ENE 206 – Fluid Mechanics

WEEK 3

- **Pressure measurement**

➢ Scales of pressure measurement:

i) Pressure is always increased with respect to same arbitrary reference

ii) These reference levels are a) zero pressure (vacuum), b) Local atmospheric pressure

When the pressure is increased with respect to absolute vacuum then it is called "**absolute pressure**". When it is measure with respect to a local atmospheric pressure, it is called "**gage pressure**". Their relation is schematically shown in Figure 2.2.

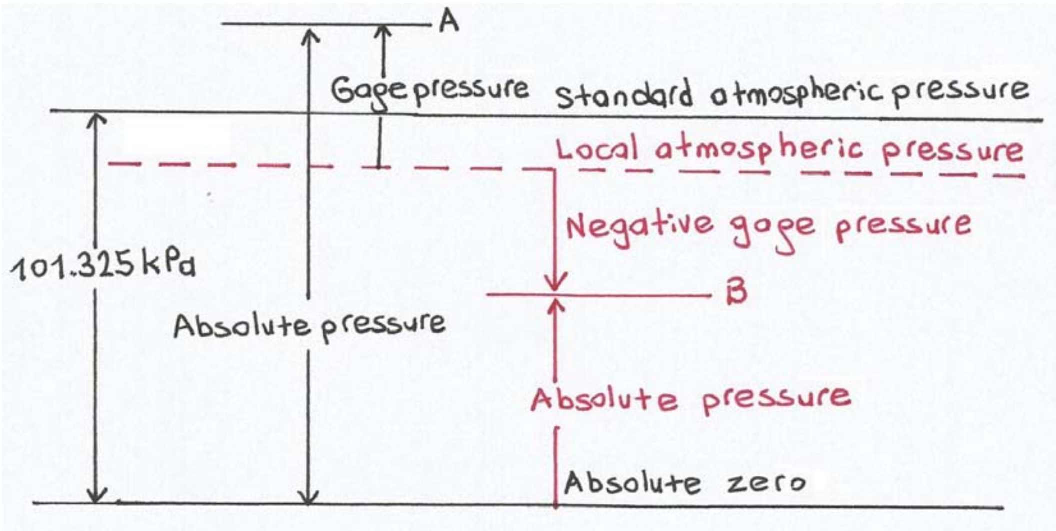


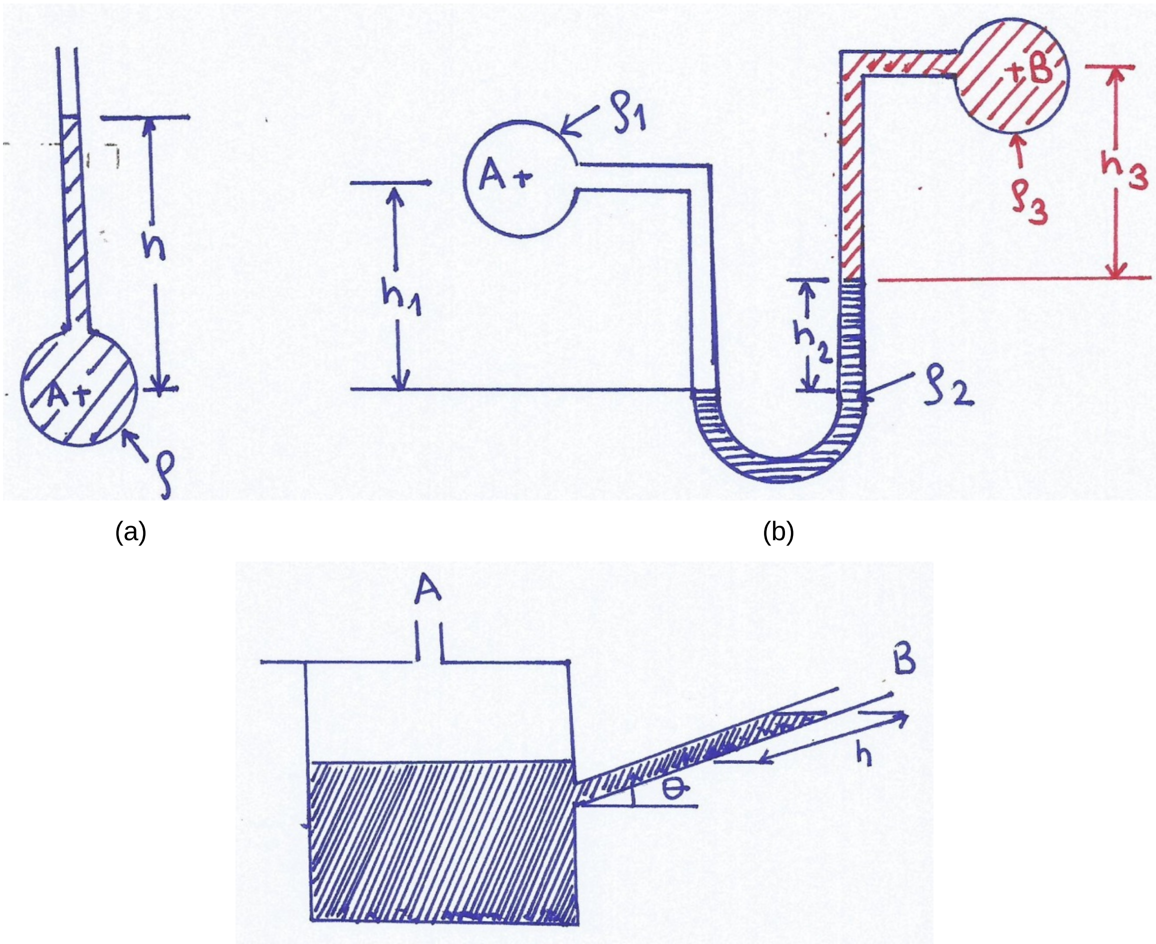
Figure 2.2. Pressure measurement

The local atmospheric pressure can be measured by means of a mercury barometer invented by Toricelli in 1643.

➢ **Pressure measurement:** It can be carried out by:

i) Mechanical pressure gages: The Aneroid barometer and the Bourdon pressure gage are in this class.

ii) Manometer: It is a device which employs liquid columns for determining pressure differences. It is classified in three groups a) Simple manometers: usually called piezometer. It is used for measuring the pressure of a liquid when it is greater than the local atmospheric pressure; b) Differential manometers in which the pressure difference between two points are measured: c) Inclined manometers: They are used for measuring small differences in gas pressures. Different manometers are shown in Figure 2.3.



(a) (b)

(c)

Figure 2.3. Examples of manometers: a) A piezometer; b) A differential manometer; c) An inclined manometer

For analyzing complex manometer problems, following rules of thumb must kept in mind:

i) Any two points within the same fluid having the same elevation poses the same pressure;

ii) Pressure increases as one goes down.

- **Capillarity:** It is defined as the tendency of a liquid in a capillary tube or absorbent material to rise or fall as a result of surface tension. When a glass tube is inserted in a wetting liquid then adhesive forces exceed the cohesive forces as a result, water rises in the tube. When the working fluid is mercury than adhesive forces are less than the cohesive forces so that the mercury is depressed.

Static forces acting on submerged plane surfaces

To specify a resultant pressure force, $\vec{F}_R$ one must determine it's a) magnitude; b) Direction and c) Line of action. The plane surface submerged in a static fluid with an inclination angle, β lies in the xy plane as shown in Figure 2.4. The static forces acting on the submerged plane is shown in the figure. Following below mathematical expressions are derived to determine the resultant pressure force, $\vec{F}_R$, its point of application known as **pressure center** and its coordinates, x_p and y_p:

$$\vec{F}_R = \int d\vec{F} = -\int_A p dA \hat{k} \tag{2.10}$$

$$x_p = \frac{1}{F_R}\int_A P x dA \tag{2.11}$$

$$y_p = \frac{1}{F_R}\int_A P y dA \tag{2.12}$$

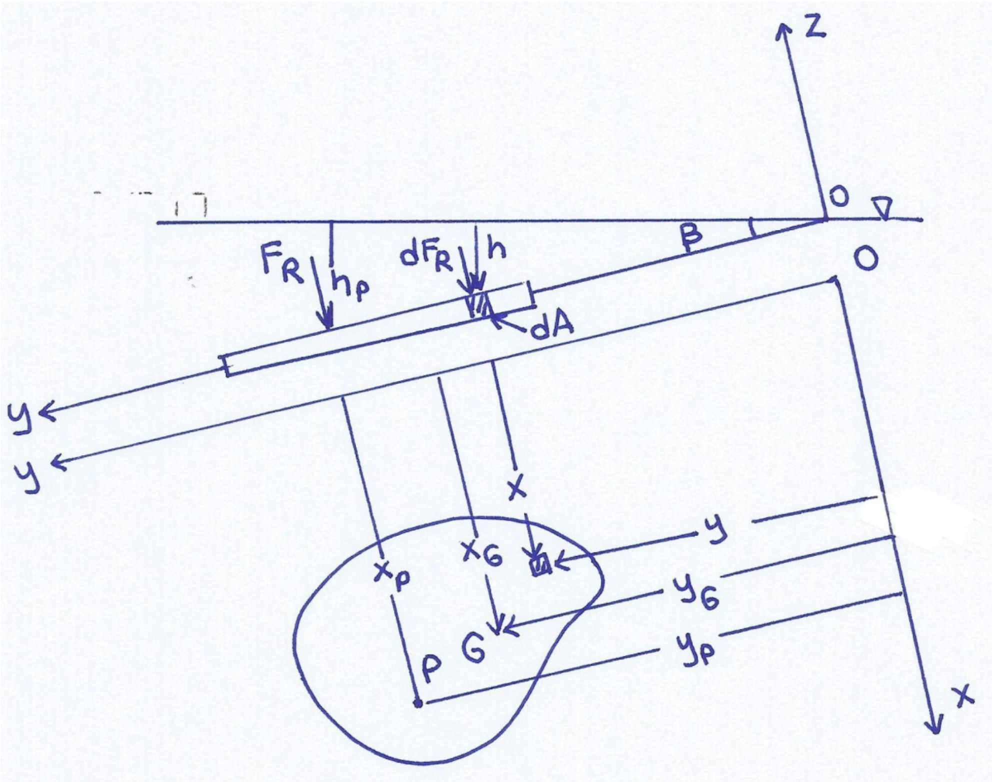


Figure 2.4. Pressure forces acting on a submerged flat plate

There are several methods for the calculation of the linearly increasing pressure:

- **Direct integration methods:** The relevant formulas as given as below:

$$\vec{F}_R = -\int_A p dA \hat{k} = -\int_A \rho g h dA \hat{k} \tag{2.13}$$

From the geometry, $x = y Sin\beta$ and, the coordinates of pressure center can be expressed as:

$$x_p = \frac{\rho g Sin\beta}{F_R}\int_A x y dA \tag{2.14}$$

$$y_p = \frac{\rho g Sin\beta}{F_R}\int_A y^2 dA \tag{2.15}$$

- **Formula method:** The coordinates for the centroid of the inclined surface is first given as :

$$x_G = \frac{1}{A}\int_A x dA \tag{2.16}$$

$$y_G = \frac{1}{A}\int_A y dA \tag{2.17}$$

By recalling Eqn. 2.13

$\vec{F}_R = -\int_A \rho g h dA\hat{k} = -\int \rho g y Sin\beta \hat{k}$ where $h = y Sin\beta$

the resultant force is determined as:

