FEEDBACK CONTROL SYSTEMS

LECTURE NOTES-12/12

ZIEGLER-NICHOLS RULES FOR TUNING PID CONTROLLERS

First Method

Type of Controller	Kp	Ti	Td
P	T/L		
PI	0.9T/L	L/0.3	
PID	(1.2)T/L	2L	0.5L

Second Method

Type of Controller	Kp	Ti	Td
P	0.5Kc		
PI	0.45Kc	(5/6)Pc	
PID	0.6Kc	0.5Pc	0.125Pc

Polar (Nyquist) Plots

Integral and Derivative Factors (jw): The polar plot of G(jw) is the negative imaginary axis.

$$G(jw) = \frac{1}{jw} = -j\frac{1}{w} = \frac{1}{w}\angle -90$$

The polar plot of G(jw)=jw is the positive imaginary axis.

First-Order Factors (1+jwT): For the sinusoidal transfer function

The values of G(jw) at w=0 and w=1/T are respectively

$$G(jw) = \frac{1}{1+jwT} = \frac{1}{\sqrt{1+w^2T^2}}\angle -\tan^{-1} wT$$

$$G(j0) = 1\angle 0$$

$$G(j\frac{1}{T}) = \frac{1}{\sqrt{2}}\angle -45$$

If w approaches infinity, the magnitude of G(jw) approaches to zero

Quadratic Factors: The low and high frequency portions of the polar plot of the following sinusoidal transfer function

$$G(jw) = \frac{1}{1 + 2\zeta\left(j\frac{w}{wn}\right) + \left(j\frac{w}{wn}\right)^2}, \zeta > 0$$

$$G(jw) = 1 + 2\zeta\left(j\frac{w}{wn}\right) + \left(j\frac{w}{wn}\right)^2$$

$$G(jw) = \left(1 - \frac{w^2}{wn^2}\right) + j\left(\frac{2\zeta w}{wn}\right)$$

The low frequency portion

$$\lim_{w \to 0} G(jw) = 1\angle 0$$

The high frequency portion

$$\lim_{w \to \infty} G(jw) = \infty\angle 180$$

General Shapes of Polar Plots

$$G(jw) = \frac{K(1+jwT_a)(1+jwT_b)\ldots}{(jw)^{\lambda}(1+jwT_1)(1+jwT_2)\ldots}$$

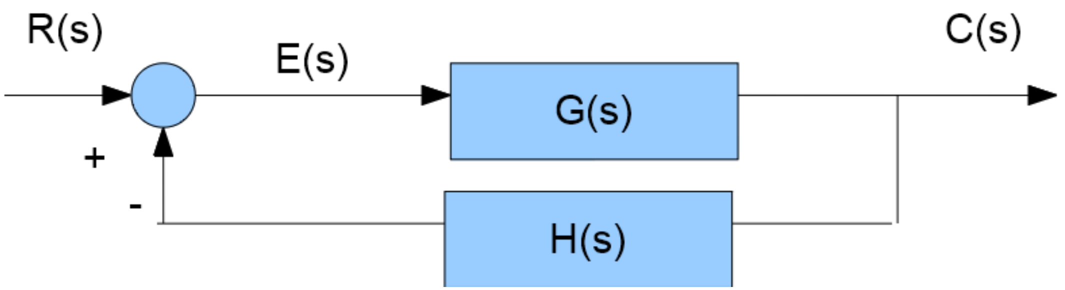
$$G(jw) = \frac{b_0(jw)^m + b_1(jw)^{m-1} + \ldots}{a_0(jw)^n + a_1(jw)^{n-1} + \ldots}$$

1. For λ=0 or type 0 systems: The starting point of the polar plot (which corresponds to w=0) is finite and is on the positive real axis. The tangent to the polar plot at w=0 is perpendicular to the real axis. The terminal point which corresponds to w=inf is at the origin and the curve is tangent to one of the axes.

