Graph Papers

a) **Arithmetic graph paper:**

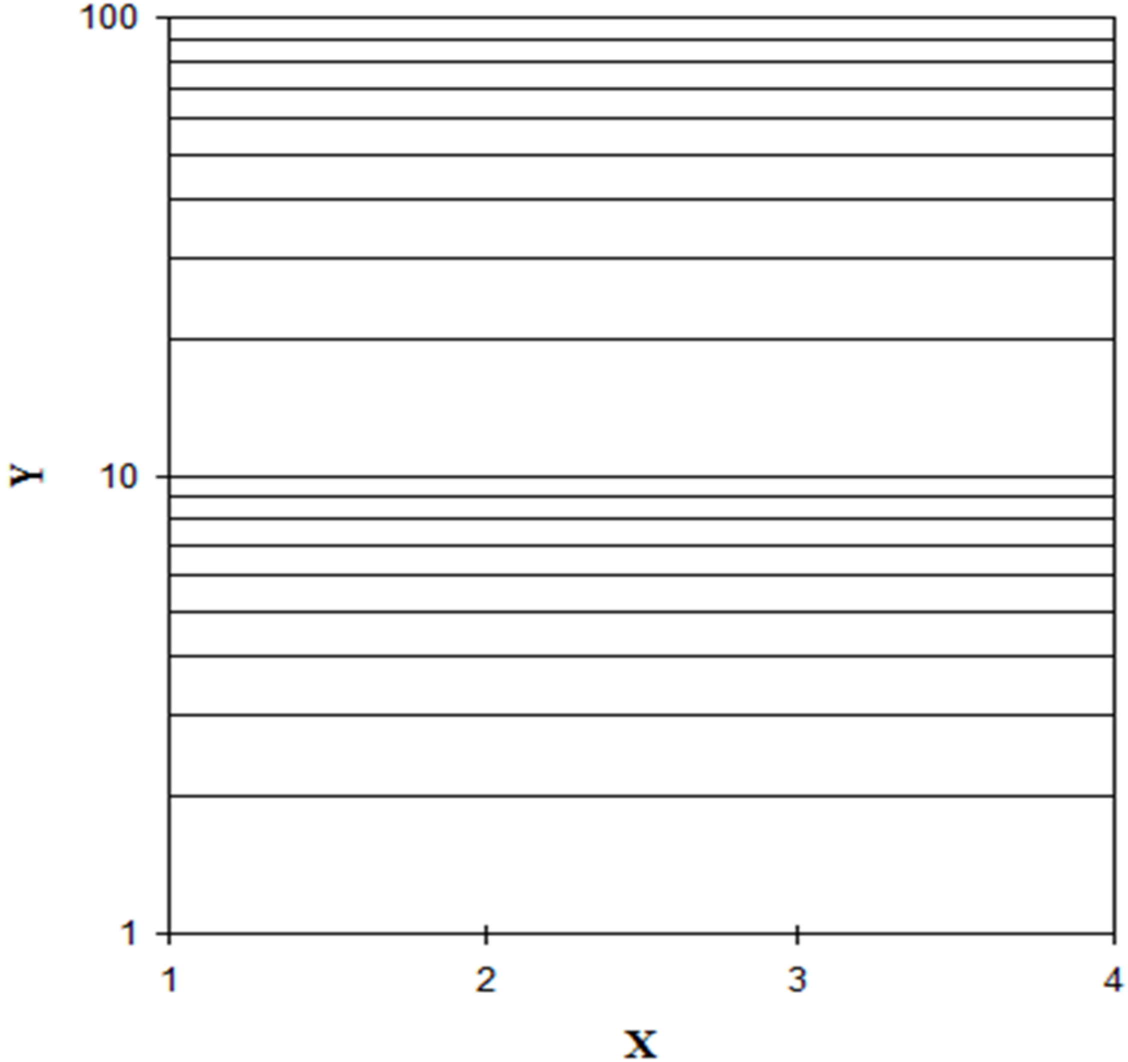
x: arithmetic scale
y: arithmetic scale

b) **Semi-logarithmic graph paper:**

x: arithmetic scale
y: logarthmic scale

c) **Full-logarithmic graph paper:**

x: logarthmic scale
y: logarthmic scale.



100
10
1
Y
1
2
3
4
X

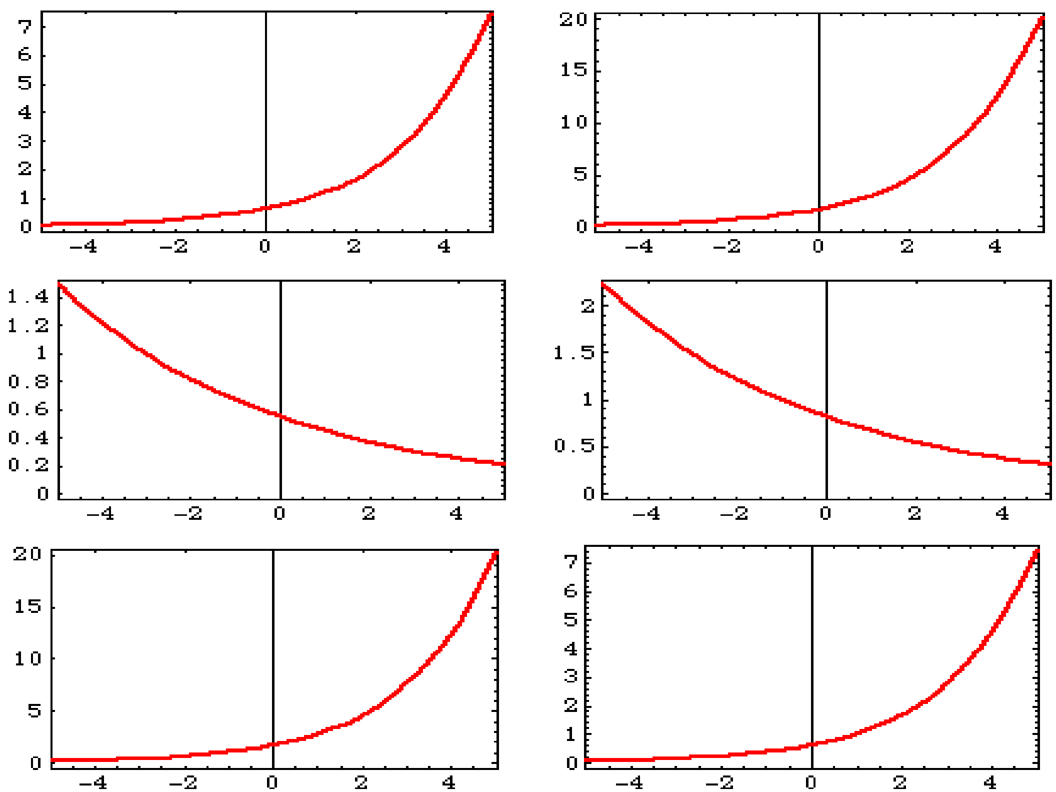
Why to use logarithmic papers?

