Lecture 5 :

Guided waves : Propagation and excitation of electromagnetic waves in hollow metallic waveguides has great practical importance. We can assume that **E** = 0 and **B** = 0 in metallic material. Hence the boundary conditions at the inner wall of the waveguide are :

$$\boldsymbol{E}_{\parallel} = 0$$
$$B_{\perp} = 0$$

Let us assume that the propagation is in positive z-direction; then electric and magnetic fields can be expressed in the general form below :

$$\text{(i)} \quad \tilde{\mathbf{E}}(x, y, z, t) = \tilde{\mathbf{E}}_0(x, y)e^{i(kz-\omega t)},$$

$$\text{(ii)} \quad \tilde{\mathbf{B}}(x, y, z, t) = \tilde{\mathbf{B}}_0(x, y)e^{i(kz-\omega t)}.$$

These field satisfy the Maxwell's equations and obey the above boundary conditions.

$$\text{(i)} \ \nabla \cdot \mathbf{E} = 0, \quad \text{(iii)} \ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\text{(ii)} \ \nabla \cdot \mathbf{B} = 0, \quad \text{(iv)} \ \nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}.$$

After inserting the field expressions in the above equations one can obtain the following result: It is sufficient to determine the longitudinal components of the electric and magnetic fields, all the other components can be expressed in terms of E_z and B_z and their spatial derivatives. E_z and B_z satisfy the following uncoupled differential equations :

$$\text{(i)} \quad \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + (\omega/c)^2 - k^2\right] E_z = 0,$$

$$\text{(ii)} \quad \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + (\omega/c)^2 - k^2\right] B_z = 0.$$

Thus one has three possibilities :

a) TE waves (transverse electric waves) $E_z = 0$ and $B_z \neq 0$
b) TM waves (transverse magnetic waves) $B_z = 0$ and $E_z \neq 0$
c) TEM waves (transverse electric and magnetic waves) $E_z = 0$ and $B_z = 0$. This mode can not ocur in hollow waveguides but might be excited, for example, in coaxial waveguides

Example 1 : TE Waves in a Rectangular Waveguide.
The TE_{mn} modes can be obtained using the boundary conditions and solving the above differential equation for B_z by the method of separation of variables : $B_z(x,y) = X(x)Y(y)$

It appears that lowest mode that can be excited is the mode TE_{10} . Also the wave number is given by

$$k = \sqrt{(\omega/c)^2 - \pi^2[(m/a)^2 + (n/b)^2]}.$$

Therefore for the frequencies ω which make the the inside of the squareroot term negative, we have imaginary k's hence attenuation of the waves. So the cut-off frequency ω_{mn} is defined as

$$\omega_{mn} = c\pi\sqrt{(m/a)^2 + (n/b)^2}$$

Group and phase velocities of the TE_{mn} waves become :

$$v_g = c\sqrt{1-(\omega_{mn}/\omega)^2}$$
$$v_p = c/\sqrt{1-(\omega_{mn}/\omega)^2}$$

TM and TEM modes can be analysed by similar mathematical methods.

If one close off the both ends of a rectangular waveguide at $z = 0$ and $z = d$ then we obtain a **resonant cavity.**

Homework : Show that TE and TM modes in such a rectangular resonant cavity the cut-off frequency is given by

$$\omega_{mnl} = c\pi\sqrt{(m/a)^2 + (n/b)^2 + (l/d)^2}$$

n, m, l are being integers.

Schumann Resonances :

The Earth and the atmosphere form an unusual resonant spherical cavity.One can work out the TE modes in such a cavity (called Schumann resonances) and obtain frequencies for the lowest modes. Lowest Schumann resonanace is approximately 8 Hz and this value may put an upper mass for the photon,

$$m_\gamma < 6\times10^{-50}\ kg$$

Schumann resonances may serve also as a global tropical thermometer !

Look at and study the J.D.Jackson's textbook "Classical Electrodynamics", Section 8.9 and Figure 8.9.