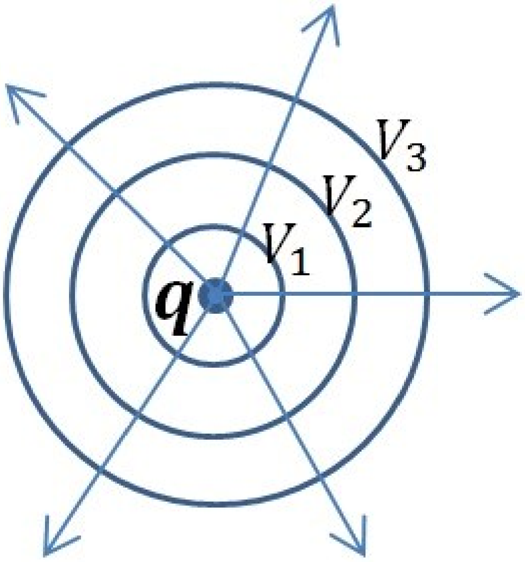
EQUIPOTENTIAL SURFACES



$$\vec{E} = -\nabla V$$

Electric field lines are perpendicular to the equipotential lines

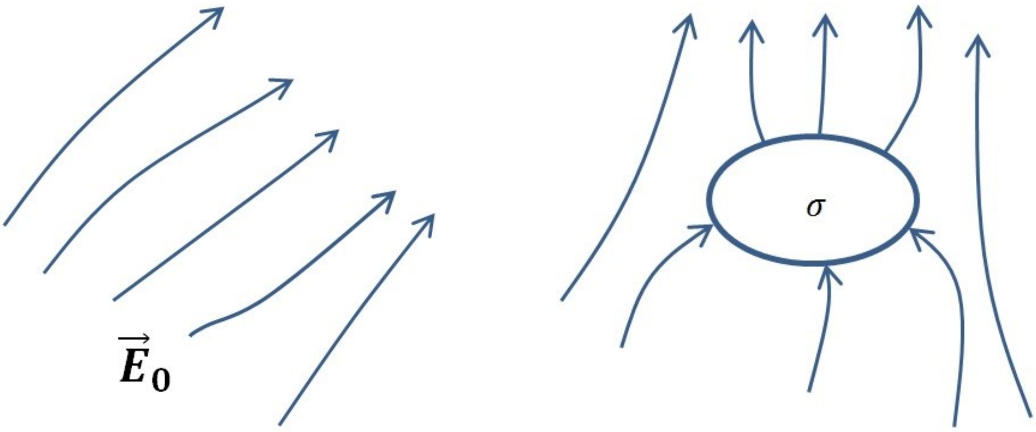
Electric field lines are opposite to the direction where maximum variation in the scalar potential occurs

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{\hat{R}}{R^2}$$

MATERIAL MEDIA IN STATIC ELECTRIC FIELDS

Fact 1

- **Static Electric field can not exist inside a conductor**



$\vec{E}_0$: Electric field before the conductor was inserted

$\vec{E}_S$: Secondary electric field created inside the conductor as a reaction to $\vec{E}_0$

$\vec{E}$: Resultant electric field after the TRANSIENT period following the insertion of the conductor, $\vec{E} = \vec{E}_0 + \vec{E}_S$

At the end of the transient period $\vec{E}_S = -\vec{E}_0$ inside the conductor

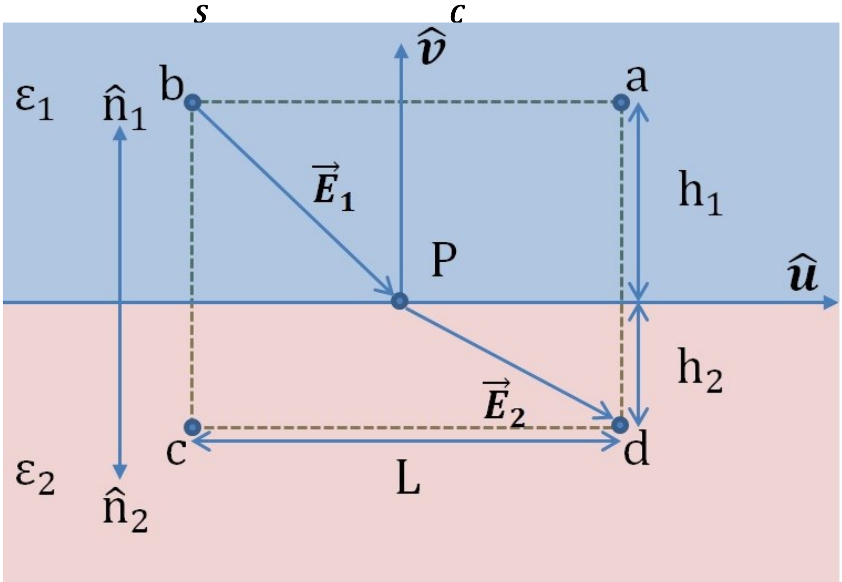
At steady state, inside the conductor, $\vec{E} = \vec{E}_0 + \vec{E}_S = \vec{E}_0 + (-\vec{E}_0) = 0$

