Lecture 10 :

Radiation : Charges at rest and steady currents do not radiate. But accelerating charges radiate. On the other hand a vibrating spherical shell type charge distribution does not radiate either due to the Gauss law, if during the vibration it keeps its spherical symmetry.

<u>*Electric dipole radiation*</u> : Let us consider two point charges +q and –q on the z-axis at

z = d/2 and z = - d/2 respectively. Assume that the value of q varies sinusoidally with time.

$q(t) = q_0 \cos \omega t$

$p(t) = p_0 \cos \omega t \ \hat{\mathbf{z}}$

$p_0 = q_0 d$

We simply replace these in the retarded scalar potential expression and need some suitable approximations in order to carry out the calculations :

a) $d \ll r$

b) $d \ll \frac{c}{\omega}$ namely $d \ll \lambda$

c) $r \gg \frac{c}{\omega}$ namely $r \gg \lambda$

The retarded scalar potential becomes :

$$V(r, \theta, t) = -\frac{p_0 \omega}{4\pi\epsilon_0 c}\left(\frac{\cos\theta}{r}\right)\sin[\omega(t - r/c)].$$

Following similar steps to obtain the retarded vector potential due to our electric dipole we get

$$\mathbf{A}(r, \theta, t) = -\frac{\mu_0 p_0 \omega}{4\pi r}\sin[\omega(t - r/c)]\,\hat{\mathbf{z}}.$$

The fields are derived from the potentials as in the usual case :

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

$$\boldsymbol{E} = -\nabla V - \frac{\partial \boldsymbol{A}}{\partial t}$$

Physically interesting quantity is the time averaged Poynting vector $\langle \boldsymbol{S} \rangle$ and it turns out that

$$\langle \boldsymbol{S} \rangle = \left\langle \frac{1}{\mu_0} \boldsymbol{E} \times \boldsymbol{B} \right\rangle = \left(\frac{\mu_0 \, p_0^2 \, \omega^4}{32\pi^2 c} \right) \frac{\sin^2 \theta}{r^2} \hat{r}$$

The total power radiated can be found by integrating it over a sphere of radius :

$$\langle P \rangle = \int \langle \boldsymbol{S} \rangle \cdot \boldsymbol{da} = \frac{\mu_0 \, p_0^2 \, \omega^4}{12\pi c}$$

Homeworks :

Study and solve the following exercises and problems from the textbook by D.Griffiths's "Introduction to Electrodynamics"

Study Exercises 11.1 (Why rhe sky is blue !)

Solve Problem 11.1

Solve Problem11.2

Solve Problem 11.3 (Radiation resistance)

Solve Problem 11.4

<u>Magnetic dipole radiation</u> : Now consider a loop of radius r on the xy- plane. An alternating current flows through the loop in counterclockwise direction.

$I(t) = I_0 \cos \omega t$

Magnetic dipole moment is **:** $\mathbf{m}(t) = \pi b^2 \, I(t) \, \hat{\mathbf{z}} = m_0 \cos \omega t \, \hat{\mathbf{z}}$ where $m_0 = \pi b^2 I_0$

Again we insert these in the retarded vector potential expression and make use of the same aproximations as above. Following very similar calculations we arrive at the intensity and total power radiated by the magnetic dipole.

$$\langle \boldsymbol{S} \rangle = \left(\frac{\mu_0 \, m_0^2 \, \omega^4}{32\pi^2 c^3} \right) \frac{\sin^2 \theta}{r^2} \hat{r}$$

$$\langle P \rangle = \int \langle \boldsymbol{S} \rangle \cdot \boldsymbol{da} = \frac{\mu_0 \, m_0^2 \, \omega^4}{12\pi c^3}$$

One can easily show that dipole radiation dominates, in our ordinary approximations

$$\frac{P_{magnetic}}{P_{electric}} \ll 1$$