- To use the **original** experimental data.
- **No need** for **any transformation** of experimental data.

- **Semi-logarithmic** graphs are used for **exponential functions** ($y = b\ a^{x}$).

- **Full logarithmic** graphs are used for **geometric functions** ($y = b\ x^{a}$).

Exponential graphs

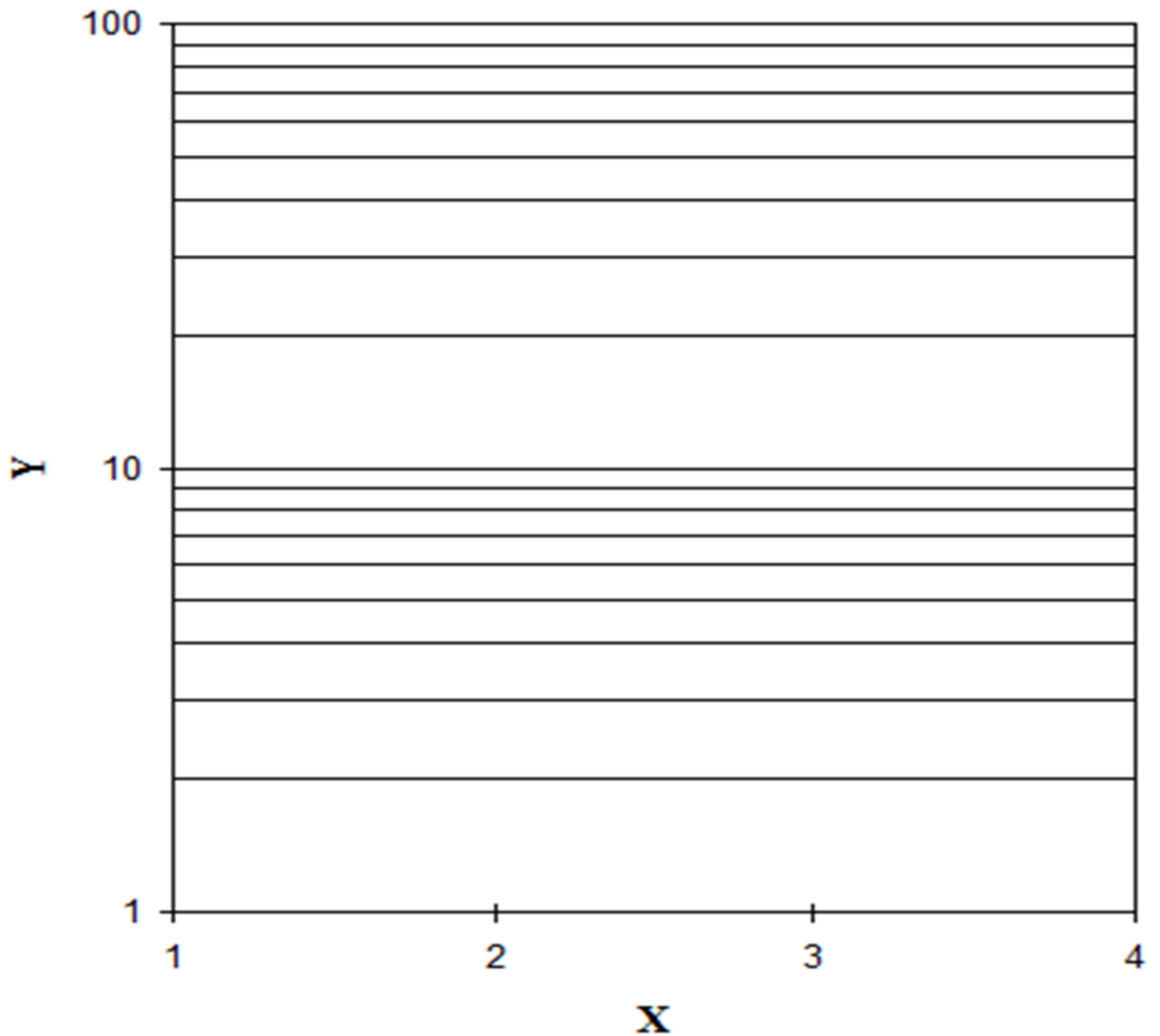


On the base of 10 of a logarithmic graphs:

- **logarithmic cycle** increases by **10** (0.001, 0.01, 0.1, 1, 10, 100, 1000 etc.).

- Each **cycle** of logarithmic scale is marked by numbers from 1 to 10.
- Distance of data from origin on log scale is determined from the following equation.

(log x) (length of one log cycle) = distance of data from origin


100
10
1
Y
1
2
3
4
X

Example 2. 8

- What would be the logarithmic cycles if the experimental data are between 0.025 and 3.02?

- What would be the logarithmic cycles if the experimental data are between 1.2 and 19.7?

Construction of logarithmic paper

- Logarithmic graph paper can be constructed using logarithmic principles.

- Following steps are used to form one logarithmic cycle.
  - The length of axis is divided in equal parts for the cycles required. For example, if the number of log cycles is 3 and axis length is 15 cm, then each log cycle should be …. cm.

- Depending on experimental values, first cycle starts with 0.01, 0.1, 1 or 10 (multiple of 10, «10’nun katları»), and ends with value of multiple of 10. For example, if the cycle starts with 0.1, it ends with 1.0.

- Intermediate values (from 1 to 10) for each cycle are determined by the equation.