Fact 2

- $\vec{E}_0$ is always perpendicular to the surface of a conductor

$$\boldsymbol{\nabla} \times \vec{\boldsymbol{E}} = \vec{0}$$

$$\int_{S} \boldsymbol{\nabla} \times \vec{\boldsymbol{E}} \bullet \overrightarrow{\boldsymbol{dS}} = \oint_{C} \vec{\boldsymbol{E}} \bullet \overrightarrow{\boldsymbol{dl}} = \mathbf{0}$$



$$\oint_{C} \vec{\boldsymbol{E}} \bullet \overrightarrow{\boldsymbol{dl}} = \sum_{i=1}^{4} \int_{C_i} \vec{\boldsymbol{E}} \bullet \overrightarrow{\boldsymbol{dl}}$$

$$\oint_{C} \vec{\boldsymbol{E}} \bullet \overrightarrow{\boldsymbol{dl}} = \int_{ab} \vec{\boldsymbol{E}} \bullet \overrightarrow{\boldsymbol{dl}} + \int_{bc} \vec{\boldsymbol{E}} \bullet \overrightarrow{\boldsymbol{dl}} + \int_{cd} \vec{\boldsymbol{E}} \bullet \overrightarrow{\boldsymbol{dl}} + \int_{da} \vec{\boldsymbol{E}} \bullet \overrightarrow{\boldsymbol{dl}} +$$

$$\int_{bc} \vec{\boldsymbol{E}} \bullet \overrightarrow{\boldsymbol{dl}} = \int_{da} \vec{\boldsymbol{E}} \bullet \overrightarrow{\boldsymbol{dl}} = \mathbf{0}$$

$\vec{\boldsymbol{E}}_2$**=0 (due to fact 1)**

$$\vec{\boldsymbol{E}}_1 = \boldsymbol{E}_1 \widehat{\boldsymbol{v}} = \boldsymbol{E}_1 \widehat{\boldsymbol{n}}_2$$

$\vec{\boldsymbol{E}}_1$ is perpendicular to boundary surface ($\vec{\boldsymbol{E}}_1 = \boldsymbol{E}_{1\perp}(-\widehat{\boldsymbol{v}}) + \mathbf{0}(\widehat{\boldsymbol{u}})$)

$\vec{\boldsymbol{E}}_1$has no tangential component ($\boldsymbol{E}_{1//} = \mathbf{0}$)

Fact 3

- Conductor surface is an equipotential surface and conductor body is an equipotential volume

Proof

As a result of Fact 2, $\boldsymbol{E_{1\perp} \neq 0}$ & $\boldsymbol{E_{1//} = 0}$

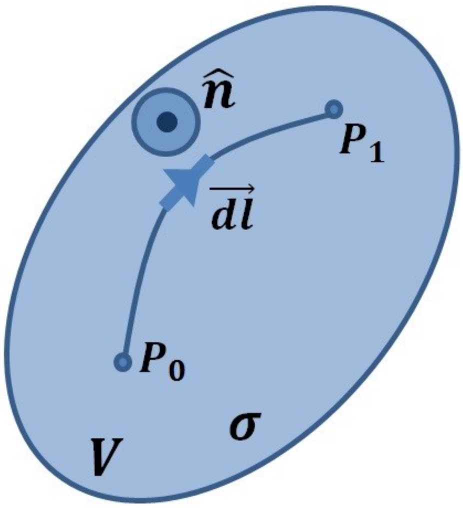
$$\& \ \vec{E} = -\nabla V \Rightarrow \vec{E} \perp S \qquad \Leftrightarrow$$

$\Rightarrow$ $\boldsymbol{S}$ is an equipotential surface

$\vec{E}$ is perpendicular to boundary surface

$\vec{dl}$ is // to the surface

$$\vec{E} \perp \vec{dl} \Rightarrow \vec{E} \cdot \vec{dl} = 0$$



$$\Delta V = V(b) - V(a)$$

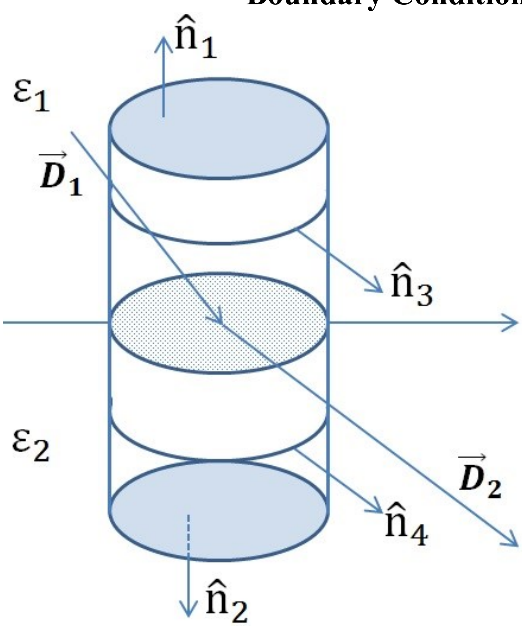
$$\Delta V = -\int_a^b \vec{E} \cdot \vec{dl}$$

$$\Delta V = 0$$

Fact 4:

$$E_\perp = \frac{\rho_s}{\varepsilon} \text{ on } E_\perp = S$$

Boundary Condition for a conductor surface



$$\oint_S \vec{E} \cdot \vec{dS} = \int_V \frac{\rho_v}{\varepsilon_0} dV$$

$$\oint_S \vec{E} \cdot \vec{dS} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$\oint_S \vec{E}\bullet\overrightarrow{dS} = \int_{S_1} \vec{E}_1\bullet\overrightarrow{dS} + \int_{S_2} \vec{E}_2\bullet\overrightarrow{dS} + \int_{S_3} \vec{E}_3\bullet\overrightarrow{dS} + \int_{S_4} \vec{E}_4\bullet\overrightarrow{dS} +$$

