\partial w} = h_w\hat{w} \Rightarrow \left|\frac{\partial \vec{R}}{\partial w}\right| = |h_w\hat{w}| = |h_w||w| = |h_w|.1 = h_w = \left|\frac{\partial \vec{R}}{\partial w}\right|$$

Kartezyen koordinat sisteminde metrik katsayılar:

$$\vec{R} = (\vec{R} \cdot \hat{x})\hat{x} + (\vec{R} \cdot \hat{y})\hat{y} + (\vec{R} \cdot \hat{z})\hat{z}$$

$$\vec{R} = \mathrm{R}\sin(\theta)\cos(\varphi)\hat{x} + \mathrm{R}\sin(\theta)\cos(\varphi)\hat{y} + \mathrm{R}\cos(\theta)\hat{z} = x\hat{x} + y\hat{y} + z\hat{z} = \vec{R}$$

Bu incelemede ve altta diğer koordinat sistemlerinde (silindirik ve küresel koordinat sisteminde) metrik katsayıları bulurken, üç boyutlu uzaya dair pozisyon vektörünü ($\vec{R}$) kartezyen koordinat sisteminin birim vektörleri cinsinden yani $\hat{x}$, $\hat{y}$ ve $\hat{z}$ yazacağız. Bundaki amaç, $\hat{x}$, $\hat{y}$ ve $\hat{z}$'nin sabit vektörler olması ve herhangi bir değişkene göre türevlerinin sıfır vermesidir:

$$\frac{\partial \hat{x}}{\partial x} = \frac{\partial \hat{x}}{\partial y} = \frac{\partial \hat{x}}{\partial z} = \frac{\partial \hat{x}}{\partial r} = \frac{\partial \hat{x}}{\partial \varphi} = \frac{\partial \hat{x}}{\partial R} = \frac{\partial \hat{x}}{\partial \theta} = \mathbf{0}$$

$$\frac{\partial \hat{y}}{\partial x} = \frac{\partial \hat{y}}{\partial y} = \frac{\partial \hat{y}}{\partial z} = \frac{\partial \hat{y}}{\partial r} = \frac{\partial \hat{y}}{\partial \varphi} = \frac{\partial \hat{y}}{\partial R} = \frac{\partial \hat{y}}{\partial \theta} = \mathbf{0}$$

$$\frac{\partial \hat{z}}{\partial x} = \frac{\partial \hat{z}}{\partial y} = \frac{\partial \hat{z}}{\partial z} = \frac{\partial \hat{z}}{\partial r} = \frac{\partial \hat{z}}{\partial \varphi} = \frac{\partial \hat{z}}{\partial R} = \frac{\partial \hat{z}}{\partial \theta} = \mathbf{0}$$

$$\frac{\partial \hat{r}}{\partial x} \neq 0, \ \frac{\partial \hat{r}}{\partial y} \neq 0, \frac{\partial \hat{r}}{\partial z} = \mathbf{0}$$

$$\frac{\partial \hat{\varphi}}{\partial x} \neq 0, \ \frac{\partial \hat{\varphi}}{\partial y} \neq 0, \frac{\partial \hat{\varphi}}{\partial z} = \mathbf{0}$$

$$\frac{\partial \hat{r}}{\partial r} = \mathbf{0}, \ \frac{\partial \hat{r}}{\partial \varphi} \neq 0, \frac{\partial \hat{r}}{\partial z} = \mathbf{0}$$

$$\frac{\partial \hat{\varphi}}{\partial r} = \mathbf{0}, \ \frac{\partial \hat{\varphi}}{\partial \varphi} \neq 0, \frac{\partial \hat{\varphi}}{\partial z} = \mathbf{0}$$

$$\frac{\partial \hat{r}}{\partial R} = \mathbf{0}, \ \frac{\partial \hat{r}}{\partial \theta} = \mathbf{0}, \frac{\partial \hat{r}}{\partial \varphi} \neq 0$$

$$\frac{\partial\boldsymbol{\hat{\varphi}}}{\partial\boldsymbol{R}} = \mathbf{0},\ \frac{\partial\boldsymbol{\hat{\varphi}}}{\partial\boldsymbol{\theta}} = \mathbf{0}, \frac{\partial\hat{\varphi}}{\partial\varphi} \neq 0$$