Construction of log paper

- (log x) x (length of one log cycle) = distance of the data from origin

if “1 log cycle = 8.4 cm”:

- log 1 x 8.4 cm = 0 cm (dist. from origin)
- log 2 x 8.4 cm = 2.5 cm
- log 3 x 8.4 cm = 4.0 cm
- log 4 x 8.4 cm = 5.1 cm
- log 5 x 8.4 cm = 5.9 cm
- log 6 x 8.4 cm = 6.5 cm
- log 7 x 8.4 cm = 7.1 cm
- log 8 x 8.4 cm = 7.6 cm
- log 9 x 8.4 cm = 8.0 cm
- log 10 x 8.4 cm=8.4 cm

- ....
- log 20 x 8.4 cm = 10.9 cm
- .
- ..
- log 100 x 8.4 cm = 16.8 cm

Figure 2.9 Two-cycle semi-log graph paper

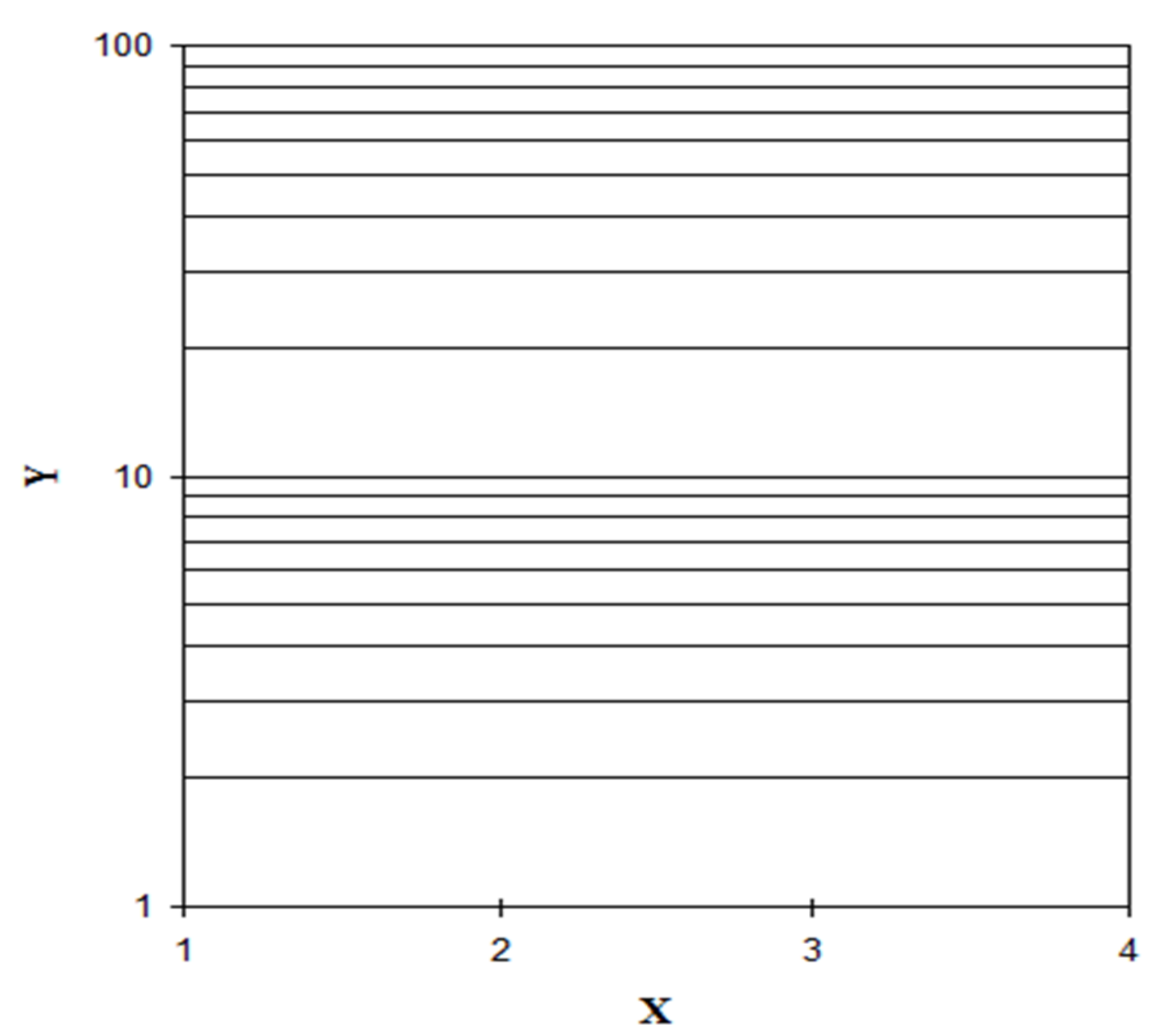
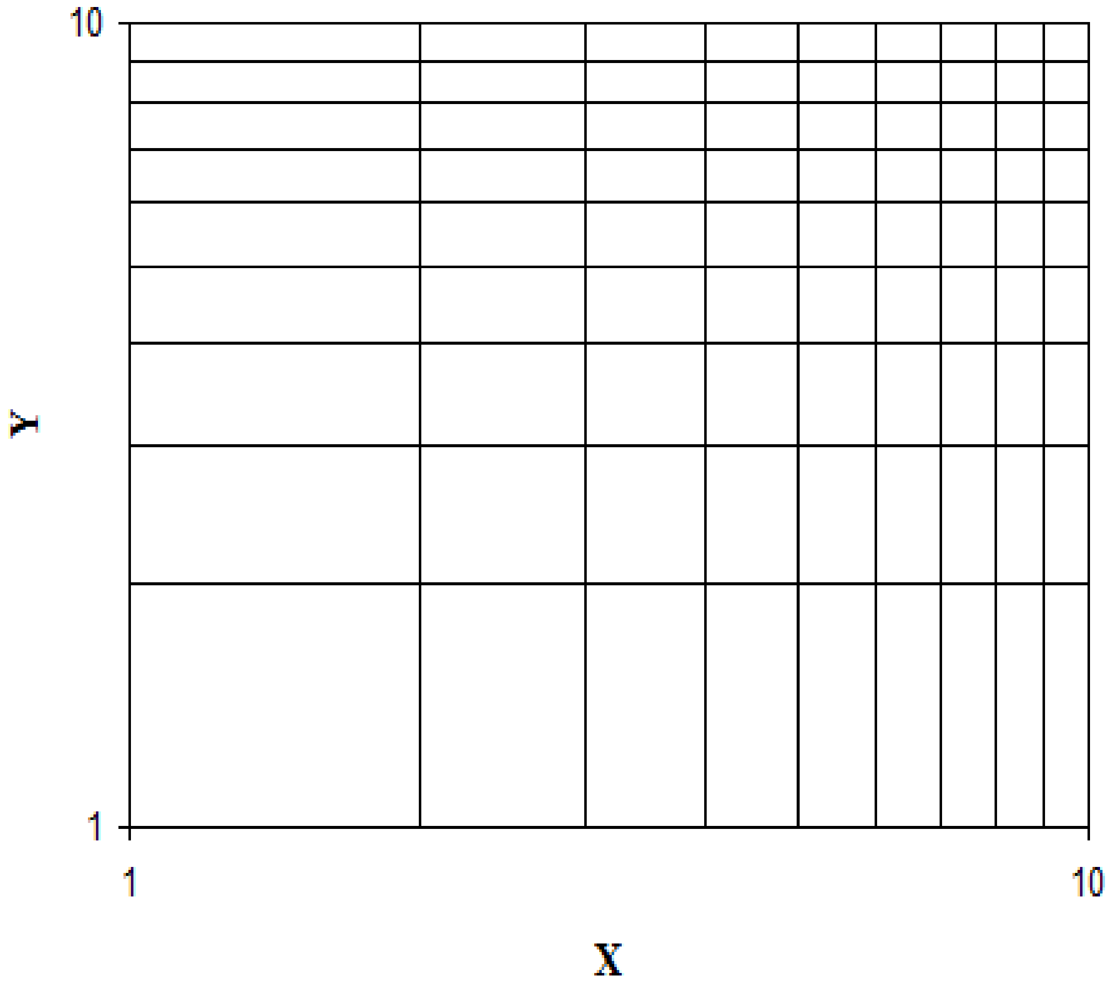