$\vec{F}_R = -\rho g Sin\beta \hat{k}\int_A y dA = -\rho g Sin\beta y_G A\hat{k}$ or

$$\vec{F}_R = -\rho g h_G A\hat{k} \tag{2.18}$$

Then the x-coordinates of the pressure center is obtained as:

$$x_p = \frac{\rho g \rho Sin\beta}{\rho g Sin\beta}\frac{I_{xy}}{Ay_G} = \frac{I_{xy}}{A_{y_G}} \tag{2.19}$$

where the product of inertia with respect to the *x* and *y* axes, I_{xy} of the submerged plane is defined as

$$I_{xy} = \int_A xy dA \tag{2.20}$$

and the y-coordinates of the pressure center is obtained as

$$y_p = \frac{\rho g \rho Sin\beta}{\rho g Sin\beta}\frac{I_{xx}}{Ay_G} = \frac{I_{xx}}{A_{y_G}} \tag{2.21}$$

where the second moment of the area inertia with respect to the *x* axis, I_{xx} of the submerged plane is defined as

$$I_{xx} = \int_A y^2 dA \tag{2.22}$$

- **Pressure prism method:** If the constant pressure on the surface is not considered, then an imaginary trapezoidal pressure distribution is generated over the submerged, inclined surface as shown in Figure 2.5 to calculate the resultant pressure force and its point of application. This imaginary volume is referred to as the **pressure prism.** The magnitude of the resultant pressure force, $\vec{F}_R$ using the pressure prism is determined as:

$$\vec{F}_R = \int_{V_p} dV_p \hat{k} = V_p \hat{k} \tag{2.23}$$

where $V_p = \int_A p dA = \int_A \rho g h dA = \int_{V_p} dV_p$

The line of action of the resultant pressure force can be also obtained as:

$$x_p = \frac{1}{F_R}\int_A P x dA = \frac{1}{V_p}\int_A \rho g h x dA = \frac{1}{V_p}\int_{V_p} x dV_p = x_G \tag{2.24}$$

$$y_p = \frac{1}{F_R}\int_A P y dA = \frac{1}{V_p}\int_A \rho g h y dA = \frac{1}{V_p}\int_{V_p} y dV_p = y_G \qquad (2.25)$$

where x_G and y_G are the coordinates of the centroid of the pressure prism.

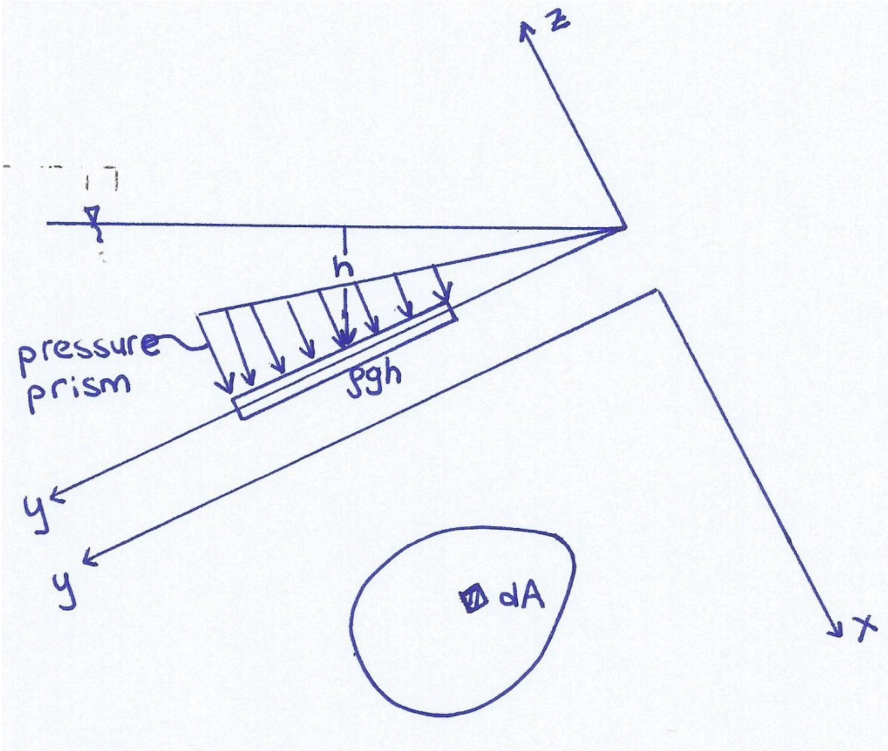


Figure 2.5. Pressure prism

- **Static forces acting on submerged curved surfaces**

  Consider the curved surface in the x-z plane such that there is no change in the y direction (Figure 2.6).