$$\frac{\partial\hat{R}}{\partial x} \neq 0,\ \frac{\partial\hat{R}}{\partial y} \neq 0, \frac{\partial\hat{R}}{\partial z} \neq 0$$

$$\frac{\partial\hat{\theta}}{\partial x} \neq 0,\ \frac{\partial\hat{\theta}}{\partial y} \neq 0, \frac{\partial\hat{\theta}}{\partial z} \neq 0$$

$$\frac{\partial\hat{\varphi}}{\partial x} \neq 0,\ \frac{\partial\hat{\varphi}}{\partial y} \neq 0, \frac{\partial\boldsymbol{\hat{\varphi}}}{\partial\boldsymbol{z}} = \mathbf{0}$$

$$\frac{\partial\boldsymbol{\hat{R}}}{\partial\boldsymbol{R}} = \mathbf{0},\ \frac{\partial\hat{R}}{\partial\theta} \neq 0, \frac{\partial\hat{R}}{\partial\varphi} \neq 0$$

$$\frac{\partial\boldsymbol{\hat{\theta}}}{\partial\boldsymbol{R}} = \mathbf{0},\ \frac{\partial\hat{\theta}}{\partial\theta} \neq 0, \frac{\partial\hat{\theta}}{\partial\varphi} \neq 0$$

$$\frac{\partial\boldsymbol{\hat{\varphi}}}{\partial\boldsymbol{R}} = \mathbf{0},\ \frac{\partial\boldsymbol{\hat{\varphi}}}{\partial\boldsymbol{\theta}} = \mathbf{0}, \frac{\partial\boldsymbol{\hat{\varphi}}}{\partial\varphi} \neq 0$$

$$\frac{\partial\hat{R}}{\partial r} \neq 0,\ \frac{\partial\hat{R}}{\partial\varphi} \neq 0\ , \frac{\partial\hat{R}}{\partial z} \neq 0$$

$$\frac{\partial\hat{\theta}}{\partial r} \neq 0,\ \frac{\partial\hat{\theta}}{\partial\varphi} \neq 0, \frac{\partial\hat{\theta}}{\partial z} \neq 0$$

$$\frac{\partial \vec{R}}{\partial x} = h_x \hat{x} \;\; =>$$

$$\left|\frac{\partial \vec{R}}{\partial x}\right| = |h_x \hat{x}| = |h_x||\hat{x}| = |h_x|.1 = h_x = \left|\frac{\partial \vec{R}}{\partial x}\right|$$

$$\frac{\partial \vec{R}}{\partial x} = \frac{\partial(x\hat{x} + y\hat{y} + z\hat{z})}{\partial x} = \frac{\partial(x\hat{x})}{\partial x} = 1\hat{x} = h_x \hat{x} =>$$

$$\left|\frac{\partial \vec{R}}{\partial x}\right| = \boldsymbol{h_x = 1}$$

$$\frac{\partial \vec{R}}{\partial y} = h_y \hat{y} \;\; =>$$

$$\left|\frac{\partial \vec{R}}{\partial y}\right| = |h_y \hat{y}| = |h_y||\hat{y}| = |h_y|.1 = h_y = \left|\frac{\partial \vec{R}}{\partial y}\right|$$

$$\frac{\partial \vec{R}}{\partial y} = \frac{\partial(x\hat{x} + y\hat{y} + z\hat{z})}{\partial y} = \frac{\partial(y\hat{y})}{\partial y} = 1\hat{y} = h_y \hat{y} =>$$

$$\left|\frac{\partial \vec{R}}{\partial y}\right| = \boldsymbol{h_y = 1}$$

$$\frac{\partial \vec{R}}{\partial z} = h_z \hat{z} \;\; =>$$

$$\left|\frac{\partial \vec{R}}{\partial z}\right| = |h_z \hat{z}| = |h_z||\hat{z}| = |h_z|.1 = h_z = \left|\frac{\partial \vec{R}}{\partial z}\right|$$

$$\frac{\partial \vec{R}}{\partial z} = \frac{\partial (x\hat{x} + y\hat{y} + z\hat{z})}{\partial z} = \frac{\partial (z\hat{z})}{\partial z} = 1\hat{z} = h_z\hat{z} =>$$

