Lecture 1 :

In this "PHYS 401 Electomagnetic Theory I" course we will mainly cover the following titles :

1. Quick overview of electromagnetic theory (Chapter 1-8 of D.Griffiths' well-known book which is adopted as our main textbook). The students are expected to know these chapters well enough, which constitute our basic groundwork.
2. Electromagnetic waves and its applications
3. Potential formulation and gauge transformations
4. Retarded potentials
5. Liénard-Wiechert potentials
6. Electromagnetic radiation (electric and magnetic dipole radiatons)
7. Radiation by an accelerated point charge, Larmor theory, Liénard generalization.
8. Radiation reaction, Abraham-Lorentz theory
9. Several special topics in electromagnetic theory (such as magnetic monopoles, topological effects in electromagnetism etc.).

   On the other hand relativistic electrodynamics is left for the Spring semester and it will be throughly elaborated in PHYS 402 starting from the first principles of the special theory of relativity.

All the necessary derivations and the details of the mathematical calculations will be given on the blackboard in the class.

Classical electromagnetic theory, together with its quantized version, quantum electrodynamics, form one of the basic core elements of the present day theoretical understanding of the physical universe. Even more, they are sometimes said to play the role of an ideal mathematical models for the other parts of the physics.

The students are expected to have mastered the vector analysis already. So we assume that they know the following mathematical notions, definitions, theorems, and their applications as well : the rectangular, polar, cylindrical and spherical coordinate systems, vector differential operators, divergence and Stokes' theorems, Green identities, line, surface and volume integrations of the vector functions, Dirac delta function, orthogonal transformations, Helmholtz theorem.

The electromagnetic theory is perfectly described by the Maxwell equations :

In vacuum :

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

In matter :

$$\nabla \cdot \mathbf{D} = \rho_f$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$$

The constitutive relations are ;

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \qquad \mathbf{P} = \epsilon_0 \chi_e \mathbf{E}, \quad \mathbf{D} = \epsilon \mathbf{E}$$

$$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} \qquad \mathbf{M} = \chi_m \mathbf{H}, \quad \mathbf{H} = \frac{1}{\mu}\mathbf{B}$$

Here :

D = Displacement vector

P = Polarization vector

H = Auxiliary magnetics field

M = Magnetization vector

Also the Lorentz force law describes the motion of a charged particle in exterior electric and magnetic fields :

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

The electric and magnetic fileds satisfy the following boundary conditions at the interfaces of the two different media.

$$D_{2n} - D_{1n} = \sigma_f \qquad B_{2n} - B_{1n} = 0$$

$$E_{2t} - E_{1t} = 0 \qquad \vec{H}_{2//} - \vec{H}_{1//} = \vec{K}_{surface} \times \hat{n}$$

Waves on a stretched string :

Using Newton's second law for a differential portion of the string, one can arrive the following wave equation for small disturbances on the string :

$$\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$$

Here the speed of propagation is v and given by :

$$v = \sqrt{\frac{T}{\mu}}$$

Where T is the tension and μ is the mass per unit length.

Its general solution can be expressed by

$f(z,t) = g(z - vt) + h\ (z + vt)$

as a linear superposition of the waves propagating in the positive and negative z directions.

At a boundary (for example a knot on the string) one must use the mathematical continuity equations for the wave f(z,t) :

a) At the both sides of the boundary the values of the function itself must be the same
b) Its derivative must also be continuous at the boundary .

These two conditions allow us to obtain the amplitudes of the reflected and transmitted waves.

Reflected and transmitted waves amplitudes differ according to the relative mass densities of the string in the left and right of the knot.

If $\mu_2 > \mu_1$ (in other words if the speed of the wave is lower in the second part than the first) then one gets :

$$A_R = \frac{v_1 - v_2}{v_2 + v_1} A_I \quad ; \quad A_T = \frac{2v_2}{v_2 + v_1} A_I$$

If on the other hand the second part is lighter than the first then we obtain :

$$A_R = \frac{v_2 - v_1}{v_2 + v_1} A_I \quad ; \quad A_T = \frac{2v_2}{v_2 + v_1} A_I$$

These waves propagating in a stretched string are transverse waves.

Polarization unit vector $\hat{n}$ defines the plane of vibrations. Since in our case here it is transverse, we have $\hat{n} \cdot \hat{z} = 0$