Lecture 4 :

The electromagnetic waves in conducting medium: In conductors one has a free curent density $\mathbf{J}_f = \sigma E$, where σ stands for the conductivity. Consequently Ampere's law of Maxwell's equations is modified with the presence of this term. We write them for a linear medium as :

$$\text{(i)}\ \nabla \cdot \mathbf{E} = \frac{1}{\epsilon}\rho_f, \quad \text{(iii)}\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\text{(ii)}\ \nabla \cdot \mathbf{B} = 0, \quad \text{(iv)}\ \nabla \times \mathbf{B} = \mu\sigma\mathbf{E} + \mu\epsilon\frac{\partial \mathbf{E}}{\partial t}.$$

Using the continuity equation and Gauss' law we can proove that the free charge density ρ_f decays very rapidly. So one can take it to be zero, $\rho_f = 0$, for good conductors, because we ara not interested in the transient behaviour . Therefore Maxwell equations reduce to the form below :

$$\text{(i)}\ \nabla \cdot \mathbf{E} = 0, \quad \text{(iii)}\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\text{(ii)}\ \nabla \cdot \mathbf{B} = 0, \quad \text{(iv)}\ \nabla \times \mathbf{B} = \mu\epsilon\frac{\partial \mathbf{E}}{\partial t} + \mu\sigma\mathbf{E}.$$

Following the same mathematical step as before one can obtain the wave equations for the fields :

$$\nabla^2\mathbf{E} = \mu\epsilon\frac{\partial^2 \mathbf{E}}{\partial t^2} + \mu\sigma\frac{\partial \mathbf{E}}{\partial t}, \quad \nabla^2\mathbf{B} = \mu\epsilon\frac{\partial^2 \mathbf{B}}{\partial t^2} + \mu\sigma\frac{\partial \mathbf{B}}{\partial t}.$$

These wave equations still admit plane wave solutions but now we have a big difference : the wave number $\tilde{k}$ is complex !

$$\tilde{k}^2 = \mu\varepsilon\omega^2 + i\mu\sigma\omega$$
$$\tilde{k} = k + i\kappa$$

The real part k determines wavelength, speed of propagation and index of refraction as usual.

$$\lambda = \frac{2\pi}{k}, \quad v = \frac{\omega}{k}, \quad n = \frac{ck}{\omega}.$$

Imaginary part κ defiens the skin depth $d = 1/\kappa$

Also the reflection and transmission coefficients can be obtained using the boundary conditions at a conducting surface.

The phase and group velocities of the eelctromagnetica waves are defined as

$$v_{phase} = \frac{\omega}{k} \quad ; \quad v_{group} = \frac{d\omega}{dk}$$

The frequency dependence of the dielectric constant ε in non-conducting medium can be derived by using a simplified model (although classical) of the electrons in dielectrics. The detailed analysis permits us to express the complex permittivity $\tilde{\varepsilon} = \varepsilon_0 (1+\tilde{\chi}_e) = \tilde{\varepsilon}_r \varepsilon_0$

$$\tilde{\epsilon}_r = 1 + \frac{Nq^2}{m\epsilon_0} \sum_j \frac{f_j}{\omega_j^2 - \omega^2 - i\gamma_j\omega}.$$

On the other hand the complex wave number can be written as :

$$\tilde{k} = \sqrt{\tilde{\varepsilon}\mu_0}\,\omega = k + i\kappa$$

Evidently the wave attenuates and the quantity $\alpha = 2\kappa$ is called the absorption coefficient.

We arrive at the following results.

$$\tilde{k} = \frac{\omega}{c}\sqrt{\tilde{\epsilon}_r} \cong \frac{\omega}{c}\left[1 + \frac{Nq^2}{2m\epsilon_0} \sum_j \frac{f_j}{\omega_j^2 - \omega^2 - i\gamma_j\omega}\right],$$

$$n = \frac{ck}{\omega} \cong 1 + \frac{Nq^2}{2m\epsilon_0} \sum_j \frac{f_j(\omega_j^2 - \omega^2)}{(\omega_j^2 - \omega^2)^2 + \gamma_j^2\omega^2},$$

$$\alpha = 2\kappa \cong \frac{Nq^2\omega^2}{m\epsilon_0 c} \sum_j \frac{f_j\gamma_j}{(\omega_j^2 - \omega^2)^2 + \gamma_j^2\omega^2}.$$

Discussion of the anomalous dispersion and Figure 9.22 of the D.Griffiths's textbook is to be done in the class.