$$\left|\frac{\partial \vec{R}}{\partial z}\right| = \boldsymbol{h_z = 1}$$

Silindirik koordinat sisteminde metrik katsayılar:

$$\vec{R} = (\vec{R}\cdot\hat{r})\hat{r} + (\vec{R}\cdot\hat{\varphi})\hat{\varphi} + (\vec{R}\cdot\hat{z})\hat{z}$$

$$\begin{aligned}
\hat{r} &= (\hat{r}\cdot\hat{x})\hat{x} + (\hat{r}\cdot\hat{y})\hat{y} + (\hat{r}\cdot\hat{z})\hat{z} \\
&= cos(\varphi)\hat{x} + sin(\varphi)\hat{y} + \cos(90)\hat{z} \\
&= cos(\varphi)\hat{x} + sin(\varphi)\hat{y}
\end{aligned}$$

$$\boldsymbol{\vec{r} = r\hat{r} = rcos(\varphi)\hat{x} + rsin(\varphi)\hat{y}} \qquad \textbf{(i)}$$

$$\begin{aligned}
\hat{\varphi} &= (\hat{\varphi}\cdot\hat{x})\hat{x} + (\hat{\varphi}\cdot\hat{y})\hat{y} + (\hat{\varphi}\cdot\hat{z})\hat{z} \\
&= cos(90+\varphi)\hat{x} + sin(\varphi)\hat{y} + \cos(90)\,\hat{z} \\
&= -sin(\varphi)\hat{x} + cos(\varphi)\hat{y}
\end{aligned}$$

$$\boldsymbol{\hat{\varphi} = -sin(\varphi)\hat{x} + cos(\varphi)\hat{y}} \qquad \textbf{(ii)}$$

$$\begin{aligned}
\vec{R} &= \mathrm{Rcos}(90-\theta)\hat{r} + \mathrm{Rcos}(90)\hat{\varphi} + \mathrm{Rcos}(\theta)\hat{z} \\
&= \mathrm{Rsin}(\theta)\hat{r} + 0\hat{\varphi} + \mathrm{Rcos}(\theta)\hat{z} \\
&= r\hat{r} + 0\hat{\varphi} + z\hat{z} \\
&= r\boldsymbol{\hat{r}} + z\hat{z}
\end{aligned}$$

$= \quad \vec{\boldsymbol{r}} + z\hat{z}$

$= \quad \boldsymbol{r}(\boldsymbol{cos}(\boldsymbol{\varphi})\hat{\boldsymbol{x}} + \boldsymbol{sin}(\boldsymbol{\varphi})\hat{\boldsymbol{y}}) + z\hat{z}$ **(iii)**

$\vec{\boldsymbol{R}} \quad = \quad \boldsymbol{rcos}(\boldsymbol{\varphi})\hat{\boldsymbol{x}} + \boldsymbol{rsin}(\boldsymbol{\varphi})\hat{\boldsymbol{y}} + \boldsymbol{z}\hat{\boldsymbol{z}}$ **(iv)**

$$\frac{\partial \vec{R}}{\partial r} = h_r \hat{r} \ =>$$

$$\left|\frac{\partial \vec{R}}{\partial r}\right| = |h_r \hat{r}| = |h_r||\hat{r}| = |h_r|.1 = h_r = \left|\frac{\partial \vec{R}}{\partial r}\right|$$

$$\frac{\partial \vec{R}}{\partial r} = \frac{\partial (rcos(\varphi)\hat{x} + rsin(\varphi)\hat{y} + z\hat{z})}{\partial r}$$

$$= cos(\varphi)\hat{x} + sin(\varphi)\hat{y} = h_r \hat{r} =>$$

$$\left|\frac{\partial \vec{R}}{\partial r}\right| = \ \boldsymbol{h_r} =>$$

$$\left|\frac{\partial \vec{R}}{\partial r}\right| = |cos(\varphi)\hat{x} + sin(\varphi)\hat{y}|$$

$$= \sqrt{(cos(\varphi)\hat{x} + sin(\varphi)\hat{y}) \cdot (cos(\varphi)\hat{x} + sin(\varphi)\hat{y})}$$