Figure 2.10 One-cycle log-log graph paper



There are two common logarithm systems in use

- A base of 10 is called as **common logarithm**:
  - $\log_{10}$ is commonly written as «log»

- a base of «e» (e=2.718) is called as **natural logarithm**:
  - $\log_e$ is commonly written as «ln»

- The two systems are interchangeable by using the factor 2.303:

$$\ln x = 2.303 \log x$$

- The logarithm of a product is the sum of the logarithms of the values.

$$\log AB = \log A + \log B$$

- The logarithm of a quotient (ratio) is the difference between the logarithms of the values.

$$\log A/B = \log A - \log B$$

Slope values in log papers

- Slope = a = $\dfrac{\log y_2 - \log y_1}{x_2 - x_1}$ (semi-log)

- Slope = a = $\dfrac{\log y_2 - \log y_1}{\log x_2 - \log x_1}$ (full-log)

- **<u>In these equations, “x" and "y" values are the original experimental data.</u>**

y-intercept (b) in log papers

- In **semi-log graphs**,
  - Found directly from the graph,
  - No need for any transformation.
  - **To find y-intercept, x axis should start from ….**

- In **full-log graphs**,
  - Found directly from the graph,
  - No need for any transformation of data,
  - **To find y-intercept, x axis should start from ….**

Arithmetic graphs can be used in plotting «log» values

- logarithms are taken and plotted in arithmetic graph.
- **<u>Slope</u>:** calculated from the following equation:

$$\text{Slope} = a = \frac{y_2 - y_1}{x_2 - x_1}$$

<u>y-intercept (b)</u>: Calculated by taking anti-log of intercept value found in graph.

Example 2.10

Intercept value was found as 3.7 from arithmetic graph constructed from logarithms of original experimental value. Find real intercept value.

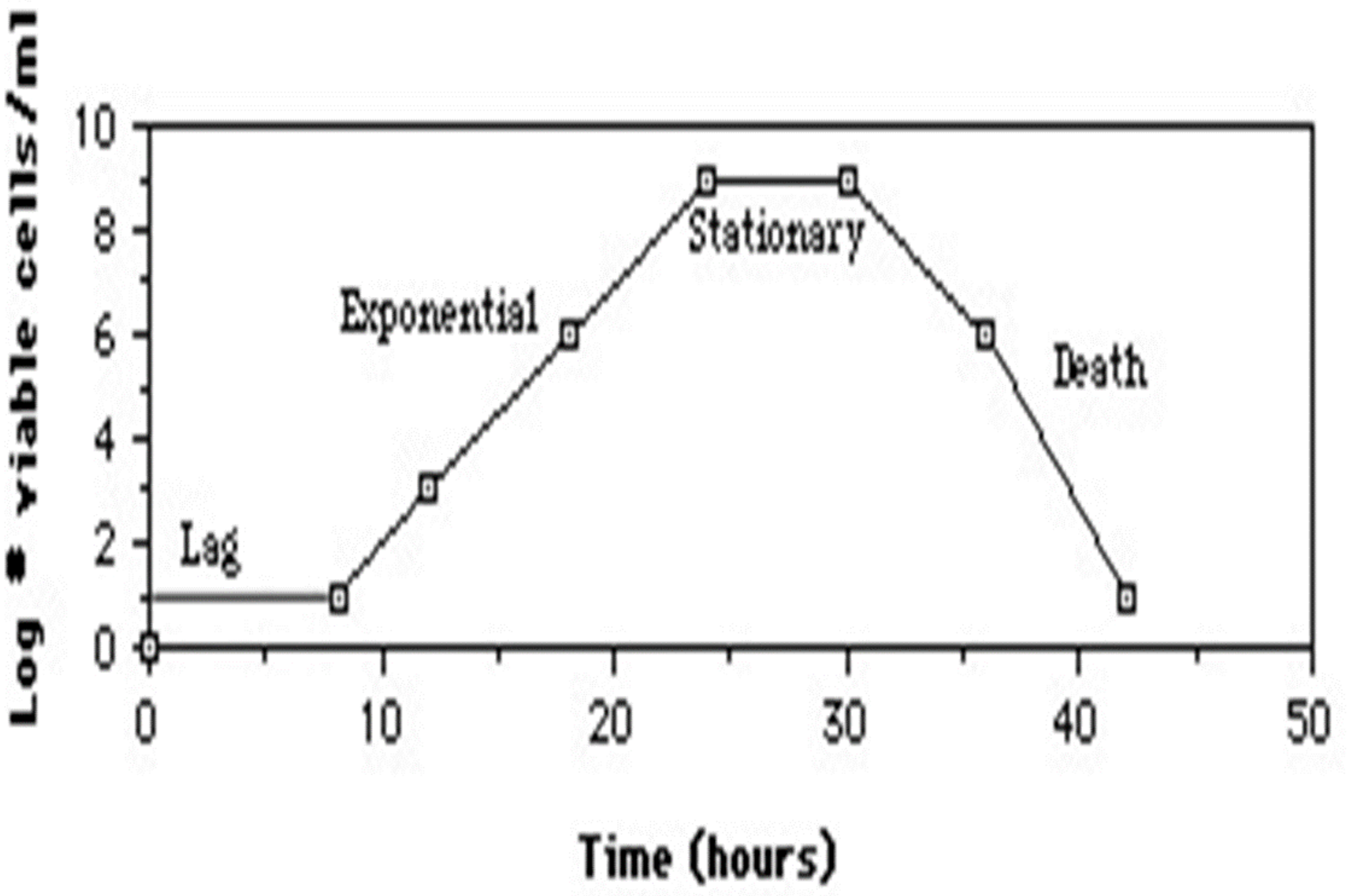
Plotting experimental data on semi-log graph paper

The following examples illustrate (show) the use of semilog graphs.