$\vec{E}_2$=$\vec{E}_4$=0 (due to fact 1)

$$\oint_S \vec{E}\bullet\overrightarrow{dS} = \int_{S_1} \vec{E}_1\bullet\overrightarrow{dS} + \int_{S_3} \vec{E}_3\bullet\overrightarrow{dS}$$

$$S_3 \to 0 => \int_{S_3} \vec{E}_3\bullet\overrightarrow{dS} \to 0$$

$$=> \oint_S \vec{E}\bullet\overrightarrow{dS} = \lim_{h\to 0} E_{1\perp}S_1 = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$E_{1\perp}S_1 = \frac{S_1 . \rho_s}{\varepsilon_0}$$

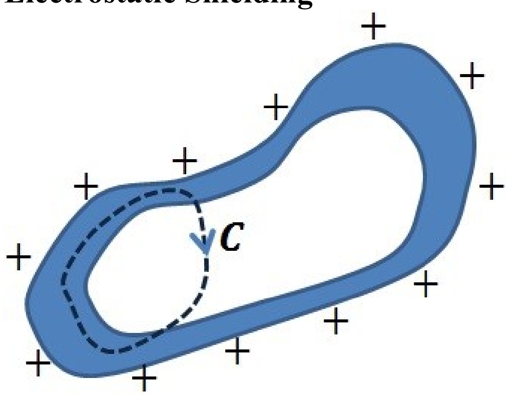
$$E_\perp = \frac{\rho_s}{\varepsilon} \text{ on } E_\perp = S$$

Fact 5:
No charge can remain within the conductor. The charges will redistribute themselves to the outer surface of the conductorwithin the transition (relaxation) time.

$$t_{relaxation,Copper} \cong 10^{-19}\, sec.$$

$$t_{relaxation,Quartz} \cong 10^{-6}\, sec.$$

Electrostatic Shielding



When external $\vec{E}$ is applied no charge can be induced in the inner conductor surface, S_{in}

We have $\vec{E}$ in the cavity.

$$\oint_C \vec{E} \cdot \vec{dl} = \int_{C_\sigma} \vec{E}_\sigma \cdot \vec{dl} + \int_{C_{cavity}} \vec{E}_{cavity} \cdot \vec{dl}$$

$$0 = 0 + \int_{C_{cavity}} \vec{E}_{cavity} \cdot \vec{dl}$$

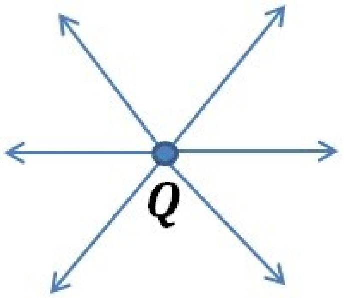
$$=> \int_{C_{cavity}} \vec{E}_{cavity} \cdot \vec{dl} = 0$$

$$=> \vec{E}_{cavity} = 0$$

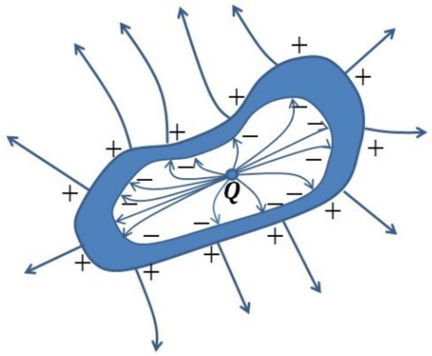
A conducting shell protects its interior from the external fields.

However, if there are charges (total of Q) inside the cavity, the exterior region to the shell will be effected.

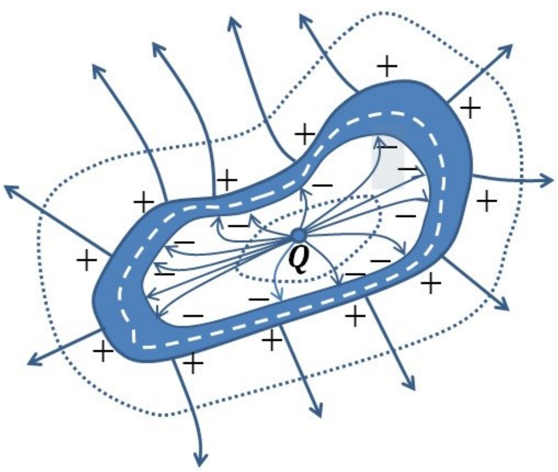
When there is no conductor



With existence of the conductor



Using Gauss' Law



For S_{cavity},

$$\oint_S \vec{E} \cdot \vec{dS} = \frac{Q}{\varepsilon}$$

For S_σ, $\vec{E} = \vec{E}_0 + \vec{E}_s = 0$

$$\oint_S \vec{E} \cdot \vec{dS} = \frac{Q + (-Q)}{\varepsilon} = 0$$

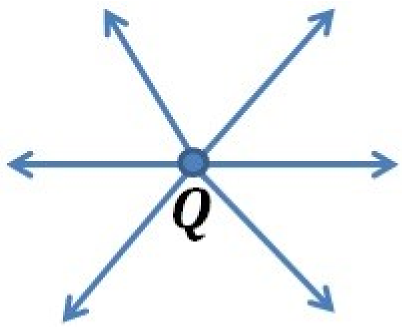
For S_{out}, $\color{red}{\vec{E} = \vec{E}_0}$

