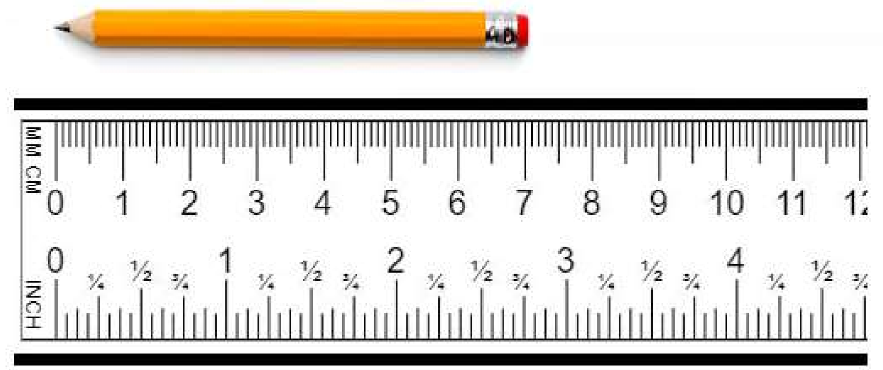
SIGNIFICANT FIGURES

The concept of significant figures has been developed to officially determine the reliability of numerical value. The important digits of a number are the numbers that can be used safely. The significant digits of numbers correspond to a certain number of digits, plus a predicted number. If we use a 1 cm ruler on the smallest scale to measure the length of a pencil. It is conventional to adjust the estimated figure in half of the smallest scale section in the measurement device.



We can say that the length of the pencil is equal to 71 mm, but not 71.15 according to the ruler we used to measure the pencil.

ACCURACY AND PRECISION

Errors in both calculations and measurements can be characterized in terms of accuracy and precision. Accuracy shows how well a calculated or measured value is in line with the true value. Sensitivity means how closely the individual calculated or measured values are close to each other.

Example: Three groups of students measure the length of the pencil as follows:
Group 1: 13.5: 13.3; 13.6; 13.7 (Accurate and precise)
Group 2: 16.4; 16.3; 16.7; 16.5 (Precise, but inaccurate)
Group 3: 10.7; 13.8; 17.1; 9.8 (Inaccurate and imprecise)

According to the measurements of the student groups; Group 1 took an accurate and precise measurement; Group 2 took a precise, but inaccurate measurement; and The last group took an inaccurate and imprecise measurement.

ERROR DEFINITIONS

The relationship between the true result and the approximation can be formulated as:

True value = approximation + true error

True Error (Et) = True value – approximation

The true percent relative error:

ϵt = true error/true value * 100

If the true value is not known

Approximate Error (Ea) = Present approximation – Past approximation

The percent relative error (ϵa) = (Current App. – Previous App.)/Current App. * 100

When performing computations, we are interested in whether the percent absolute value is lower than a prespecified percent tolerance.

Example: The exponential function can be computed using the following infinite serie.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \frac{x^n}{n!}$$

Calculate e^x for x=1. The percent tolerance (ϵs) is equal to 0.5%

Solution: The first estimate is done with a single term

$e^1 = 1$

The second estimate is done by adding the second term

$e^1 = 1 + 1 = 2$ → The percent relative error (ϵa) = (2 – 1)/2x100 = 50%

The third estimate is done by adding the third term

$e^1 = 1 + 1 + ½ = 2.5$ → The percent relative error (ϵa) = (2.5 – 2)/2.5x100 = 20%

The fourth estimate is done by adding the fourth term

$e^1 = 1 + 1 + ½ + 1/6 = 2.67$ → The percent relative error (ϵa) = (2.67 – 2.5)/2.67x100 = 6.37%

The 5th estimate is done by adding the 5th term

$e^1 = 1 + 1 + ½ + 1/6 + 1/24 = 2.71$ → The percent relative error (ϵa) = (2.71 – 2.67)/2.71x100 = 1.47%

The 6th estimate is done by adding the 6th term

$e^1 = 1 + 1 + ½ + 1/6 + 1/24 + 1/120 = 2.72$ → The percent relative error (ϵa) = (2.72 – 2.71)/2.72x100 = 0.367%

After six terms are added, the approximate error falls below the percent tolerance (ϵs), which is equal to 0.5%.

The true vale of e^1 is equal to 2.7183 with five significant digits.

Example: Determine the number of terms necessary to approximate cos(x) using the Maclaurin series approximation

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots.$$

Calculate the approximation using a value of x = 0.3Π. The percent tolerance (ϵs) is equal to 0.05%. Note with 3 significant figures.

Solution: The first estimate is done with a single term

1st estimate → cos (0.3Π) = 1

The second estimate is done by adding the second term

2nd estimate → cos (0.3Π) = 1 – $(0.3Π)^2/2$ = 0.556 → The percent relative error (ϵa) = (0.556 – 1)/0.556x100 = 79.9%

The third estimate is done by adding the third term

3rd estimate → cos (0.3Π) = 1 – $(0.3Π)^2/2$ + $(0.3Π)^4/4!$= 0.589 → The percent relative error (ϵa) = (0.589 – 0.556)/0.589x100 = 5.60%

The fourth estimate is done by adding the fourth term

4th estimate → cos (0.3Π) = 1 – $(0.3Π)^2/2$ + $(0.3Π)^4/4!$ - $(0.3Π)^6/6!$ = 0.588 → The percent relative error (ϵa) = (0.588 – 0.589)/0.588x100 = 0.17%

The fifth estimate is done by adding the fifth term

5th estimate → cos (0.3Π) = 1 – $(0.3Π)^2/2$ + $(0.3Π)^4/4!$ - $(0.3Π)^6/6!$ + $(0.3Π)^8/8!$ = 0.588 → The percent relative error (ϵa) = (0.588 – 0.588)/0.588x100 = 0%

Due to 3 significant figures limitation, the percent relative error (ϵa) is rounded off and obtained the same value.

With fifth term, we obtain the approximate cos (0.3Π).

RERENCES:

S.C. Chapra and R.P. Canale, “Numerical Methods for Engineers”, 6th ed., McGraw-Hill,, NY, 2010