Example 2.11

The generation time (g) of m.o.'s (time needed for doubling of m.o.'s) is an index of the rate of growth. It is defined as the average time between two consecutive generations of m.o. In logarithmic phase of microbial growth, the number of viable m.o.'s (N) changes with time of growth (t) according to the following equation:

Bacterial growth curve



$$N = N_o [2]^{t/g}$$

Where;

N_o : Number of viable m.o. at the beginning of a time interval,

N : Number of viable m.o. at the end of the time interval,

t : Time interval (time between N_o and N)

g : Generation time (time between two consecutive generations).

Table 2.13 Numbers of bacteria as a function of time of growth

Numbers (N)	Time of growth (min)
980	0
1700	10
4000	30
6200	40

a) Find out the generation time of this bacteria by graphic method using «freehand method of curve fitting.»

b) Calculate the generation time of this bacteria by linear regression method. Then, draw the regression line.

Homework

- Calculate the R^2 for this equation

  (Answer: R^2= 0.9989)

Solution by graphical method

- First, linearize the equation.

$$N = N_o\,[2]^{t/g}$$

- What is «x» and what is «y»?

- According to this “linearized” equation, if **number of viable m.o’s (log y)** is **plotted against growth time (x)** in a semi-log graph paper, straight line is obtained.

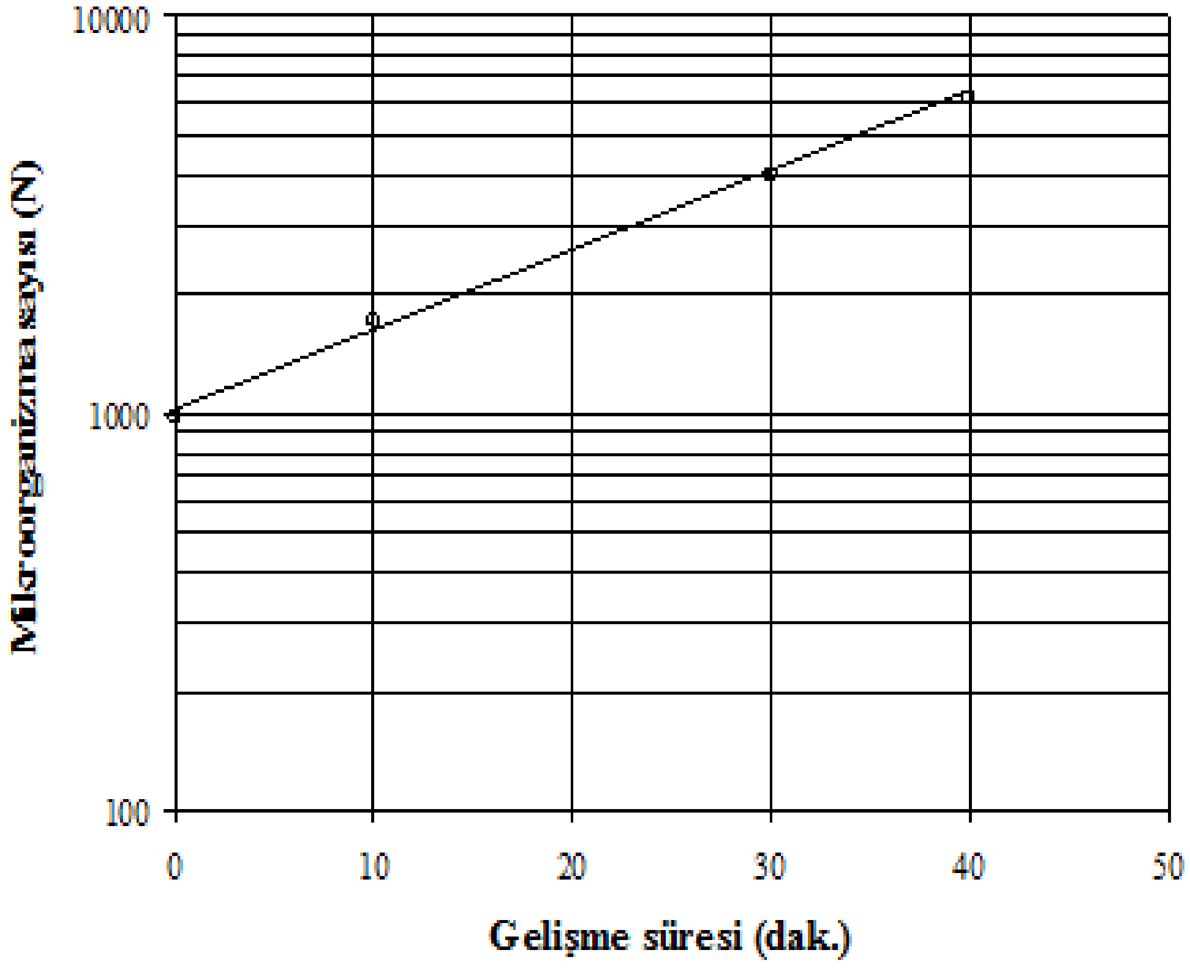
- **Slope (a)** = $\frac{\log [2]}{g}$

- **Intercept (b)** = $\log N_o$

- Original experimental data are plotted on a semi-log graph paper without any transformation (Figure 2.11).

- Straight line reflecting growth of m.o's is drawn by **freehand method of curve fitting.**

Figure 2.11 Growth curve of m.o's



Calculation of slope and generation time

- Choose **two points** on the straight line (x_1=5 and x_2=35) and find out the corresponding «y» values.

- Calculate the **slope**.

- Find out **generation time**.

Intercept and the equation

- Find the **intercept value**.