$$\oint_S \vec{E} \cdot \vec{dS} = \frac{Q + Q + (-Q)}{\varepsilon}$$

$$= Q$$

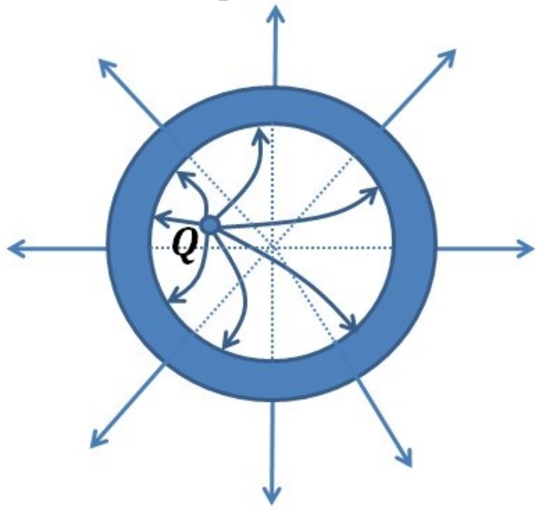
It can be proven that the distribution of Q_{out} is not effected by the distribution of Q

Without conductor



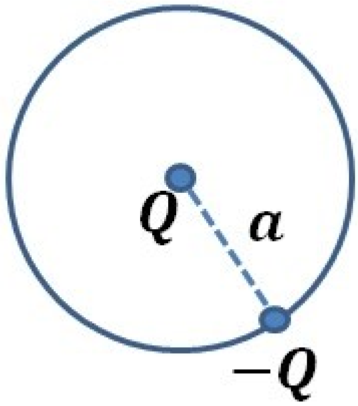
$$\vec{E} = \frac{Q}{4\pi\varepsilon R^2}, \qquad everywhere$$

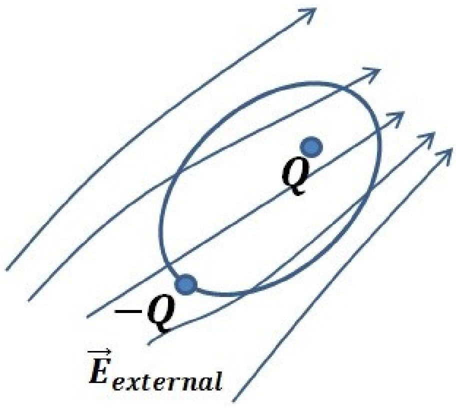
With conductor sphere, off-center



$$\vec{E} = \begin{cases} \neq 0, & 0 < r < a \\ 0, & a < r < b \\ \dfrac{Q}{4\pi\varepsilon R^2}, & r > b \end{cases}$$

Behaviour of dielectrics in Static Electric Field

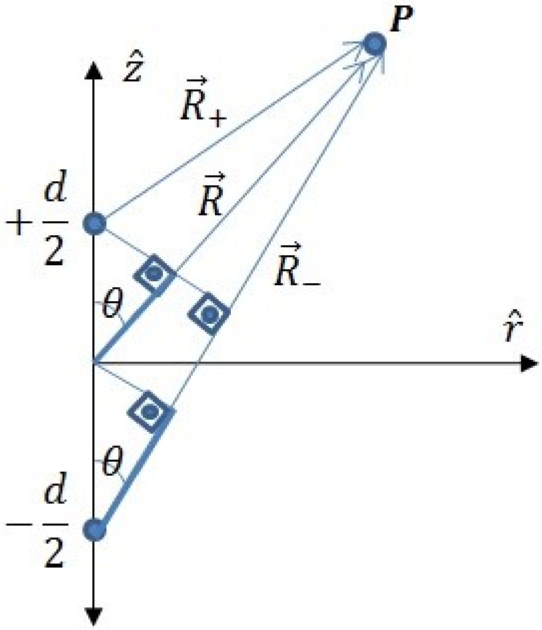




=>

A dipole is created by $\vec{E}_{external}$

Ex: Electric field due to a dipole:



$$\vec{R}_+ = \vec{R} - \frac{d}{2}\hat{z}$$

$$\vec{R}_+ = R\hat{R} - \frac{d}{2}\hat{z}$$

$$R_+ = R - \frac{d}{2}cos(\theta)$$

$$\vec{R}_- = \vec{R} + \frac{d}{2}\hat{z}$$

$$R_- = R + \frac{d}{2}cos(\theta)$$

$$\vec{R}_- = R\hat{R} + \frac{d}{2}\hat{z}$$

$$\vec{E}(P) = \frac{q}{4\pi\varepsilon_0}\left(\frac{R\hat{R} - \frac{d}{2}\hat{z}}{\left(R - \frac{d}{2}cos(\theta)\right)^3} - \frac{R\hat{R} + \frac{d}{2}\hat{z}}{\left(R + \frac{d}{2}cos(\theta)\right)^3}\right)$$

$$\vec{E}(P) =$$

$$\frac{q}{4\pi\varepsilon_0}R\hat{R}\left(\frac{R^3 + 3R^2\frac{d}{2}cos\theta + 3R\left(\frac{d}{2}\right)^2.cos^2\theta + \left(\frac{d}{2}\right)^3.cos^3\theta}{R^6}\right)$$

$$-\frac{q}{4\pi\varepsilon_0}R\hat{R}\left(\frac{R^3 - 3R^2\frac{d}{2}cos\theta + 3R\left(\frac{d}{2}\right)^2.cos^2\theta - \left(\frac{d}{2}\right)^3.cos^3\theta}{R^6}\right)$$

$$-\frac{q}{4\pi\varepsilon_0}\frac{d}{2}\hat{z}\left(\frac{R^3 - 3R^2\frac{d}{2}cos\theta + 3R\left(\frac{d}{2}\right)^2.cos^2\theta - \left(\frac{d}{2}\right)^3.cos^3\theta}{R^6}\right)$$

$$-\frac{q}{4\pi\varepsilon_0}\frac{d}{2}\hat{z}\left(\frac{R^3 - 3R^2\frac{d}{2}cos\theta + 3R\left(\frac{d}{2}\right)^2.cos^2\theta - \left(\frac{d}{2}\right)^3.cos^3\theta}{R^6}\right)$$

$$=\frac{q}{4\pi\varepsilon_0}\left[\frac{2R\hat{R}\left(3R^2\frac{d}{2}cos\theta+\left(\frac{d}{2}\right)^3.cos^3\theta\right)-2\frac{d}{2}\hat{z}\left(R^3+R\left(\frac{d}{2}\right)^2.cos^2\theta\right)}{R^6}\right]$$

$$=\frac{q}{4\pi\varepsilon_0}\left[\frac{\hat{R}\left(d.2.cos\theta.R^3+d.2.cos\theta.R^3\right)-d.\hat{z}\left(R^3\right)}{R^6}\right]$$

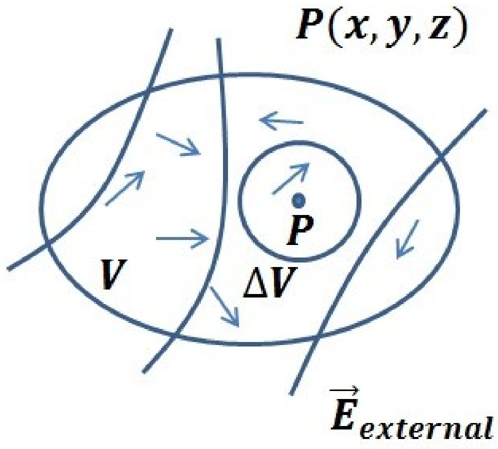
$$=\frac{q}{4\pi\varepsilon_0}\left[\frac{\left(d.2.cos\theta.R^3\right)}{R^6}\hat{R}+\frac{\left(d.cos\theta.R^3\right)}{R^6}\hat{R}-\frac{\left(d.R^3\right)}{R^6}\hat{z}\right]$$

$$=\frac{q}{4\pi\varepsilon_0}\left[\frac{\left(d.2.cos\theta.R^3\right)}{R^6}\hat{R}+\frac{\left(d.cos\theta.R^3\right)}{R^6}\hat{R}-\frac{\left(d.R^3\right)}{R^6}\hat{z}\right]$$

$$=\frac{q}{4\pi\varepsilon_0}\left[\frac{(d.2.cos\theta)}{R^3}\hat{R}+\frac{d}{R^3}[\hat{R}cos\theta-\hat{z}]\right]$$

$$=\frac{qd}{4\pi\varepsilon_0 R^3}[2.cos\theta\hat{R}+sin\theta\hat{\theta}]$$

Polarization Vector $\vec{P}$



Polarized dielectric

$$\vec{P}=\lim_{\Delta V\to 0}\frac{\sum\vec{p}_i}{\Delta V}$$

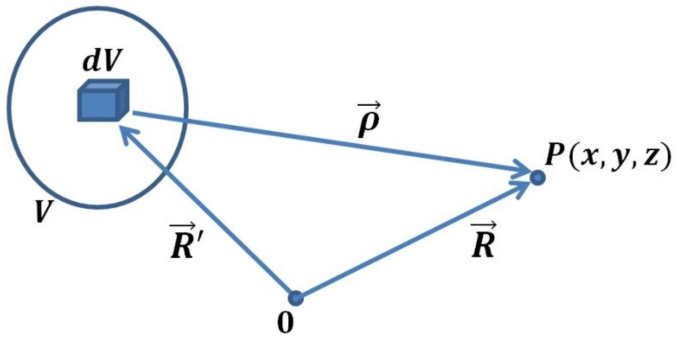
where

P or *P(x,y,z)* is the observation point

$\vec{p}_i$ is the elemental dipole moment (Coul.m)

$\vec{P}$ is the polarization vector is the volme average of the vector sum of the elemental dipole moments and its unit is (Coul.m/m^3= Coul/m^2)

Microsopic effect of millions of electric dipoles contained in a differential volume dV of a polarized dielectric can be represented by a single macroscopic electric dipole moment vector, $\vec{P}dV$.



