Lecture 3 :

The electromagnetic waves in dielectric medium: Wave equations for the electric and magnetic fields in matter can be derived again starting from the Maxwell's equations. In a dielectric medium with no free charge and free current sources, we write them as :

$$\text{(i)} \quad \nabla \cdot \mathbf{D} = 0, \quad \text{(iii)} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\text{(ii)} \quad \nabla \cdot \mathbf{B} = 0, \quad \text{(iv)} \quad \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}.$$

For the linear media we have the linear relations between **E** and **D** (**B** and **H** fields) :

$$\mathbf{D} = \epsilon \mathbf{E}, \quad \mathbf{H} = \frac{1}{\mu}\mathbf{B},$$

If the dielctric constant and permeability do not vary in space, one get easily

$$\text{(i)} \quad \nabla \cdot \mathbf{E} = 0, \quad \text{(iii)} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\text{(ii)} \quad \nabla \cdot \mathbf{B} = 0, \quad \text{(iv)} \quad \nabla \times \mathbf{B} = \mu\epsilon \frac{\partial \mathbf{E}}{\partial t},$$

Taking the curl of the third equation, and using the vector identity for the product rule for del operators and also inserting the first and fourth equations one can easily arrive at the wave equation for electric field in dielectric medium. Following similar steps starting with Ampere's + Maxwell law the wave equation for the magnetic field can be obtained. This time the speed of the electromagnetic wave turns out to be :

$$v = \sqrt{\frac{1}{\varepsilon\mu}} = \frac{c}{n} v = \sqrt{\frac{1}{\varepsilon_0\mu_0}} \quad \text{where } n = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} \text{ is the index of refraction}$$

All of the previous expressions for the vacuum case are now enlarged to the dielectric medium :

$$u = \frac{1}{2}\left(E^2 + \frac{1}{\mu} B^2\right)$$

$$\boldsymbol{S} = \frac{1}{\mu} \boldsymbol{E} \times \boldsymbol{B}$$

$$I = \frac{1}{2} \varepsilon v E_0^2$$

Reflection and Transmission at Normal and Oblique Incidences

One can investigate the behaviour of the electromagnetic waves at the interface of two different medium, such as between air and glass etc. The boundary conditions are :

$$\text{(i)}\ \epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp}, \quad \text{(iii)}\ \mathbf{E}_1^{\parallel} = \mathbf{E}_2^{\parallel},$$

$$\text{(ii)}\ B_1^{\perp} = B_2^{\perp}, \quad \text{(iv)}\ \frac{1}{\mu_1}\mathbf{B}_1^{\parallel} = \frac{1}{\mu_2}\mathbf{B}_2^{\parallel}.$$

Using the above boundary conditions one can get the reflection coefficient R and transmission coefficient T can be derived in case of normal incidence as follows :

$$R = \frac{I_{\text{Reflected}}}{I_{Incident}} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

$$T = \frac{I_{Transmitted}}{I_{Incident}} = \frac{4n_1 n_2}{(n_1 + n_2)^2}$$

$$R + T = 1$$

On the other hand, in the richer case of oblique incidence one can obtain using the same boundary conditions :

a) Snell's law for the reflection and refraction

b) Fresnel equations for the field amplitudes (for parallel and normal polarizations)

c) Brewester angle

d) Polarization by reflection

e) Total internal reflection

Further discussions of the Fresnel equations and also the Figures 9.16 and 9.17 from the D.Griffiths' textbook will be done in the class.