2. For λ=1 or type 1 systems: the jw term in the denominator contributes -90 to -180 to the total phase angle of G(jw). At w=0, the magnitude of G(jw) is infinity, and the phase angle is equal to -180. At low frequencies the polar plot may be asymptotic to the negative real axis. At w=inf the magnitude becomes zero and the curve is tangent to one of the axes.

NYQUIST STABILITY CRITERION

The Nyquist stability criterion determines the stability of a closed loop system from its open loop frequency response and open loop poles



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$F(s) = 1 + G(s)H(s) = 0$$

$$G(s)H(s) = \frac{2}{s-1}$$

$$F(s) = 1 + G(s)H(s)$$

$$F(s) = 1 + \frac{2}{s-1} = \frac{s+1}{s-1} = 0$$

For example, if s=2+j1 the F(s) becomes

$$F(2+j) = \frac{2+j+1}{2+j-1} = 2-j$$

Thus point s=2+j1 in the s plane maps into point 2-j1 in the F(s) plane

For the characteristic equation F(s) the conformal mapping of the lines w=0,+1,-1,+2,-2,.. And the lines yield circles in the F(s) plane.

If the contour in the s plane encloses equal number of poles and zeros, then the corresponding closed curve in the F(s) plane does not encircle the origin of the F(s) plane. The foregoing discussion is a graphical explanation of the mapping theorem which is the basis for the Nyquist stability criterion

If the contour in the s plane encloses the pole of F(s), there is one encirclement of the origin of the F(s) plane by the locus of F(s) in the counterclockwise direction

If the contour in the s plane encloses the zero of F(s), there is one encirclement of the origin of the F(s) plane by the locus of F(s) in the clockwise diretion

The contour in the s plane encloses both the zero and the pole or if the contour encloses neither the zero nor the pole then there is no encirclement of the origin of the F(s) plane by the locus of F(s)

1. There is no encirclement of -1 point. This implies that the system is stable if there are no poles of G(s)H(s) in the right half s plane; otherwise the system is unstable

2. There are one or more counterclockwise encirclements of the -1 point. In this case the system is stable if the number of counterclockwise encirclements in the same as the number of poles of G(s)H(s) in the right half s plane; otherwise the system is unstable

3. There are one or more clockwise encirclements of the -1 point. In this case the system is unstable

Consider a closed loop system whose open loop transfer function is given by

$$G(s)H(s) = \frac{K}{(T_1 s + 1)(T_2 s + 1)}$$

Examine the stability of the system: Since G(s)H(s) does not have any poles in the right half s plane and the -1 point is not encircled by the G(jw)H(jw) locus this system is stable for any positive values of K, T1 and T2

Consider the system with the following open loop transfer function:

$$G(s)H(s) = \frac{K}{s(T_1 s + 1)(T_2 s + 1)}$$

Determine the stability of the system for two cases: 1) the gain K is smaller and 2) the K is large.

The number of poles of G(s)H(s) in the right half s plane is zero

For small values of K there is no encirclement of the -1 point. Hence the system is stable for small values of K. For large values of K, the locus of G(s)H(s) encircles the -1 point twice in the clockwise direction, indicating two closed loop poles in the right half s plane, and the system is unstable. (For good accuracy K should be large)

The stability of a closed loop system with the following open loop transfer function depends on the relative magnitudes of T1 and T2.

$$G(s)H(s) = \frac{K(T_2 s + 1)}{s^2 (T_1 s + 1)}$$

Plots of the root locus G(s)H(s) for three cases T1<T2, T1=T2, and T1>T2. For T1<T2 the locus of G(s)H(s) does not encircle the -1 point and the closed loop system is stable. For T1=T2 the G(s)H(s) locus passes through the –1 point which indicates that closed loop poles are located on the jw axis. For T1>T2 the locus of G(s)H(s) encircles the -1 point twice in the clockwise direction. The close loop system has two closed loop poles in the right half s plane and the system is unstable.