The infinitesimal electrostatic potential at observation point P, created by the electric dipole moment $\vec{P}dV$

$$d\Phi = \frac{1}{4\pi\varepsilon_0}\frac{\vec{P}dV \bullet \hat{\rho}}{\rho^2}$$

The total electrostatic potential

$$\Phi = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\vec{P}dV \bullet \hat{\rho}}{\rho^2} = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\vec{P} \bullet \hat{\rho}}{\rho^2}dV$$

$$\vec{R} : Vector\ from\ origin\ to\ the\ observation\ point\ P$$

$$\vec{R}' : Vector\ from\ origin\ to\ the\ source\ point$$

$$\vec{\rho} = \vec{R} - \vec{R}'$$

$$\hat{\rho} = \frac{\vec{\rho}}{\rho} = \frac{\vec{R} - \vec{R}'}{|\vec{R} - \vec{R}'|}$$

$$\frac{\vec{R} - \vec{R}'}{|\vec{R} - \vec{R}'|^3} = \frac{\vec{R} - \vec{R}'}{|\vec{R} - \vec{R}'|^1}\frac{1}{|\vec{R} - \vec{R}'|^2} = \frac{\hat{\rho}}{1}\frac{1}{\rho^2} = \frac{\hat{\rho}}{\rho^2}$$

$$\Phi(\vec{R}) = \frac{1}{4\pi\varepsilon_0}\int_{V'} \vec{P} \bullet \frac{\hat{\rho}}{\rho^2}dV = \frac{1}{4\pi\varepsilon_0}\int_{V'} \vec{P}(\vec{R}') \bullet \frac{\vec{R} - \vec{R}'}{|\vec{R} - \vec{R}'|^3}dV$$

Polarization Charges:

An equivalent charge distribuiton can be assumed to model the polarization of the dielectric:

$$R = \sqrt{\vec{R} \bullet \vec{R}} = \sqrt{(x\hat{x} + y\hat{y} + z\hat{z}) \bullet (x\hat{x} + y\hat{y} + z\hat{z})}$$

$$R = \sqrt{x^2 + y^2 + z^2}$$

$$\nabla\left(\frac{1}{R}\right) = \nabla\left(\frac{1}{\sqrt{x^2 + y^2 + z^2}}\right) = -\frac{x\hat{x} + y\hat{y} + z\hat{z}}{(x^2 + y^2 + z^2)^{3/2}}$$

$$\nabla\left(\frac{1}{R}\right) = -\frac{\vec{R}}{R^3} = -\frac{R\hat{R}}{R^3} = -\frac{\hat{R}}{R^2}$$

$$\vec{R} - \vec{R}' = (x\hat{x} + y\hat{y} + z\hat{z}) - (x'\hat{x} + y'\hat{y} + z'\hat{z})$$

$$\vec{R} - \vec{R}' = (x - x')\hat{x} + (y - y')\hat{y} + (z - z')\hat{z}$$

$$|\vec{R} - \vec{R}'| = \sqrt{(\vec{R} - \vec{R}') \bullet (\vec{R} - \vec{R}')}$$

$$|\vec{R} - \vec{R}'| = \sqrt{(x - x')\hat{x} + (y - y')\hat{y} + (z - z')\hat{z} \bullet (x - x')\hat{x} + (y - y')\hat{y} + (z - z')\hat{z}}$$

$$\left|\vec{R}-\vec{R}'\right| = \sqrt{(x-x')^2+(y-y')^2+(z-z')^2}$$

$$\nabla\left(\frac{1}{\left|\vec{R}-\vec{R}'\right|}\right) = \left(\frac{\partial}{\partial x}\hat{x}+\frac{\partial}{\partial y}\hat{y}+\frac{\partial}{\partial z}\hat{z}\right)\left(\frac{1}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}\right)$$

$$\nabla\left(\frac{1}{\left|\vec{R}-\vec{R}'\right|}\right) = -\frac{(x-x')\hat{x}+(y-y')\hat{y}+(z-z')\hat{z}}{\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{3/2}}$$

$$\nabla\left(\frac{1}{\left|\vec{R}-\vec{R}'\right|}\right) = -\frac{\vec{R}-\vec{R}'}{\left|\vec{R}-\vec{R}'\right|^3}$$

$$\nabla'\left(\frac{1}{\left|\vec{R}-\vec{R}'\right|}\right) = \left(\frac{\partial}{\partial x'}\hat{x}+\frac{\partial}{\partial y'}\hat{y}+\frac{\partial}{\partial z'}\hat{z}\right)\left(\frac{1}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}\right)$$

$$\nabla'\left(\frac{1}{\left|\vec{R}-\vec{R}'\right|}\right) = +\frac{(x-x')\hat{x}+(y-y')\hat{y}+(z-z')\hat{z}}{\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{3/2}}$$