$$= \sqrt{(cos(\varphi)\hat{x} \cdot cos(\varphi)\hat{x} + sin(\varphi)\hat{y} \cdot sin(\varphi)\hat{y})}$$

$$= \ \sqrt{cos^2(\varphi) + sin^2(\varphi)}$$

$$= \ \boldsymbol{h_r} = \boldsymbol{1}$$

$$\frac{\partial \vec{R}}{\partial \varphi} = h_\varphi \hat{\varphi}$$

$$=> \left|\frac{\partial \vec{R}}{\partial \varphi}\right| = \left|h_\varphi \hat{\varphi}\right| = \left|h_\varphi\right| |\hat{\varphi}| = \left|h_\varphi\right|.1 = h_\varphi = \left|\frac{\partial \vec{R}}{\partial \varphi}\right|$$

$$\frac{\partial \vec{R}}{\partial \varphi} = \frac{\partial (rcos(\varphi)\hat{x} + rsin(\varphi)\hat{y} + z\hat{z})}{\partial \varphi}$$

$$= r(-sin(\varphi)\hat{x} + rcos(\varphi)\hat{y})$$

$$= h_\varphi \hat{\varphi} =>$$

$$\left|\frac{\partial \vec{R}}{\partial \varphi}\right| = \boldsymbol{h_\varphi}$$

$$\left|\frac{\partial \vec{R}}{\partial \varphi}\right| = |r(-sin(\varphi)\hat{x} + cos(\varphi)\hat{y})|$$

$$= \sqrt{r(-sin(\varphi)\hat{x} + cos(\varphi)\hat{y}) \cdot \mathrm{r}(-sin(\varphi)\hat{x} + cos(\varphi)\hat{y})}$$

$$\left|\frac{\partial \vec{R}}{\partial \varphi}\right| = \sqrt{r(-sin(\varphi))\hat{x} \cdot r(-sin(\varphi))\hat{x} + rcos(\varphi)\hat{y} \cdot r\cos(\varphi)\hat{y}}$$

$$= \sqrt{r^2(cos^2(\varphi) + sin^2(\varphi))}$$

$$\left|\frac{\partial\vec{R}}{\partial\varphi}\right| = \sqrt{r^2.1} = \boldsymbol{h_\varphi} = \boldsymbol{r}$$

$$\frac{\partial\vec{R}}{\partial z} = h_z\hat{z} \;\; =>$$

$$\left|\frac{\partial\vec{R}}{\partial z}\right| = |h_z\hat{z}| = |h_z||\hat{z}| = |h_z|.1 = h_z = \left|\frac{\partial\vec{R}}{\partial z}\right|$$

$$\frac{\partial\vec{R}}{\partial z} = \frac{\partial(rcos(\varphi)\hat{x} + rsin(\varphi)\hat{y} + z\hat{z})}{\partial z}$$

$$= \frac{\partial(z\hat{z})}{\partial z} = 1\hat{z} = h_z\hat{z} =>$$

$$\left|\frac{\partial\vec{R}}{\partial z}\right| = |1\hat{z}| = 1 = |h_z\hat{z}| = |h_z||\hat{z}| = h_z.1 =>$$

$$\boldsymbol{h_z = 1}$$

Küresel koordinat sisteminde metrik katsayılar:

$$\vec{R} = (\vec{R}\cdot\hat{R}\,)\hat{R} + (\vec{R}\cdot\hat{\theta}\,)\hat{\theta} + (\vec{R}\cdot\hat{\varphi})\hat{\varphi}$$

$$\hat{R} = (\hat{R}\cdot\hat{x})\hat{x} + (\hat{R}\cdot\hat{y})\hat{y} + (\hat{R}\cdot\hat{z})\hat{z}$$

$$= sin(\theta)\cos(\varphi)\hat{x} + sin(\theta)\sin(\varphi)\hat{y} + cos(\theta)\hat{z}$$

$$\vec{R} = R\hat{R} =>$$

$$\vec{R} = Rsin(\theta)\cos(\varphi)\hat{x} + Rsin(\theta)\sin(\varphi)\hat{y} + Rcos(\theta)\hat{z} \qquad \textbf{(v)}$$

