Lecture 8 :

Liénard-Wiechert potentials : Let us find the potentials due to a point charge q moving on a specified trajectory **w**(t). Hence we replace the position vector of the source **r'** by **w**(t) in the following expression for the retarded time :

$$t_r = t - \frac{|\boldsymbol{r}-\boldsymbol{r}'|}{c}$$

Due to the retardation effects a multiplicative factor of $(1-\hat{\boldsymbol{\imath}}\cdot\boldsymbol{v}/c)^{-1}$ is needed in the volume integrations.

Thus we can write :

$$\int \rho(r',t_r)\, d^3 r' = \frac{q}{(1-\hat{\boldsymbol{\imath}}\cdot\boldsymbol{v}/c)}$$

Then one can write the scalar potential due to a moving charge as

$$V(\boldsymbol{r},t) = \frac{1}{4\pi\varepsilon_0}\frac{qc}{(\boldsymbol{\imath}c-\boldsymbol{\imath}\cdot\boldsymbol{v})}$$

Since the current at a retarded time is $\rho(r',t_r)\boldsymbol{v}(t_r)$ one can obtain the retarded vector potential due to a moving charge as follows :

$$A(\boldsymbol{r},t) = \frac{\mu_0}{4\pi}\frac{q\,c\,\boldsymbol{v}}{(\boldsymbol{\imath}-\hat{\boldsymbol{\imath}}\cdot\boldsymbol{v}/c)}$$

Homeworks :

Study and solve the following exercises and problems from the textbook by D.Griffiths's "Introduction to Electrodynamics"

Study Exercises 10.3

Solve Problem 10.13

Solve Problem10.14

Solve Problem10.15

Fields of a moving charge : In order obtain expressions for the electric and magnetic fields due to a moving point charge one can make use of the retarded potentials already obtained above and insert them in the usual relationsfor ***E*** and ***B*** .

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

$$\boldsymbol{E} = -\nabla V - \frac{\partial \boldsymbol{A}}{\partial t}$$

The calculational steps are quite tedious because of the presence of the retarded time factor in the expressions for the potentials.

One gets :

$$\boxed{\mathbf{E}(\mathbf{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{\boldsymbol{\mathscr{r}}}{(\boldsymbol{\mathscr{r}} \cdot \mathbf{u})^3} [(c^2 - v^2)\mathbf{u} + \boldsymbol{\mathscr{r}} \times (\mathbf{u} \times \mathbf{a})].}$$

$$\boxed{\mathbf{B}(\mathbf{r}, t) = \frac{1}{c} \hat{\boldsymbol{\mathscr{r}}} \times \mathbf{E}(\mathbf{r}, t).}$$

Here

$$u = c\hat{\boldsymbol{\mathscr{r}}} - \boldsymbol{v}$$
$$\boldsymbol{a} = \dot{\boldsymbol{v}}$$

Study and solve the following exercises and problems from the textbook by D.Griffiths's "Introduction to Electrodynamics"

Homework:

Study Exercises 10.4

Solve Problem 10.24

Finally we can obtain the Lorentz force between a charge Q and q moving with velocities **V** and **v** respectively.

$$\begin{aligned} \mathbf{F} &= \frac{qQ}{4\pi\epsilon_0} \frac{\boldsymbol{\mathscr{r}}}{(\boldsymbol{\mathscr{r}} \cdot \mathbf{u})^3} \Big\{ [(c^2 - v^2)\mathbf{u} + \boldsymbol{\mathscr{r}} \times (\mathbf{u} \times \mathbf{a})] \\ &+ \frac{\mathbf{V}}{c} \times \Big[\hat{\boldsymbol{\mathscr{r}}} \times [(c^2 - v^2)\mathbf{u} + \boldsymbol{\mathscr{r}} \times (\mathbf{u} \times \mathbf{a})] \Big] \Big\}, \end{aligned}$$

All the velocities and positions are evaluated at a retarded time. This expression is the relativistic generalization of the Coulomb's law ! See. P.439 of D.Griffiths textbook.