$$\nabla'\left(\frac{1}{\left|\vec{R}-\vec{R}'\right|}\right) = +\frac{\vec{R}-\vec{R}'}{\left|\vec{R}-\vec{R}'\right|^3} = -\nabla\left(\frac{1}{\left|\vec{R}-\vec{R}'\right|}\right)$$

$$\Phi(R) = \frac{1}{4\pi\varepsilon_0}\int_{V'} \vec{P}(\vec{R}')\bullet\nabla'\left(\frac{1}{\left|\vec{R}-\vec{R}'\right|}\right)dV'$$

$$a = a(x,y,z)$$

$$\vec{F} = F_x(x,y,z)\hat{x}+F_y(x,y,z)\hat{y}+F_z(x,y,z)\hat{z}$$

$$\nabla\bullet\left(a\vec{F}\right) = \nabla a\bullet\vec{F} + a\nabla\bullet\vec{F}$$

$$\vec{F}\bullet\nabla a = \vec{P}\left(\vec{R}'\right)\bullet\nabla'\left(\frac{1}{\left|\vec{R}-\vec{R}'\right|}\right)$$

$$\nabla a\bullet\vec{F} = \vec{F}\bullet\nabla a = \nabla\bullet\left(a\vec{F}\right) - a\nabla\bullet\vec{F}$$

$$\vec{P}(\vec{R}')\bullet\nabla'\left(\frac{1}{\left|\vec{R}-\vec{R}'\right|}\right) = \nabla'\bullet\left(\frac{1}{\left|\vec{R}-\vec{R}'\right|}\vec{P}(\vec{R}')\right) - \frac{1}{\left|\vec{R}-\vec{R}'\right|}\nabla'\bullet\vec{P}(\vec{R}')$$

$$\Phi$$

$$\Phi(\vec{R}) = \frac{1}{4\pi\varepsilon_0}\int_{V'} \nabla'\bullet\left(\frac{1}{\left|\vec{R}-\vec{R}'\right|}\vec{P}(\vec{R}')\right) - \frac{1}{\left|\vec{R}-\vec{R}'\right|}\nabla'\bullet\vec{P}(\vec{R}')dV$$

$$\Phi = \frac{1}{4\pi\varepsilon_0}\int_{V'} \nabla' \bullet \left(\frac{1}{\left|\vec{R}-\vec{R}'\right|}\vec{P}(\vec{R}')\right) dV' + \frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{-1}{\left|\vec{R}-\vec{R}'\right|}\nabla' \bullet \vec{P}(\vec{R}')dV'$$

$$\Phi(\vec{R}) = \frac{1}{4\pi\varepsilon_0}\oint_{S'} \frac{1}{\left|\vec{R}-\vec{R}'\right|}\vec{P}(\vec{R}') \bullet \overrightarrow{dS'} + \frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{-\nabla' \bullet \vec{P}(\vec{R}')}{\left|\vec{R}-\vec{R}'\right|}dV'$$

$$\overrightarrow{dS'} = \hat{n}dS'$$

$$\Phi(\vec{R}) = \frac{1}{4\pi\varepsilon_0}\oint_{S'} \frac{\vec{P}(\vec{R}') \bullet \hat{n}}{\left|\vec{R}-\vec{R}'\right|}\overrightarrow{dS'} + \frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{-\nabla' \bullet \vec{P}(\vec{R}')}{\left|\vec{R}-\vec{R}'\right|}dV'$$

$$\Phi(\vec{R}) = \frac{1}{4\pi\varepsilon_0}\oint_{S'} \frac{\rho_{sp}}{\left|\vec{R}-\vec{R}'\right|}\overrightarrow{dS'} + \frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{\rho_{vp}}{\left|\vec{R}-\vec{R}'\right|}dV'$$

$$\rho_{sp} = \vec{P}(\vec{R}') \bullet \hat{n}$$

$$[\rho_{sp}] = \frac{Coulomb}{m^2}$$

$$\rho_{vp} = -\nabla' \bullet \vec{P}(\vec{R}')$$

$$[\rho_{vp}] = \frac{Coulomb}{m^3}$$

Electric Flux Density & Dielectric Constant

Assume both free charges & polarized dielectric are given:
Free charges: ρ_v, ρ_s, ρ_l **externally put**
Polarization charges: ρ_{vp}, ρ_{sp}