- Write out the quation defining the straight line:

$$\log y = 0.0203\,(x) + \log 1020$$

Solution by regression analysis

- Since straight line drawn by **<u>freehand method of curve fitting</u>** changes from person to person, slope and intercept values of these lines will also be different.

- To prevent personal differences, regression analysis is applied to experimental data.

$$\mathbf{a} = \frac{\Sigma\, xy - (\Sigma x\, \Sigma y / n)}{\Sigma\, x^2 - [(\Sigma x)^2 / n)]}$$

$$\mathbf{b} = \frac{\Sigma\, y\, \Sigma\, x^2 - \Sigma x\, \Sigma xy}{n\, (\Sigma\, x^2 - [(\Sigma x)^2 / n)])}$$

Before calculation of «a» and «b» values, we need to take into consideration of two important points:

- Since regression equations are aritmetic and our «**y**» values are lograitmic, then we need to ………..
- The calculated intercept value (b) should be equal to ……….

$$\log N = \frac{\log 2}{g}(t) + \log No$$

Table 2.15 Data for regression analysis

X	log Y	X^2	$(\log Y)^2$	X logY
0	2.991			
10	3.230			
30	3.602			
40	3.792			
$\Sigma X =$	$\Sigma Y =$	$\Sigma X^2 =$	$\Sigma Y^2 =$	$\Sigma XY =$

Table Data for regression analysis

X	log Y	X^2	$(\log Y)^2$	X logY
0	2.991	0	8.946	0
10	3.230	100	10.433	32.30
30	3.602	900	12.974	108.06
40	3.792	1600	14.379	151.68
$\Sigma X = 80$	**$\Sigma Y = 13.615$**	**$\Sigma X^2 = 2600$**	**$\Sigma Y^2 = 46.732$**	**$\Sigma XY = 292.04$**

Calculation of slope and intercept

- a = 0.01974 min^{-1}

- log b = 3.0090 (shift log 3.0090 =)

  or,

  b = $10^{3.0090}$ (10 ^ 3.0090 =)

  b = 1021 number mL^{-1}

Equation based on common logarithm ($\underline{\log_{10}}$)

$$\log y = 0.01974\ (t) + \log 1021$$

or;

$$\log y = 0.01974\ (t) + 3.0090$$

Calculation of generation time

$$\log N = \frac{\log 2}{g}(t) + \log N_o$$

- **Generation time (g)** is calculeted from **slope**.

$$\text{slope} = \frac{\log [2]}{g}$$

g = 15.25 min

Drawing regression line

Place **lowest** and **highest** «time» values (**x**) in regression equation to find the "**corrected y**" values.