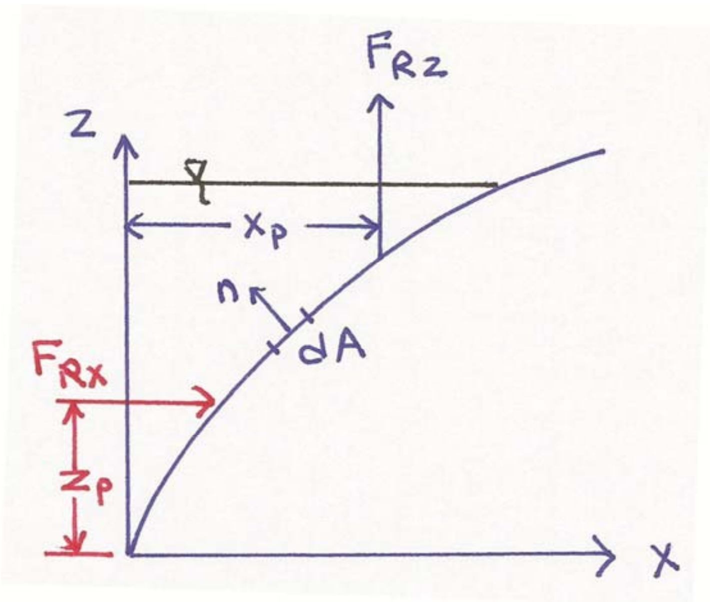


Figure 2.6. Pressure force pairs on a submerged curved surface

The components of the resultant pressure force in the x and z directions are calculated separately as:

$$F_{Rx} = \int dF_{Rx} = \int_{A_x} p dA_x \qquad (2.26)$$

and

$$F_{Rz} = \int dF_{Rz} = \int_{A_z} p dA_z \qquad (2.27)$$

In the above equations, dA_x and dA_z are the projected area of dA in the yz and xz planes, respectively.

The line of action of the resultant pressure force is then determined in components as below:

$$x_p = \frac{1}{F_{Rz}} \int_{A_z} p x dA_z \qquad (2.28)$$

$$z_p = \frac{1}{F_{Rx}} \int_{A_z} p z dA_x \qquad (2.29)$$

The line of action of the resultant pressure force components on the curved surface do not coincide and no single resultant force is found.

- ## Buoyancy

It should be noted here that the net horizontal force acting on a submerged body in a static fluid is zero. The net vertical pressure force or the buoyant force on a submerged body is equal to the weight of the fluid displaced by the submerged body (Archimedes principle).

- ## References

1. Aksel, M.H., 2016, “Notes on Fluids Mechanics”, Vol. 1, METU Publications

2. Fox, R.W. and McDonald, A.T., 1994, “Introduction to Fluid Mechanics”, 4th Edition, John Wiley and Sons, Inc., New York

3. Prandtl, L., and Tietjens, O.G., 1957, “Fundamental of Hydro and Aero-mechanics”, Dover Publications, Inc., New York

4. Roberson, J.A. and Crowe, T.C., 1957, “Fundamentals of Hydro and Aero-mechanics”, Dover Publications, Inc., New York

5. Shames, I.H., 1982, “Mechanics of Fluids”, 2md Edition, McGraw Hill Book Company, New York

6. Street, R.L, WATTERS; G.Z. and Vennard, J.K., 1996, “Elementary Fluid Mechanics”, 7th Edition, John Wiley and Sons, Inc., New York

7. Vennard, J.K. and Street, R.L., 1982, “Elementary Fluid Mechanics”, 7th Edition, John Wiley and Sons, Inc., New York