For this case, the Gauss' Law:

$$\oint_S \vec{E} \bullet \overrightarrow{dS} = \frac{Q_{total}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int_V \rho_v + \rho_{vp} dV$$

$$= \frac{1}{\varepsilon_0}\int_V \rho_v dV + \frac{1}{\varepsilon_0}\int_V \rho_{vp} dV$$

$$= \frac{1}{\varepsilon_0}\int_V \rho_v dV + \frac{1}{\varepsilon_0}\int_V -\nabla \bullet \vec{P} dV$$

$$= \frac{1}{\varepsilon_0}\int_V \rho_v dV - \frac{1}{\varepsilon_0}\int_V \nabla \bullet \vec{P} dV$$

$$= \frac{1}{\varepsilon_0}\int_V \rho_v dV - \frac{1}{\varepsilon_0}\oint_S \vec{P} \cdot \overrightarrow{dS}$$

$$= \frac{1}{\varepsilon_0}\int_V \nabla \cdot \vec{D} dV - \frac{1}{\varepsilon_0}\oint_S \vec{P} \cdot \overrightarrow{dS}$$

$$= \frac{1}{\varepsilon_0}\oint_{S'} \vec{D} \cdot \overrightarrow{dS} - \frac{1}{\varepsilon_0}\oint_{S'} \vec{P} \cdot \overrightarrow{dS}$$

$$= \frac{1}{\varepsilon_0}\oint_{S'} \varepsilon\vec{E} \cdot \overrightarrow{dS} - \frac{1}{\varepsilon_0}\oint_{S'} \vec{P} \cdot \overrightarrow{dS}$$

$$\oint_S \vec{E} \cdot \overrightarrow{dS} = \frac{1}{\varepsilon_0}\oint_{S'} (\varepsilon\vec{E} - \vec{P}) \cdot \overrightarrow{dS}$$

$$\oint_S \varepsilon_0\vec{E} \cdot \overrightarrow{dS} = \oint_{S'} (\varepsilon\vec{E} - \vec{P}) \cdot \overrightarrow{dS}$$

$$\varepsilon_0\vec{E} = (\varepsilon\vec{E} - \vec{P})$$

$$(\varepsilon - \varepsilon_0)\vec{E} = \vec{P}$$

$$\oint_S \vec{E} \cdot \overrightarrow{dS} = \frac{1}{\varepsilon_0}\int_V \rho_v dV - \frac{1}{\varepsilon_0}\oint_S \vec{P} \cdot \overrightarrow{dS}$$

$$\oint_S (\varepsilon_0\vec{E} + \vec{P}) \cdot \overrightarrow{dS} = \int_V \rho_v dV$$

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$$

$$\oint_S \vec{D} \cdot \overrightarrow{dS} = \int_V \rho_v dV \qquad GGL_{Integral\ Form}$$

$$\nabla \cdot \vec{D} = \rho_v \qquad GGL_{Differential\ Form}$$

$$[\nabla \cdot \vec{D}] = [\rho_v] = \frac{Coulomb}{m^3} \qquad [\vec{D}] = \frac{Coulomb}{m^2}$$

Note: Sources of $\vec{D}$ are only free charges: ρ_v, ρ_s, ρ_l

Note: Sources of $\vec{E}$ are both free charges and polarization charges

$\vec{D}: \begin{cases} Electric\ Flux\ Density \\ Displacement\ vector \end{cases}$ $\qquad \frac{Coulomb}{m^2}$

$\vec{E}: Electric\ Field\ Intensity\ Density$ $\qquad \frac{Volt}{m}$

$$\oint_S \vec{D} \cdot \overrightarrow{dS} = \int_V \rho_v dV = Q_{free}$$

For an arbitrary dielectric

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

For a linear dielectric

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$

χ_e: **electric susceptibility** **Dimensionless**

$$\chi_e \geq 0$$

Dielectric	χ_e is independent of
Homogeneous	Space coordinates
Linear	$\lvert\vec{E}\rvert$
Isotropic	Direction of $\vec{E}$

For a linear dielectric ***

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \vec{E} + \varepsilon_0 \chi_e \vec{E}$$

$$\vec{D} = \varepsilon_0 (1 + \chi_e) \vec{E}$$

$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$$

$$\vec{D} = \varepsilon \vec{E}$$

$$\varepsilon = \varepsilon_0 \varepsilon_r$$ **(Absolute) permittivity**

$$\varepsilon_r = (1 + \chi_e)$$ **Relative permittivity**

$$\varepsilon_r \geq 1$$

$$\varepsilon \geq \varepsilon_0$$ **always**

$$\varepsilon_{r,vacuum} = 1$$

$$\varepsilon_{r,air} = 1.00059$$

$$\varepsilon_{air} = \varepsilon_0 \varepsilon_{r,air} = 1.00059 \varepsilon_0 \cong \varepsilon_0$$

Medium is simple if it is homogeneous, linear and isotropic

ε_r: *constant for a simple medium*

Some material (some crystals) are anistropic

$\boldsymbol{\varepsilon_r}$ depends on direction

$\vec{E}$ and $\vec{D}$ are in different directions for anistropic medium

Dielectric Strength

An electric field causes small displacements in the bound charges of the dielectric yielding polarization

If the E-field strong enough, it can pull electrons out of their in the atoms/molecules yielding permanent damage in the material

Avalanche effect of ionization may occur resulting in very large currents; the dielectric turns into a conductor. This phenomenon is known as Dielectric Breakdown.

The maximum electric field that can be applied onto the dielectric before the breakdown occurs is called as the Dielectric Strength.

Air breaks down for $\boldsymbol{E} \geq 3000\ V/m$ at atmospheric pressure
Massive ionization of air molecules, sparking, corona discharge occur

$\boldsymbol{E}$ is larger at the conductor surfaces with larger curvature (at the tip of the conductor)

Wet air close to the tip of the lightning rod can be easily ionized and electric charges in the clouds can be easily discharged to the ground via the lightning rod.