$$\log y = 0.01974\ (t) + 3.0090$$

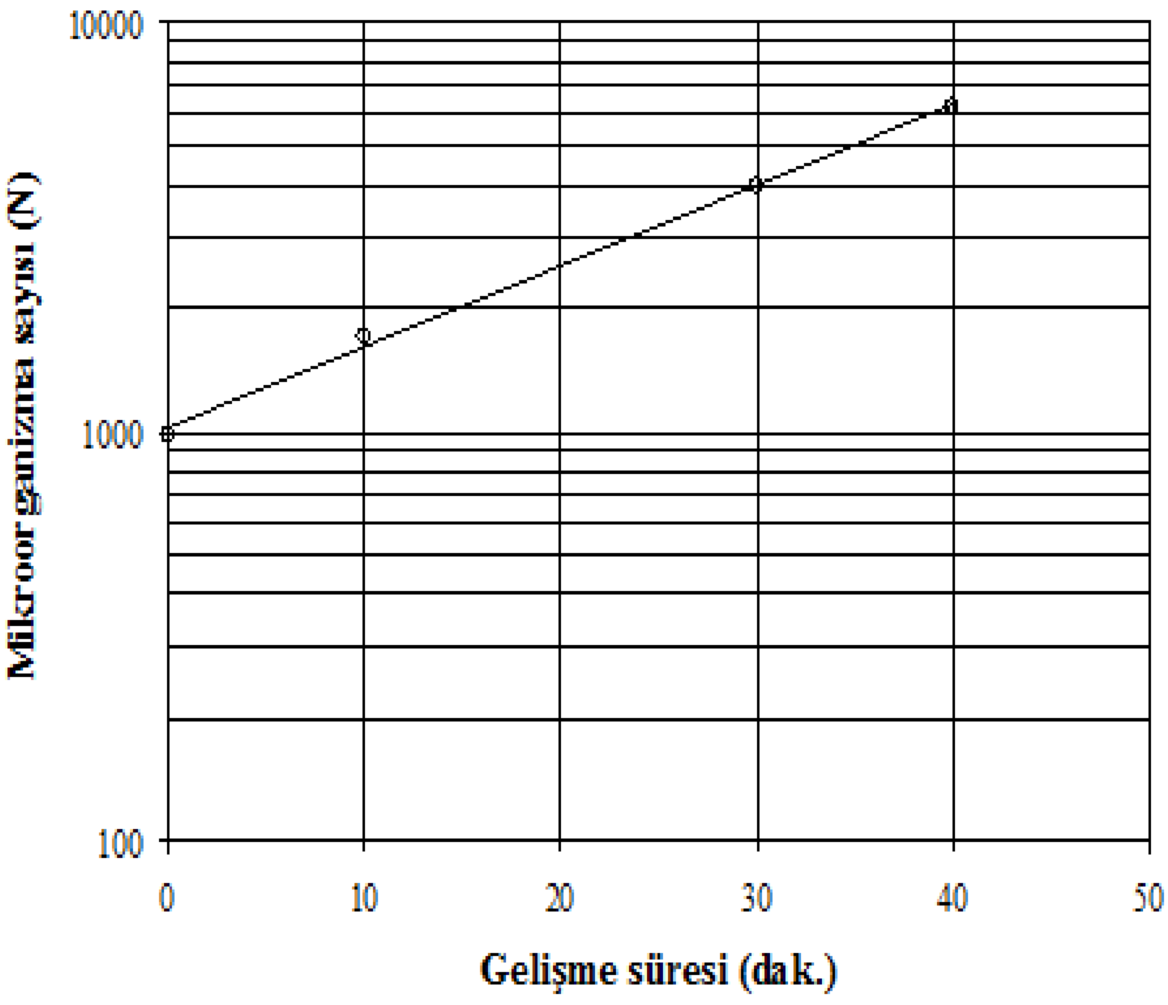
$$\log y = 0.01974\ (t) + 3.0090$$

$$x_1 = 0 \rightarrow \mathbf{y_1 = 1021}$$

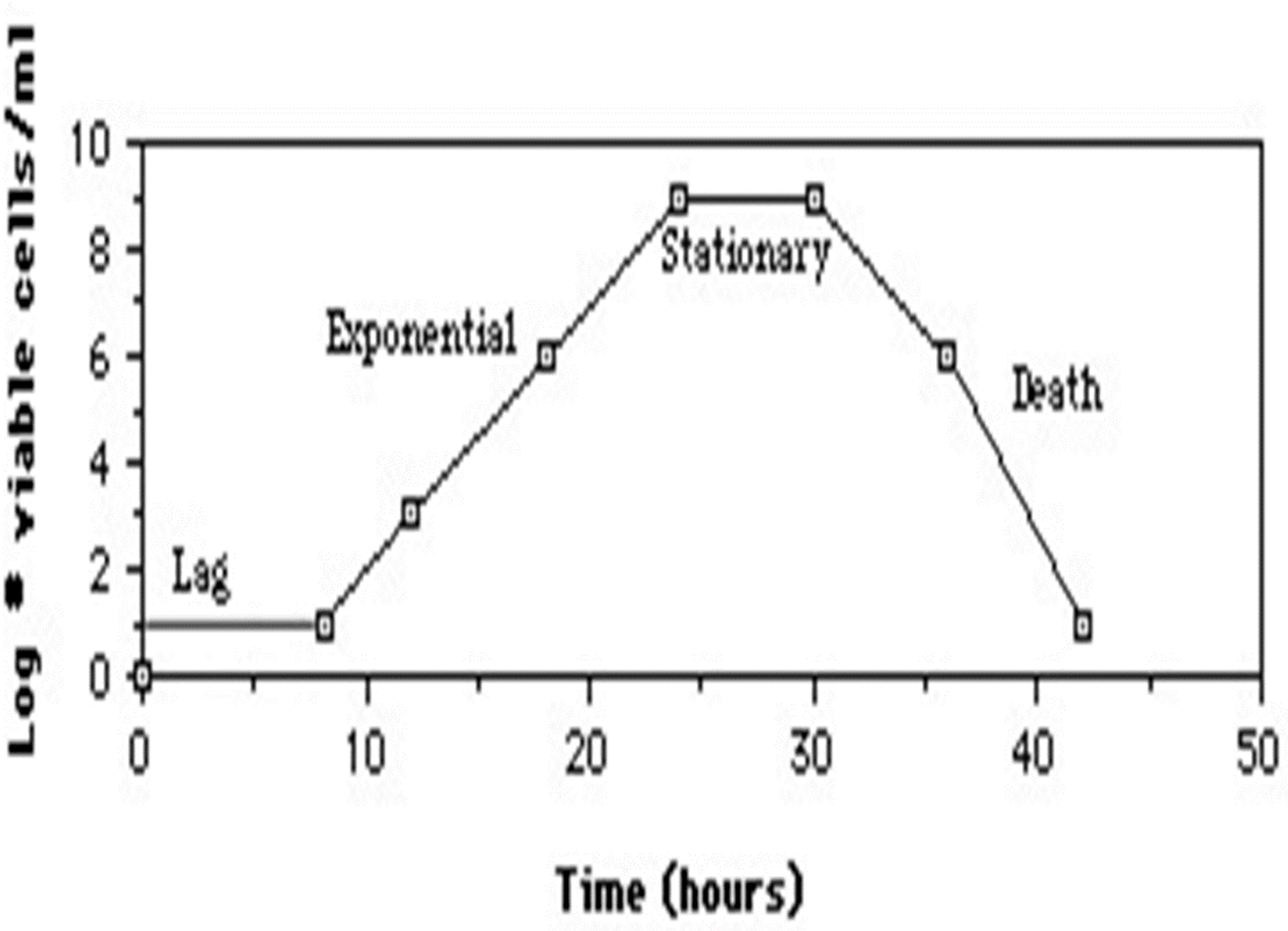
$$x_4 = 40 \rightarrow \mathbf{y_4 = 6289}$$

- Place the following two data points on the graph: **(0, 1021)** and **(40, 6289).**
- Then, connect these two points to draw the **regression line**.

Figure 2.12 Growth curve of m.o.s



Bacterial growth curve



Lag phase

- Cells grow in volume or mass, synthesizing enzymes, proteins etc.,
- No apparent cell division occurs,
- The length of the lag phase is **usually 1 h**.

Exponential (log) phase

- All cells divide regularly by geometric progression.
- Plenty of nutrients.
- Cells doubles their number each generation time (**usually 20 min**).

Stationary phase

- Population growth is limited by one of three factors:
  - exhaustion of nutrients,
  - accumulation of inhibitory metabolites
  - exhaustion of space,

- During this phase, cell population stops growing and dividing.

Death phase

- Number of viable cells decreases exponentially,
- No more nutrients,
- A lot of metabolites present.

Example 2.12

What is the generation time of a bacterial population that increases from 10,000 cells to 10,000,000 cells during growth of this bacteria in Figure 2.13?

$$N = N_o [2]^{t/g}$$

Where;