$$\frac{\partial\vec{R}}{\partial R} = h_R\hat{R} \;=>$$

$$\left|\frac{\partial\vec{R}}{\partial R}\right| = |h_R\hat{R}| = |h_R||\hat{R}| = |h_R|.1 = \boldsymbol{h_R} = \left|\frac{\boldsymbol{\partial\vec{R}}}{\boldsymbol{\partial R}}\right|$$

$$\frac{\partial\vec{R}}{\partial R} = \frac{\partial(Rsin(\theta)\cos(\varphi)\,\hat{x} + Rsin(\theta)\sin(\varphi)\,\hat{y} + Rcos(\theta)\hat{z})}{\partial R}$$

$$= sin(\theta)\,cos(\varphi)\,\hat{x} + sin(\theta)\,sin(\varphi)\,\hat{y} + cos(\theta)\hat{z} = h_R\hat{R} =>$$

$$\left|\frac{\partial\vec{R}}{\partial R}\right| = \boldsymbol{h_R}$$

$$= \sqrt{sin(\theta)\cos(\varphi)\,\hat{x}\cdot sin(\theta)\cos(\varphi)\hat{x} + sin(\theta)\sin(\varphi)\hat{y}\cdot sin(\theta)\sin(\varphi)\hat{y} + cos(\theta)\hat{z}\cdot cos(\theta)\hat{z}}$$

$$= \sqrt{sin^2(\theta)cos^2(\varphi) + sin^2(\theta)sin^2(\varphi) + cos^2(\theta)}$$

$$= \sqrt{sin^2(\theta)(cos^2(\varphi) + sin^2(\varphi)) + cos^2(\theta)}$$

$$= \sqrt{sin^2(\theta) + cos^2(\theta)} = \mathbf{1} = \boldsymbol{h_R}$$

$$\frac{\partial \vec{R}}{\partial \theta} = h_\theta \hat{\theta} \ =>$$

$$\left|\frac{\partial \vec{R}}{\partial \theta}\right| = \left|h_\theta \hat{\theta}\right| = |h_\theta|\left|\hat{\theta}\right| = |h_\theta|.1 = \boldsymbol{h_\theta} = \left|\boldsymbol{\frac{\partial \vec{R}}{\partial \theta}}\right|$$

$$\frac{\partial \vec{R}}{\partial \theta} = \frac{\partial(Rsin(\theta)\cos(\varphi)\,\hat{x} + Rsin(\theta)\sin(\varphi)\,\hat{y} + Rcos(\theta)\hat{z})}{\partial \theta}$$

$$\frac{\partial \vec{R}}{\partial \theta} = Rcos(\theta)\,cos(\varphi)\,\hat{x} + Rcos(\theta)\,sin(\varphi)\,\hat{y} + (-sin(\theta)\hat{z}) = h_\theta \hat{\theta} =>$$

$$\left|\frac{\partial \vec{R}}{\partial \theta}\right| = \ \boldsymbol{h_\theta}$$

$$= \sqrt{\begin{matrix} Rcos(\theta)\,cos(\varphi)\,\hat{x} \cdot Rcos(\theta)\,cos(\varphi)\hat{x} \\ +Rcos(\theta)\,sin(\varphi)\hat{y} \cdot Rcos(\theta)\,sin(\varphi)\hat{y} \\ +R(-sin(\theta))\hat{z} \cdot \widehat{R(-sin(\theta))z} \end{matrix}}$$

$$= \sqrt{R^2cos^2(\theta)cos^2(\varphi) + R^2cos^2(\theta)sin^2(\varphi) + R^2cos^2(\theta)}$$

$$= \sqrt{R^2cos^2(\theta)(cos^2(\varphi) + sin^2(\varphi)) + R^2sin^2(\theta)}$$

$$= \sqrt{R^2cos^2(\theta) + R^2sin^2(\theta)} = \boldsymbol{R} = \boldsymbol{h_\theta}$$

$$\frac{\partial \vec{R}}{\partial \varphi} = h_\varphi \hat{\varphi} \;\; =>$$

$$\left|\frac{\partial \vec{R}}{\partial \varphi}\right| = \left|h_\varphi \hat{\varphi}\right| = \left|h_\varphi\right| |\hat{\varphi}| = \left|h_\varphi\right|.1 = h_\varphi = \left|\frac{\partial \vec{R}}{\partial \varphi}\right|$$

