FEEDBACK CONTROL SYSTEMS

LECTURE NOTES-7/12

Stability

A linear time invariant system is stable if natural response approaches to zero as time goes to infinity.

A linear time invariant system is unstable if the natural response grows without bound as time approaches infinity

A linear time invariant system is marginally stable if the natural response neither decays nor grows but remains constant or oscillates as time approaches to infinity

A system is stable if every bounded input yields a bounded output.

We call this statement the bounded input bounded output (BIBO) stability definition of stability

a. A system is stable if every bounded input yields a bounded output
b. A system is unstable if any bounded input yields an unbounded output.

Poles at lhp yields pure exponential decay then negative real part which means
Stable systems have closed-loop transfer function with poles only in the left half-plane

Routh-Hurwitz Criterion:

Find the number of poles without the coordinates

$$\frac{N(s)}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	$\frac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} = b_1$	$\frac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & 0 \end{vmatrix}}{a_3} = b_2$	$\frac{-\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3} = 0$
s^1	$\frac{-\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1} = c_1$	$\frac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$	$\frac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$
s^0	$\frac{-\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1} = d_1$	$\frac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = d_1$	$\frac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = d_1$

PROBLEM: Make the Routh table for the system

$$\frac{1000}{s^3+10s^2+31s+1030}$$

s^3	1	31	0
s^2	1	103	0
s^1	$\frac{-\begin{vmatrix}1 & 31\\1 & 103\end{vmatrix}}{1}=-72$	$\frac{-\begin{vmatrix}1 & 0\\0 & 0\end{vmatrix}}{1}=0$	$\frac{-\begin{vmatrix}1 & 0\\1 & 0\end{vmatrix}}{1}=0$
s^0	$\frac{-\begin{vmatrix}1 & 103\\-72 & 0\end{vmatrix}}{-72}=103$	$\frac{-\begin{vmatrix}1 & 0\\-72 & 0\end{vmatrix}}{-72}=0$	$\frac{-\begin{vmatrix}1 & 0\\-72 & 0\end{vmatrix}}{-72}=0$

Third degree polynomial means 3 poles
Two rows change sign means 2 in rhp and 1 in lhp

Number of roots in rhp = number of sigh change at first column

Routh-Hurwitz Special Cases:

Case 1: Zero only in the first column

$$T(s) = \frac{10}{s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3}$$

s^5	1	3	5
s^4	2	6	3
s^3	ε	7/2	0
s^2	$(6\varepsilon-7)/\varepsilon$	2	0
s^1	$(42\varepsilon-49-6\varepsilon^2)/(12\varepsilon-14)$	0	0
s^0	3	0	0

s^5	1	+	+
s^4	2	+	+
s^3	ε	+	-
s^2	$(6\varepsilon-7)/\varepsilon$	-	+
s^1	$(42\varepsilon-49-6\varepsilon^2)/(12\varepsilon-14)$	+	+
s^0	3	+	+

Example: Determine the stability of the closed loop transfer function

$$T(s) = \frac{10}{s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3}$$

$$D(s) = s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3$$

s^5	3	6	2
s^4	5	3	1
s^3	4.2	1.4	0
s^2	1.33	1	0
s^1	-1.75	0	0
s^0	1	0	0

2 rhp
3 lhp

Case 2: Entire Row is Zero

$$T(s) = \frac{10}{s^5 + 27 + 6s^3 + 42s^2 + 8s + 56}$$

s^5	1	6	8
s^4	1	6	8
s^3	1	3	0

$$P(s) = s^4 + 6s^2 + 8$$

$$\frac{dP(s)}{ds} = 4s^3 + 12s$$

s^5	1	6	8
s^4	1	6	8
s^3	1	3	0
s^2	3	8	0
s^1	1/3	0	0
s^0	8	0	0

Example: For the transfer function given below; obtain the number of poles at right half-plane, left- half plane and jw axis.

$$T(s) = \frac{10}{s^8 + s^7 + 12s^6 + 22s^5 + 39s^4 + 59s^3 + 48s^2 + 38s + 20}$$

S^5	1	12	39	48	20
S^7	1	22	59	38	0
S^6	-1	-2	1	2	0
S^5	1	3	2	0	0
s^4	1	3	2	0	0
s^3	2	3	0	0	0

S^5	1	12	39	48	20
S^7	1	22	59	38	0
S^6	-1	-2	1	2	0
S^5	1	3	2	0	0
s^4	1	3	2	0	0
s^3	2	3	0	0	0
s^2	3	4	0	0	0
s^1	1/3	0	0	0	0
s^0	4	0	0	0	0

s8 to s4, two sign change → 2 rhp and 2lhp

$$P(s) = s^4 + 3s^2 + 2$$

No sign change from s4 to s0 → No rhp
Then 4 poles on jw-axis

$$\frac{dP(s)}{ds} = 4s^3 + 6s$$

Example: Find the number of poles

$$T(s) = \frac{100}{s^4 + 6s^3 + 11s^2 + 6s + 200}$$

s^4	1	11	200
s^3	1	1	0
s^2	1	20	0
s^1	-19	0	0
s^0	20	0	0

Example: Find the number of poles

$$T(s) = \frac{10}{2s^5 + 3s^4 + 2s^3 + 3s^2 + 2s + 1}$$

s^5	2	2	2
s^4	3	3	1
s^3	ε	4/3	0
s^2	$(3^{\varepsilon}-4)/\varepsilon$	1	0
s^1	$(12\varepsilon-16-32\varepsilon^2)/(9^{\varepsilon}-12)$	0	0
s^0	1	0	0

s^5	1	3	3
s^4	2	2	2
s^3	2	2	0
s^2	ε	2	0
s^1	$(2\varepsilon-4)/\varepsilon$	0	0
s^0	2	0	0