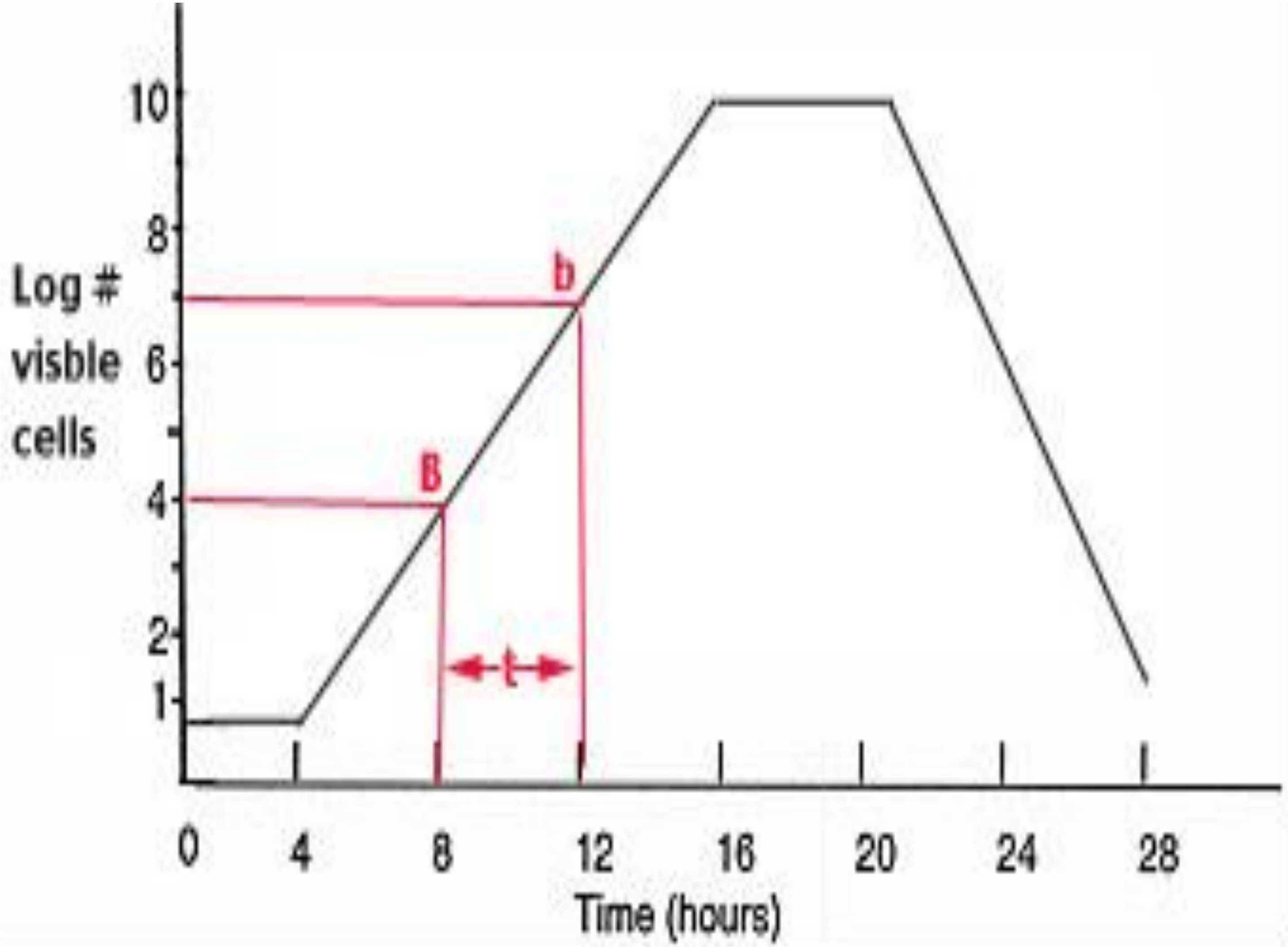
N_o : Number of m.o. at the beginning of a time interval,

N : Number of m.o. at the end of the time interval,

t : Time interval (time between N_o and N)

g : Generation time (time between two consecutive generations).

Figure 2.13 Bacterial growth curve



Answer

$$\log N = \frac{\log [2]}{g} t + \log N_o$$

$$\log N = \frac{\log [2]}{g}\, t + \log N_o$$

$$7 = \frac{\log [2]}{g}\, 240 \text{ min} + 4$$

g = 24 min

Example 2.13 (homework)

Degradation of anthocyanins in sour cherry juice heated at 80°C was studied and experimental data was given in Table 2.14.

Note: Anthocyanin degradation during heating gives staright line in a semi-log graphic paper.

a) Calculate the **the regression coefficients** and **R^2** with units (if applicable),

b) Draw the regression line,

c) How much of anthocyanins (%) would be degraded after 25 h of heating at 70^o and 80^{o}C.

Table 2.14 Anthocyanin contents of sour cherry juice heated at 80°C for various time.

Time (h)	Anthocyanin content (mg/L)	Time (h)	Anthocyanin content (mg/L)
0	160	12	60
2	136	14	55
4	115	16	45
6	98	18	37
8	90	20	33
10	67	–	–

Answers

a= –0.0348 units?

b= 2.207 units?

R^2 =0.9960 units?

***Interpretation*:** Very close R^2 value shows that experimental data excellently fit straight line.

Regression equation???

- ??????????????

Regression equation

$$\log y = -0.0348\,(t) + 2.207$$

At 70^{o}C???

At 80^{o}C???

$$\log y = -0.0348\,(t) + 2.207$$

At 80°C???

$$\log y = -0.0348\,(t) + 2.207$$

86.42% of anthocyanins was degraded after 25 h of heating at 80°C

Example 2.14

The term half-life is an index used to express the stability of a compound and is **defined as the time required for the concentration to drop to half the original value.** The equation for half-life value is:

$$C = C_o\ [2]^{-t/t1/2}$$

where;

C_o: Concentration at the beginning of a time interval,

C: Concentration at the end of a time interval,

t: Time interval,

$t_{1/2}$: Half life period.

If orange juice packaged aseptically initially contained 60 mg ascorbic acid/100 mL, what should be declared on the label so that after 10 weeks the declared level is at least 90% of the actual value. The half-life of ascorbic acid is known to be 30 weeks.

How much of aa (in %) would be degraded after 24 days of storage?

Solution

The question can be both solved with **equation** and **graph**.

Solution by equation

- First, organize the equation.

……………………………………

Solution by equation

$$C = C_o [2]^{-t / t1/2}$$

$$\log C = \log C_o + -\frac{t}{t_{1/2}} \log [2]$$

$$\log C = -\frac{\log [2]}{t_{1/2}} (t) + \log C_o$$

$$\updownarrow \qquad \updownarrow \qquad \updownarrow \qquad \updownarrow$$

$$y = a \quad (x) + b$$

Solution by equation

$$\log C = -\frac{\log[2]}{t_{1/2}}(t) + \log C_o$$

$t_{1/2} = 30$ weeks,

$C_o = 60$ mg 100 mL^{-1}

$t = 10$ weeks

After 10 weeks of storage, aa content will be:

C = 47.62 mg/100 mL

After 10 weeks of storage, aa which should be declared on the label is:

47.62 mg/100 mL (0.90)

= **42.86 mg/100 mL**

Graphical solution

- Coordinates of first point: ………….
- Coordinates of second point: …………

- Coordinates of first point: **(0, 60)**

- Coordinates of second point: **(30, 30)**

Which graph paper should you use?

- ??????

Which graph paper should you use?

Of course, semi-log!!!!!!!