$$\frac{\partial \vec{R}}{\partial \varphi} = \frac{\partial (Rsin(\theta)\cos(\varphi)\,\hat{x} + Rsin(\theta)\sin(\varphi)\,\hat{y} + Rcos(\theta)\hat{z})}{\partial \varphi}$$

$$= Rsin(\theta)\,(-sin(\varphi))\,\hat{x} + Rsin(\theta)\,cos(\varphi)\,\hat{y} + 0\hat{z}$$

$$= h_\varphi \hat{\varphi} => \left|\frac{\partial \vec{R}}{\partial \varphi}\right| = \boldsymbol{h_\varphi}$$

$$\left|\frac{\partial \vec{R}}{\partial \varphi}\right| = |Rsin(\theta)\,(-sin(\varphi))\,\hat{x} + Rsin(\theta)\,cos(\varphi)\,\hat{y}|$$

$$= \sqrt{[Rsin(\theta)\,(-sin(\varphi))\,\hat{x} + Rsin(\theta)\,cos(\varphi)\,\hat{y}] \cdot [Rsin(\theta)\,(-sin(\varphi))\,\hat{x} + Rsin(\theta)\,cos(\varphi)\,\hat{y}]}$$

$$\left|\frac{\partial \vec{R}}{\partial \varphi}\right| = \sqrt{Rsin(\theta)(-sin(\varphi))\hat{x} \cdot \mathrm{R}sin(\theta)(-sin(\varphi))\hat{x} + Rsin(\theta)cos(\varphi)\hat{y} \cdot Rsin(\theta)\cos(\varphi)\hat{y}}$$

$$= \sqrt{R^2 sin^2(\theta)\left(cos^2(\varphi) + sin^2(\varphi)\right)} = \left|\frac{\partial \vec{R}}{\partial \varphi}\right|$$

$$= \sqrt{R^2 sin^2(\theta).1} = \boldsymbol{h_\varphi} = \boldsymbol{Rsin(\theta)}$$

Kartezyen Koordinat Sistemi

Eksen	Metrik katsayı	İnfinitesimal uzunluk
x̂	$h_x = 1$	$dl_x = h_x dx = 1.dx$
ŷ	$h_y = 1$	$dl_y = h_y dy = 1.dy$
ẑ	$h_z = 1$	$dl_z = h_z dz = 1.dz$

İnifinitesimal yer değiştirme vektörü

$$\overrightarrow{dl} = \overrightarrow{dl}_x + \overrightarrow{dl}_y + \overrightarrow{dl}_z$$

$$\overrightarrow{dl} = (h_x dx)\hat{x} + (h_y dy)\hat{y} + (h_z dz)\hat{z}$$

$$\overrightarrow{dl} = (1.dx)\hat{x} + (1.dy)\hat{y} + (1.dz)\hat{z}$$

Kartezyen Koordinat Sistemi

İnifinitesimal alan büyüklüğü

$$ds_x = dl_y.dl_z = (h_y dy)(h_z dz)$$

$$ds_y = dl_x.dl_z = (h_x dx)(h_z dz)$$

$$ds_z = dl_x.dl_y = (h_x dx)(h_y dy)$$

İnifinitesimal alan vektörü

$$\overrightarrow{ds_x} = ds_x\hat{x} = dl_y.dl_z\,\hat{x} = (h_y dy)(h_z dz)\,\hat{x} = (1.dy)(1.dz)\,\hat{x}$$

$$\overrightarrow{ds_y} = ds_y\hat{y} = dl_x.dl_z\,\hat{y} = (h_x dx)(h_z dz)\,\hat{y} = (1.dx)(1.dz)\,\hat{y}$$

$$\overrightarrow{ds_z} = ds_z\hat{z} = dl_x.dl_y\hat{z} = (h_x dx)(h_y dy)\,\hat{z} = (1.dx)(1.dz)\,\hat{z}$$

İnifinitesimal hacim büyüklüğü

$$dv = dl_x dl_y dl_z = (h_x dx)(h_y dy)(h_z dz)$$

$$dv = dl_x dl_y dl_z = (1.dx)(1.dy)(1.dz) = (dx.dy.dz)$$

Silindirik Koordinat Sistemi